APPROACHES AND POSSIBLE IMPROVEMENTS IN THE AREA OF MULTIBODY DYNAMICS MODELING

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(NASA-CR-179227) APPROACHES AND POSSIBLE IMPROVEMENTS IN THE AREA OF MULTIBODY DYNAMICS MODELING (Honeywell) 45 p CSCL 01A

N88-14067

Unclas
G3/02 0103654

Prepared for NASA Marshall Space Flight Center Under Contract NAS8-34588 by Honeywell Space and Strategic Avionics Divison, Clearwater, Florida

Date: 30 October 1987

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IN-02

Acknowledgement

Subcontracting support was provided by DYNACS Engineering of Clearwater, Florida and is reflected primarily in Sections 4.4, 5.1 and 5.3. Dr. R. Singh served as principal investigator.

ABSTRACT

This study takes a wide ranging look at issues involved in the dynamic modeling of complex, multibodied orbiting space systems. Capabilities and limitations of two major codes (DISCOS, TREETOPS) are assessed and possible extensions to the CONTOPS software are outlined. In addition recommendations are made concerning the direction future development should take in order to achieve higher fidelity, more computationally efficient multibody software solutions.

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1.0 INTRODUCTION

Planned space systems (large space antennas, Space-Based-Radar, directed energy weapons) may be called upon to provide a precision in maneuverability and in pointing which is orders of magnitude beyond that possible to date.[1] While the performance requirements are becoming more stringent, the spacecraft themselves are becoming more complex, larger and more flexible and consequently are more difficult to model.

As well, spacecraft configurations are characteristically quite diverse from one another. It is for this reason that "generic" multibody codes have been developed [2,3] to model dynamic behavior and for use in control design. Reference 4 provides a good general overview, from a controls viewpoint, of the difficulties involved in the analysis of large flexible space structures (LFSS). The emphasis here, however, is on fidelity of the multibody modeling process and on computational efficiency. The spirit of the investigation is to initiate a fresh look at the major codes and to not be dedicated to any one methodology a priori.

The open literature is reviewed to establish the state-of-the-art in formulation of multibody dynamic equations of motion and in multibody solution methodologies. Fundamental issues relate to the system topology are examined. Pros and cons of two existing codes (DISCOS, TREETOPS) are compared and the suitability of present structural modeling are looked at. Effects of other influences such as fluid motions, environmental loads and implications of maneuvering are also discussed briefly. As a consequence of the literature review, several useful extensions to CONTOPS are evident ("free-free" flex modes, deployment, foreshortening) and an outline is provided for their implementation. To complete the study, some general conclusions are reached and suggestions are made on what might be done to generate more efficient, higher fidelity models in the future.

2.0 MULTIBODY DYNAMIC SIMULATION

2.1 Role of Simulation, Analysis Tools

Closed form analytic solutions for dynamic response exist only for the simplest of systems: mass particles, translating finite masses, rotations of symmetric (or at least partly symmetric) rigid bodies, etc. As system complexity grows due to number of bodies, types of bodies (fluid, rigid, flex,...), geometric irregularity, range of motions (linear, nonlinear), the only recourse one has is a numerical solution. The specific list of assumptions grows longer as one strives for an evermore generic representation and can seem endless with the growth in demand for higher accuracies and the ensuing need to more fully capture all interaction effects.

Ultimately situations can arise for which it is not possible to generate simulations capable of providing the desired accuracies. Nevertheless the analytic exercise remains an important one. Best-available-engineering models can be constructed and used to gain an understanding of important aspects of the problem. The modeling process itself develops insight into much of the basic physics involved and thus, in addition, is valuable training for the sorting out and interpreting of any measured data. From the viewpoint of controls design, this expertise forms an important building block in the pursuit of a workable control methodology.

2.2 Nature of the Multibody Problem

A multibody system is made up of any collection of interconnected bodies. In general, these bodies will have finite dimensions and will have mass distributed throughout the spatial domain. The motion dynamics tends to become quite complex because of kinematic/load coupling introduced via the interconnections. Major analytic efforts to date have been concentrated on determining a systematic manner for computing and keeping track of the forces of inertia associated with the accelerating masses. Comparatively, much less work has gone into establishing consistent phenomenological force models (elastic strain energy, dissipative mechanisms, etc.) and representations of environmental loads as applied to multibody configurations.

2.3 Derivation of Equations

All derivation procedures rely on some combination of D'Alembert, Newton, Euler, Hamilton or Lagrange principles for establishing conditions for dynamic equilibrium. It is taken as prerequisite that the individual procedures result in equations which are equivalent if not identical in mathematical form. Consequently this topic is not dealt with further in this study. Discussion and evaluations of derivation techniques can be found, for example, in References 5-14.

2.4 Status of Multibody Code Development

Earlier efforts in development of multibody formalisms and computer codes have already been reviewed or commented on in References 5-7, 15, 16. Reference 17 reviews developments from a user perspective. Overall assessments by Jerkovsky (1978) [15] and by Covington, et. al. (1984)[16] are summarized in the Appendix (Tables 1 and 2, respectively). What emerges is evidence of an evolutionary path being followed in the search of ever more realistic and hence more useful formulations. The configurations modeled range from the cluster and chain categories shown in Figures 1(a), 1(b) through to the open and closed tree topologies of Figures 1(c), 1(d). Some important factors addressed include: Degree of rigidity (100% rigid? 1,2, ... DOF vibrations?), type of state variable (displacement? velocity? momenta?), choice of reference coordinates (relative? inertial?), choice of reference points (center of mass? fixed hinge?) and character of the interconnection (point? finite domain? #DOF? constraints?).

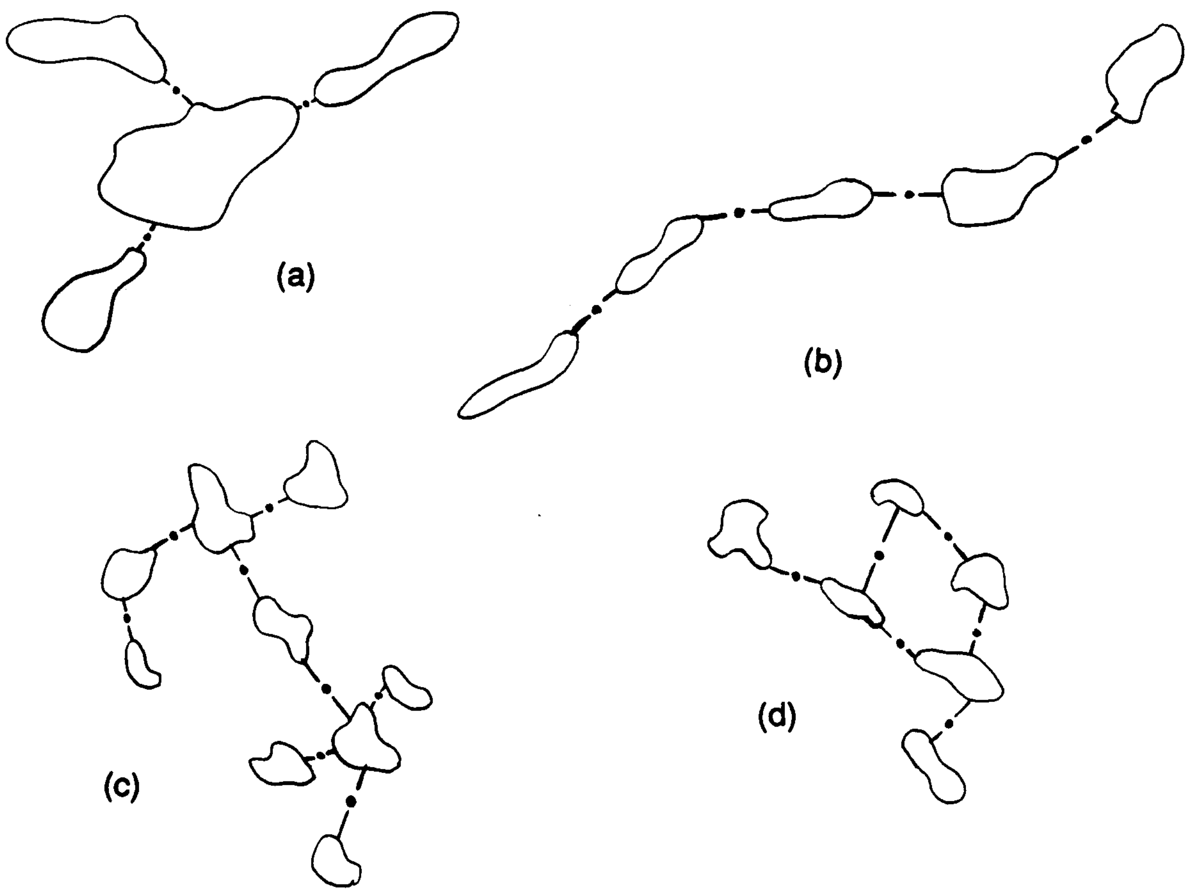

Figure 1 Classes of multibody configurations: (a) Cluster
(b) Chain
(c) Open-loop tree
(d) Closed-loop

The question of which formalism to adopt from the basic laws of mechanics is also an important one, but the exact method does not appear to be a critical consideration since all methodologies end up providing equivalent sets of equations.[5,12] Suffice it to say that for multibody applications it simplifies the derivation process considerably to not include nonworking forces of constraint.

Another key issue in the realm of multibody modeling is centered around methods for dealing with the "system" aspects of the topology. For example, is it always advantageous to work through the barycenter of augmented bodies[18] or is the direct path approach as advanced by Ho[7] more beneficial all round? Hughes[6] states that, while the use of augmented bodies does help uncouple the system translational motion from rotations, the price paid is a more complicated form for the rotational dynamics. Furthermore, Reference 6 points out that although the translations/rotations are coupled in the direct path procedure, the constraint forces are eliminated at the outset by choosing interbody hinge points as reference points and more straightforward interpretations are possible for quantities such as the inertia coefficients.

All in all, a consensus (References 6,8) appears to be building in favor of the direct path method. This, by the way, is consistent with the methodology employed in the TREETOPS algorithm.[3] Still other attempts at organizing the system kinematic, dynamic topology is the "vector-networking" of Reference 19 and the "bond-graph" strategy followed in Reference 20. Such procedures are well suited to efficient implementation on a digital computer.

It has been commonly assumed that to solve the governing multibody equations one must ultimately face the task of numerically inverting a mass matrix. There is some recent development which introduces recursive algebra early in the formulation to avoid this situation (References 21-23).

2.5 Evaluation of Representative Multibody Software

Rather than duplicate any assessments carried out to date, the focus here will be on comparing and contrasting essential characteristics of two of the more contemporary multibody codes resident in the public domain. Both DISCOS[2] and TREETOPS[3] capture the fully coupled, large angle, 6DOF translational rotational dynamics of a contiguous collection of rigid, flexible bodies. Vibrational displacements are handled via an Assumed-Mode type of expansion. In DISCOS, bodies are modeled independently and constrained through the kinematics and Lagrange multipliers, which also allows for loop closure. The net result is a fairly high order system which can be computationally sensitive if there are a large number of constraint forces. Reference 24 points out a specific problem of the closed loop formulation of DISCOS is the requirement that at least one member in the loop be modeled as elastic. TREETOPS makes use of a direct path methodology, thus minimizing presence of constraint forces and, in addition, the order of the system is adjusted according to the number of degrees of freedom specified by the user. To encompass closed loop topologies, the TREETOPS has been modified to include specific classes of equality (e.g. prescribed velocities) and inequality (e.g. hardstops) constraints. This version is referred to as CONTOPS[25] and makes use of Singular Value Decompostion[26] to further minimize system degrees of freedom in the presence of constraints.

There are other differences between these two codes. DISCOS allows for local body spin or locally stored momentum which TREETOPS does not. In DISCOS, one of the bodies can be made to follow a prescribed orbital trajectory whereas it is not clear that TREETOPS has this capacity. Neither does TREETOPS embody the aerodynamic, gravitational or thermal environmental loading, although this development is understood to be underway. Also, in DISCOS, the reference points are taken to be at the center of mass or at "hard" points, whereas in TREETOPS only the inner hinge point need be fixed. Common to both algorithms is the lack of provision for component mass flow (which would be needed in study of tether problems) or provision for coupling of axial load effects with transverse vibrations (see Kane, et. al.[27] or Lips[28]) and, in both cases, body interconnections are assumed to be point-like. More recently some effort has been made to modify DISCOS so that it might include foreshortening.[29,30]

A significant extension to this class of multibody model is work of Reference 31 which allows for relative translation along not a rigid, straight path, but rather along a flexible, curved path.

This discussion shows how far multibody simulations have progressed. On the European scene, the MEDYNA[32] software is also of this calibre. As well, a fairly sophisticated but computationally intensive code LATDYN has been developed by NASA Langley[33] to deal specifically with the case of deployment, unfolding of the joint-dominated structures.

2.6 Symbolic Manipulation, Parallel Processing

Generating equations governing multibody dynamics and the development of the associated computer software tends to be labor intensive and error prone. This situation has spawned a new approach in which the computer is harnessed to do both tasks using symbolic manipulation. In the United States, for example, SD/EXACT[34] is a symbolic code dedicated to multibody dynamics whereas in Europe NEWEUL[35] and MESA VERDE[36] have been applied to multibody spacecraft dynamics.

Another attempt aimed at improving computational efficiencies of a multibody simulation is the processing of dynamic events in parallel.[37]

3.0 ADEQUACY OF STRUCTURAL MODELS

The question of exactly how to best represent the structural flexibility of an individual body is indeed a very large subject in its own right. It is undergoing constant and considerable revision and we will not attempt here to deal with all aspects in detail. Rather the intent is to leave the reader with a sense of what some of the main issues are at present and to indicate current approaches to solutions.

3.1 Basic Mathematical Descriptions for Flexibility

The most common manner of representing flexibility is to express the displacement field as superposition of an infinite number of assumed-modes. These modes are normally derivable from an eigenvalue description of the mass, elastic properties and amounts to a standing wave model. The approach ultimately leads to discrete equations which can be truncated, thus reducing system order and rendering a transient dynamic analysis tractable. Truncation criteria used to be a matter of selecting the first few lower frequency, higher energy modes (this also eliminates high frequency modes which may or may not be desirable). Currently, however, the literature contains more sophisticated approaches which take into account, for example, the contributions of a given mode to rigid/flex coupling (References 38-41). These modal cost assignment techniques have been used to arrive at reduced models for entire systems (see References 42-44). This approach can be particularly significant to a control designer faced with a high order finite element system model.

The question of exactly which modal set to adopt is not clear in a multibodied setting because of the dynamic coupling and because of the complicated and not always well defined boundary condition. Ideally, the modes chosen would be the eigenfunctions, but they are usually not easily arrived at so that simplified sets of dynamic situations (e.g. constant spin) are imposed as reviewed in Section 1.2 of Reference 45. Perhaps it is not essential to work with the closest approximation to the eigenfunctions. Meirovitch, et. al.[46] conclude that for any linear gyroscopic system it is sufficient to use any set of admissible functions provided the set is complete. For a function to be considered admissible, it must satisfy geometric boundary conditions and be differentiable to order P for a system of order 2P.

Finite Element Models (FEM) (e.g. NASTRAN) are commonly used to generate modal data for cases having nonuniform geometry and nonuniform mass, stiffness distributions. The way in which this data is used in a multibody code such as DISCOS has been referred to as inconsistent according to References 47,48. Other subtleties embodied in FEM are discussed in Reference 49. When dealing with very large structures,the order of a FEM approach can become quite large. References 50-52 discuss the feasibility of adopting equivalent lower order configuration models for those situations when a structure is built up from a series of repetitive subelements. Truncated discrete standing wave solution is not the only modeling option. No truncation is necessary if one works in the frequency domain as suggested by Poelart 45, 54, but this is not possible for a nonlinear system. A travelling wave approach is discussed in References 55, 56. Alternately, given the uncertainty surrounding structural characteristics, are stochastic models (as described in Reference 57) really more appropriate?

3.2 Specialized Considerations

3.2.1 Energy Dissipation

Historically there has been, and continues to be, considerable seat-of-the-pants judgment exercised when arriving at a value to be assigned the damping factor of a given mode. This parameter is vital to response stability and settling times (particularly at high frequency) and cannot be left to chance if high accuracy control is mandatory. Fortunately research of a fundamental nature is underway as outlined in Reference 58. The objective is to measure damping of a small material piece of a structure, relate it, via a viscoelastic modeling process and time domain realization, to the full sized structure. Evidence of the continuing effort to rethink and experimentally quantify damping parameters is contained in the work of References 51,59-62, for example. Some success has been had in measuring damping in-orbit as cited in References 63,64 for systems having solar arrays (Communications Technology Satellite and Orbiter-Based-SAFE experiment) but for some modes the damping is much different than the predicted levels.

3.2.2 Mass Flow Deployment

By now we have gained some appreciation as to the complexities associated with configuration geometries and vehicle elasticity. Analysis becomes even more involved during extension or retraction of rigid and/or flexible components. This type of deployment introduces a variable mass distribution (and hence variable moments of inertia) together with additional relative velocities and accelerations, all factors that affect transient dynamic behavior. Perhaps because of its inherent complexity the problem has received relatively little attention. In general, available investigations tend to be more limited in scope than those dealing with nondeploying flexible structures as reviewed in Section 1.2.5 of Reference 21. One of the more significant studies (Reference 65) considers deployment of a single beam. Reference 66 investigates the characteristics of a spinning, deploying beam in-orbit under the influence of the gravity gradient. One of the more general multibody software developments to consider individual body mass flow is the cluster configuration investigated by Lips (Reference 21) where it is shown that deployment-related Coriolis loads can be significant. More recently, Keat, et. al (References 16,67) are developing a multibody code applicable to spacecraft with unfolding and with mass flow and, as mentioned, Housner, et. al. (References 33,68) have developed such a code for joint-dominant structures.

Later in the report a look is taken at what the implications are of extending a code such as TREETOPS/CONTOPS to include deployment.

3.2.3 Reduced Dimension Models and the Foreshortening Effect

Reference 27 asserts that a fundamental flaw exists in multibody codes such as DISCOS and TREETOPS (which is not the case for the clustered appendage configuration of Reference 21 or the model in Reference 66). Concern appears to be centered around the absence of the flex-foreshortening-induced dynamic coupling between the axial loading and transverse vibration. Concern over this issue was raised earlier by Lips (Reference 28). In all cases the problem is described for a one-dimensional beam-like structure - a simplified model sometimes used to represent space trusses or astromast types of booms.

Here, the literature in this area is reviewed and an attempt is made to determine the relative significance of this type of coupling. An indication of the main steps needed to introduce such an effect into the TREETOPS formulation follows later in the report.

As shown in Figure 2(a), axial foreshortening refers to the translational displacement which a mass element experiences along the axial direction (*x*) due to elastic deformation (*v*) normal to that direction. Differential line element theory can be used to relate the deformed element dimension ds to its projected undeformed length *dx* as seen from Figure 2(b). This concept dates back to at least 1964.[69] Over the years it has been introduced directly as a working displacement, [69,70] indirectly during evaluation of spatial integrals,[71,72] explicitly as an added component of translation,[21,66,73,74] and implicitly as a nonlinear strain displacement effect.[75]

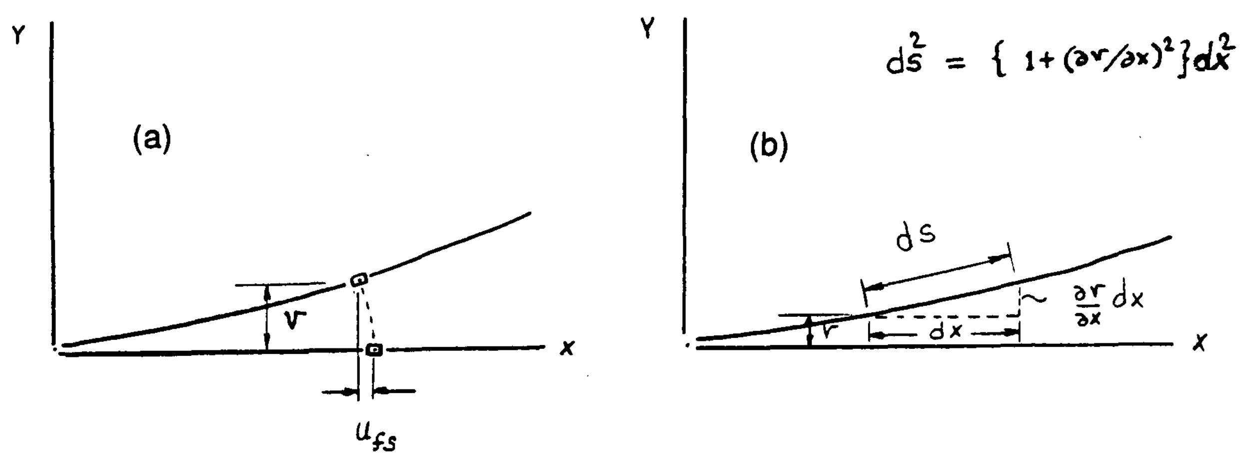

Figure 2 Concept of axial foreshortening showing: a) deformation (v) along a transverse direction together with the constrained translation (u_{fs}) along the axial direction; and (b) length of deformed element (ds) and its shortened length (dx) as projected along the undeformed x axis.

Kane, et. al. (1987)[27] introduce effects of foreshortening explicitly as an added translational velocity. However, in addition to the subtleties associated with foreshortening, one must deal with the terminology and methodology unique to the Kane approach. To the uninitiated this tends to obscure somewhat the handling of foreshortening and the coupling induced through it. The problem is not helped by the absence of any reference to the term foreshortening or to differential line element theory. The handling of this problem is a new application for the Kane methodology but, at least as far as beam foreshortening is concerned, it does not qualify as a "new" theory. Nevertheless, Reference 27 is correct in pointing out that the foreshortening - related effects have not been incorporated into the existing multibody codes.

The question of how significant the coupling terms are is addressed, in part, in Reference 76, where an approximate result is given relating the foreshortening - corrected frequency of a spinning beam with that of a non-spinning beam. The results are plotted in Figure 3 where it is evident that foreshortening-induced effects can be expected to become significant if spin rates are comparable or much higher than that frequency characterizing the non-spinning structure.

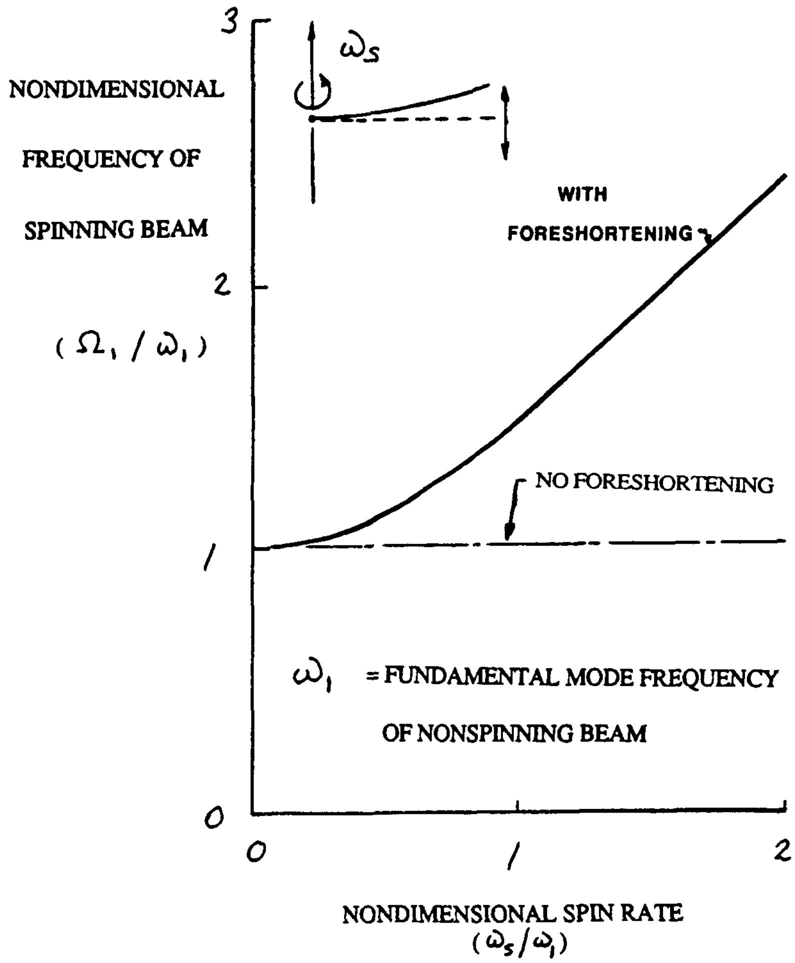

Figure 3 Foreshortening-induced spin stiffening effect on a uniform beam

3.2.4 Joint Idealization

Large flexible frame structures in space will be characterized by large numbers of joints each of which can present the analyst with a host of unknowns. Although idealized as points, joints in fact involve finite physical domains (a factor also in the setup of actuators, sensors). There can be sloppiness in joints, different friction regimes, considerable nonlinear behavior and flexibility. Such features could make the cost of computing the response prohibitive. Only recently has attention been focused on this area. Perhaps the most advanced effort to date is the Large Angle Transient Dynamics (LATDYN) software under development at NASA Langley (References 33,68). Independent work of Reference 77 suggests that the presence of joints may not alter modal frequencies significantly provided the overall structure is not too short. Some work is also being done to identify energy dissipation, taking place at the joint (see References 78-80).

4.0 OTHER FACTORS IMPACTING FIDELITY OF MULTIBODY SIMULATION

4.1 Fluid Interaction

Just as elastic systems make it difficult to quantify response, the presence of fluids can present an even more vexing modeling problem being particularly sensitive to temperature, pressure. With the advent of liquid apogee motors, the liquid/solid ratio can now be as high as 55%, and hence the problem is commanding much more serious attention both in Europe and in the U.S.A., not to mention SDI applications. An overview of the state-of-the-art can be had by reviewing the papers given at the colloquium dedicated to this problem and cited in Reference 81.

A number of presentations concentrated on development of fundamental models applicable to rotating, free-surface, fluid behavior. However, due to the complex nature of the problem only restricted approximate solutions, in the form of one homogeneous eigenmode for example[81a] are forthcoming. As well as analysis, a significant amount of the work is dedicated to experimental testing of ground-based gyro-fluid systems (e.g. Vanyo, Reference 81b). While the test data appears to be reliable, it is not certain that fluid character observed at ground level will be the same in orbit. To address this question, ESA has observed fluid behavior in orbit on the Shuttle SPACELAB 1 mission[81c].

An experimental method which appears to give good agreement (better than 20%) with in orbit measurement, is the ground-based "Drop-Test" [81d]. A dynamically scaled model of the satellite/fluid system is allowed to free-fall. Nutation data is transmitted continuously throughout the duration of the drop.

In addition to generating analytic fluid models and measuring fluid/system behavior, serious efforts have been made to construct overall system models [81a, 81e]. McIntyre of Hughes Aircraft outlines an approach that is analogous to that used when dealing with flexibility effects. Attitude equations are formulated for the spacecraft which couple directly to fluid motions which, in turn, are characterized in terms of discrete coordinates (derived from a truncated Assumed-Mode expansion of the fluid displacement). Required is a set of representative fluid mode shapes which can be arrived at either analytically, experimentally or in combination. Such a method, when generalized, can also model fluid/flexibility interactions (platform asymmetry, etc.). Another effect which can become large as fluid mass ratios increase, and which can effect the system dynamics, is a shift in center of mass location.

Attempts to come to grips with this complex phenomenon are continuing at both the experimental and theoretical levels as evident from References 82, 83 which deal with coning cylindrical fluids and with clusters of gaseous bubbles contained within an oscillating column of liquid, respectively.

4.2 Environmental Loads

An analyst may go to great lengths to generate a high fidelity model of the inertia-related forces of a vibrating gyroscopic system only to find that external forces applied by the environment are known only very approximately in some instances. References 84, 85 are among the more recent efforts aimed at improving atmospheric density modeling, whereas Reference 86 is one of the latest attempts to come to grips with solar-radiation-pressure-induced loads. To develop the best possible environmental modeling, one should also consult with the comprehensive bibliography on the subject contained in Reference 87. To date, generic multibody environmental models are not available since the description used is linked closely to the kinematic gendre adopted within a given formalism.

4.3 Requirement for Controlled Maneuvers

It is expected that upcoming space systems will be required to exhibit a higher degree of maneuverability. For example, the entire spacecraft (or part of it) may have to be re-oriented rapidly or it may have to be spun up or spun down in the shortest time while keeping vibrations to a minimum. Reference 88 discusses open loop strategies for vibration control of an Orbiter-based experiment having long (up to 150m), highly flexible booms (Note that in this case flex foreshortening is included at the outset.). References 89-92 also point out strategies for slewing flexible systems so as to minimize the post-maneuver oscillation. References 93-94, on the other hand, deal with optimal attitude control for large angle maneuvering of rigid vehicles. None of these studies yet rely on any of the major multibody codes.

4.4 Numerical Considerations in the Solution of Mixed Differential-Algebraic Equations

Several methods have been developed to solve sets of mixed Differential-Algebraic Equations, hereafter abbreviated DAE. Gear [95,96] has shown that his stiff numerical integration algorithm can be used to solve certain kinds of DAE. Certain other DAE can be converted into equivalent forms that are solvable by Gear's method. There exist, however, systems of DAE that cannot be solved by this method. In References 97-100, Gear's idea has been successfully employed for analysis and optimal design of mechanical systems and electrical networks. Mathematical models of dynamic mechanical systems, using Gear's method of solving DAE, make the equations of motion stiff, thus involving considerable computational cost and time. Furthermore, this formulation requires several problem dependent derivative expressions that make automatic computer generation of the equations of motion difficult [98,99] In fact, Reference 101 shows that DAE systems are not ordinary differential equations at all.

A second approach, due to Baumgarte, [102] uses ideas from feedback control theory to construct a modifed differential equation that implicitly accounts for the constraint equations. A difficulty, however, is the selection of factors in this method that greatly influence accuracy of solutions. Proper values of these factors are not known and experts are in disagreement over suitable values. Furthermore, there is no way to impose positive error control on constraint violations for this method. Work is in progress to develop methods to determine these factors, based on constraint violations that ensure that the constraint violations at the next time step are minimized. The major drawback of these methods is that all n generalized coordinates must be integrated, whereas only n-m of them are independent. Furthermore, m is often large and n-m is small. Thus, it would be attractive to integrate for only n-m independent generalized coordinates. Solution for the remaining m dependent generalized coordinates from constraint equations can then ensure positive error control on constraint violations. The n-m generalized coordinates that are computed by integrating the equations of motion are called independent generalized coordinates and the remaining generalized coordinates are called dependent generalized coordinates.

Independent generalized coordinates can be picked from the set of n generalized coordinates that describe the system configuration. Wehage and Haug [103,104] developed an algorithm to pick an acceptable choice of independent coordinates, using LU decomposition of the constraint Jacobian matrix to identify independent and dependent coordinates. This algorithm performs satisfactorily, but occasionally leads to poorly conditioned matrices. It is then required that LU factorization be repeated and a new set of independent generalized coordinates be selected. The result is an increase in computing time and greater propagation of integration error than desired. In Reference [26] a method based on singular value decomposition is used to solve the equations of motion of dynamic systems. This method, however, does not provide physical insight into the solution process and is not convenient for automatic generation of the equations of motion for general mechanical systems in a general purpose computer program.

A broader class of vectors p of independent generalized coordinates can be

$$\underset{(n-m)\times 1}{p} = \underset{(n-m)\times n}{VI} \times \underset{n\times 1}{q}$$

where VI is a transformation matrix. In the algorithm of References [103,104], matrix VI is a Boolean matrix. Therefore, only individual generalized coordinates are chosen as independent. Use of a Boolean transformation matrix to pick independent generalized coordinates from q is rather restrictive. It is desirable to enlarge the set of possible independent generalized coordinates by allowing matrix VI to be a more general, possibly nonsparse matrix. Such a choice of VI can allow linear combinations of physical generalized coordinates to be selected as independent. If the rows of VI are mutually orthogonal, the resulting linear combinations of individual generalized coordinates will be mutually independent. Specific forms of such a transformation can be chosen, based on properties of the DAE under consideration and desired properties of the transformed coordinates.

5.0 EXTENSIONS TO THE CONTOPS ALGORITHM

This section discusses revisions that, if fully implemented, could increase both range of applicability and efficiency of the CONTOPS code.

5.1 Generalization of the CONTOPS Hinge

Currently, there are two restrictions imposed in CONTOPS.

(i) The reference point h_k (Figure 4) of the reference body is its mass center,

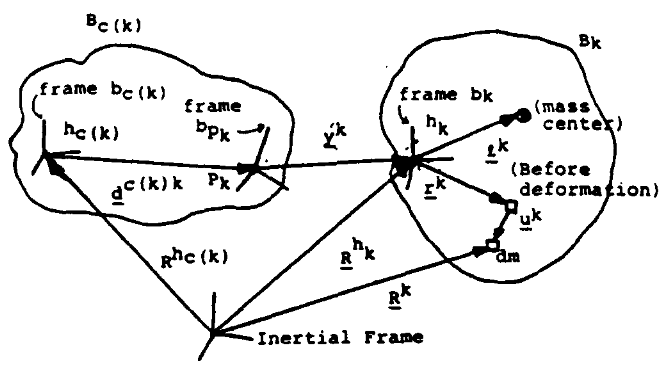

Figure 4 Geometry of deformable body *j*.

(ii) Modal displacements of reference point h_k and modal rotations of frame b_k (Figure 4) are assumed to be zero for all bodies. For the reference body the second restriction is naturally satisfied if the free-free mode shapes are utilized in defining its elastic deformations. For all other bodies in the system the second restriction is naturally satisfied by the fixed-free modes of the individual body (fixed at point h_k in Figure 4). CONTOPS results may not yield desired accuracy if the modeshapes are obtained with boundary conditions other than those described above.

Because the mass center is taken as the reference point for the reference body two vector quantities $\underline{h}^{ki}$ and $\underline{l}^k$ defined by equations 18,22 of Reference 3 (see attached copy) have been assumed null (for free-free modes of the reference body this is a valid assumption). With this exception the form of the equations is same for all bodies in the system. The first restriction may be removed by inserting non-null vectors for $\underline{h}^{ki}$ and $\underline{l}^k$ for the reference body B_k.

Removal of the second restriction is not an easy matter. It is pointed out in Reference 3 that the elastic deformation, $\underline{u}^k$, is measured with respect to frame b_k. Furthermore, it is required that the elastic deformations remain small. Thus the frame b_k and point h_k cannot be arbitrarily defined. In what follows the formulation of Reference 3 will be modified which will allow a CONTOPS user to:

(i) define reference point and reference frame for individual bodies independent of the definition of hinge.

(ii) generalize the existing hinge definition to accommodate a wider class of boundary conditions for the mode descriptions of the substructures.

Figure 5 describes the kinematics of body B_k and the kth hinge. Frames p_k and q_k and vector $\underline{y}^k$ fully define the kth hinge. The reference point c_k for B_k is arbitrarily chosen.

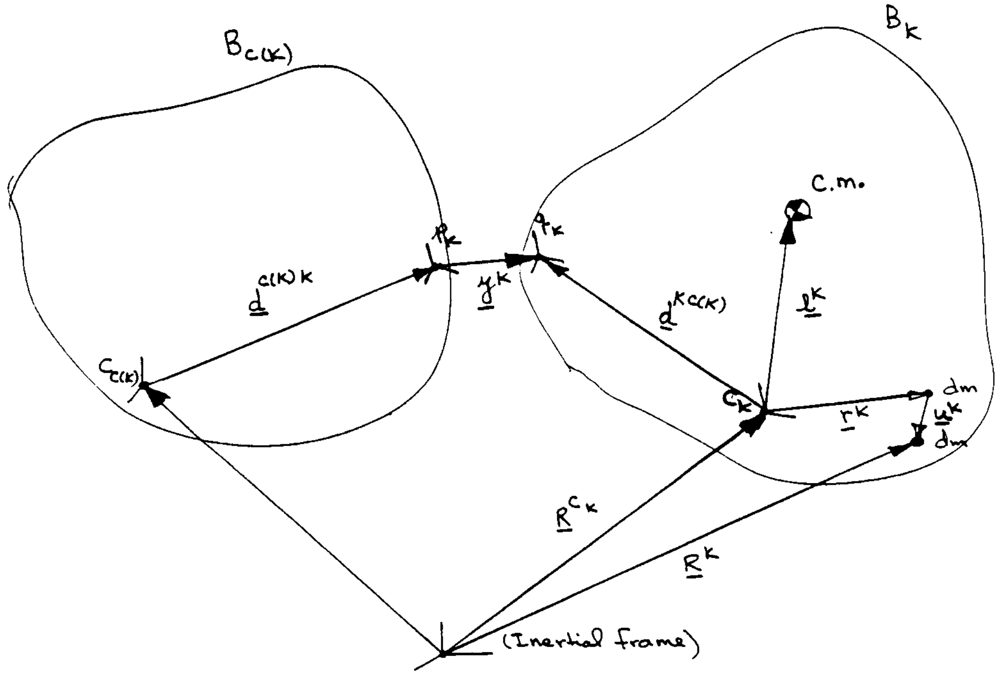

Figure 5 k^{th} hinge and k^{th} body kinematics

Following the notations of Reference 3 from Figure 5.

$$\underline{R}^{K} = \underline{R}^{C_K} + \underline{r}^{K} + \underline{u}^{K} \tag{1}$$

$$\underline{R}^{C_K} = \underline{R}^{C_{C(K)}} + \underline{d}^{C(K)K} + \underline{y}^{K} - \underline{d}^{KC(K)} \tag{2}$$

$$\underline{d}^{C(K)K} = \underline{D}^{C(K)K} + \sum_{j=1}^{N_{C(K)}} \underline{\Phi}_j^{C(K)}(P_K)\, \eta_j^{C(K)}$$

$$\underline{\omega}^{K} = \underline{\omega}^{C(K)} + \underline{\omega}^{P_K C(K)} + \underline{\omega}^{q_K P_K} + \underline{\omega}^{K q_K}$$

or,

$$\underline{\omega}^{K} = \underline{\omega}^{C(K)} + \sum_{j=1}^{N_{C(K)}} \underline{\Phi}_j'^{\,C(K)}(P_K)\, \dot{\eta}^{C(K)} + \sum_{j=1}^{NR_K} \dot{\theta}_j^{K}\, \underline{\ell}_j^{K} - \sum_{j=1}^{N_K} \underline{\Phi}_j'^{\,K}(q_K)\, \dot{\eta}_j^{K} \tag{3}$$

the inertial derivative of equation (1) yields

$$\dot{\underline{R}}^{K} = \dot{\underline{R}}^{C_K} + \underline{\omega}^{K} \times (\underline{r}^{K} + \underline{u}^{K}) + \mathring{\underline{u}}^{K} \tag{4}$$

$$\begin{aligned}\dot{\underline{R}}^{C_K} = {} & \dot{\underline{R}}^{C_{C(K)}} + \underline{\omega}^{C(K)} \times \underline{d}^{C(K)K} \\ & + \sum_{j=1}^{N_{C(K)}} \underline{\Phi}_j^{C(K)}(P_K)\, \dot{\eta}_j^{C(K)} + \mathring{\underline{y}}^{K} \\ & + \Big[\underline{\omega}^{C(K)} + \sum_{j=1}^{N_{C(K)}} \underline{\Phi}_j'^{\,C(K)}(P_K)\, \dot{\eta}_j^{C(K)}\Big] \times \underline{y}^{K} \\ & - \underline{\omega}^{K} \times \underline{d}^{KC(K)} - \sum_{j=1}^{N_K} \underline{\Phi}_j^{K}(q_K)\, \dot{\eta}_j^{K}\end{aligned} \tag{5}$$

Equations 3,4,5 are sufficient for identifying the coefficients of generalized speed (equations 26,27,28 of Reference 3).

Figure 6 here would replace Figure 4 of Reference 3.

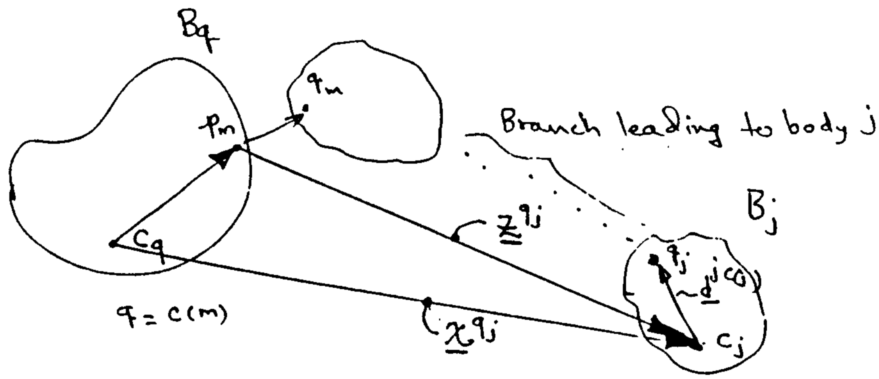

Figure 6 Vectors defining coefficients of generalized speed.

Replace equation 26 of Reference 3 by

$$
\begin{aligned}
\underline{V}_p^{c_j} &= \underline{l}^q_m \times \underline{x}^{qj} && j \in E(q) \\
&= \underline{0} && j \notin E(q) \\
\underline{\omega}^j_p &= \underline{l}^q_m && j \in P(j) \\
&= \underline{0} && \\
\underline{V}_p^{\eta_j} &= \underline{0} &&
\end{aligned}
\qquad (6)
$$

Replace equation 27 of Reference 3 by

$$
\begin{aligned}
\underline{V}_p^{c_j} &= \underline{g}_n^{q} \qquad\qquad j \in P(q) \\
&= \underline{0} \\
\underline{\omega}_p^{j} &= \underline{0} \\
\underline{V}_p^{\eta_j} &= \underline{0}
\end{aligned}
\qquad (7)
$$

Replace equation 28 of Reference 3 by

$$
\begin{aligned}
\underline{V}_p^{c_j} &= \underline{\phi}_\ell^{q}(P_m) + \underline{\phi}_\ell'^{q}(P_m) \times \underline{\underline{z}}^{q_j} \qquad j \in E(q) \\
&= \underline{0} \\
\underline{\omega}_p^{j} &= \underline{\phi}_\ell'^{q}(P_m) \qquad j \in E(q) \\
&= \underline{0} \\
\underline{V}_p^{\eta_j} &= \underline{\phi}_\ell^{q}(\underline{r}^{q}) - \underline{\phi}_\ell^{q}(\underline{d}^{qc(q)}) + \underline{\phi}_\ell'^{q} \times [\underline{d}^{qc(q)}] \qquad j = q \\
&= \underline{0} \qquad j \neq q
\end{aligned}
\qquad (8)
$$

Equations 14,15 of Reference 3 remain intact except superscript h_k is replaced by c_k.

5.2 Implications of Deployment and Flex Foreshortening

As currently formulated, the CONTOPS multibody algorithm does not allow for mass deployment or retraction of individual members nor does it take into account the geometric foreshortening that accompanies flex deformation. In light of this, the existing recursive equations are reviewed once again. This report discusses some basic considerations involved in modifying CONTOPS to incorporate such effects.

5.2.1 Existing CONTOPS Kinematics (Reference 3)

The kth body (B_k), shown earlier in Figure 4, is in the deformed state. Elemental mass dm in B_k is located relative to an inertial reference by the vector (using Equation 1):

$$\underline{R}^k = \underline{R}^{h_k} + \underline{r}^k + \underline{u}^k \tag{9}$$

$\underline{R}^{h_k}$ locates the hinge point relative to inertial

$\underline{r}^k$ is fixed in B^k

$\underline{u}^k$ represents elastic deformation

As already implied, in order to arrive at a solution in terms of a set of ordinary differential equations (and to pave the way for truncation to an approximate

$$\underline{u}^k = \sum_{l} \underline{\phi}_l^k (\underline{r}^k) \, \eta_l^k(t) \tag{10}$$

Overall inertial velocity of dm then is given by

$$\dot{\underline{R}}^k = \dot{\underline{R}}^{h_k} + \underline{\omega}^k \times (\underline{r}^k + \underline{u}^k) + \mathring{\underline{u}}^k \tag{11}$$

where

$(\dot{\ })$ = inertial time derivative.

$(\mathring{\ })$ = differentiation in Bk - fixed frame.

5.2.2 Modifications due to Changing Mass

It is common in satellite design to store elements in as compact a manner as possible prior to flights and then to unfurl the device once in orbit, allowing it to extend to full size. Such Storable Extendable Members (STEM) complicate dynamic studies by introducing a time variable mass feature into the analysis which becomes even more complex when components are flexible. To allow for this in the CONTOPS type of multibody code implies the following (Reference 21,28):

1. A structural mass element can be viewed asbeing convected by a prescribed rate of deployment ($\underline{V}_D$) in a manner analogous to a fluid continuum model. Then any local time derivative must be augmented so that

$$\left.\frac{d(\ )}{d_t}\right|_{\substack{\text{kth}\\ \text{body}}} = \left.\frac{\partial(\ )}{\partial t}\right|_{\substack{\text{no spin}\\ \text{no deployment}}} + (\underline{V}_D \cdot \nabla)\ (\ ) \qquad (12)$$

2. When differentiating the discrete expansion given by Equation 10, it must be recognized that $\mathring{u}^k$ is not zero but now is implicitly time dependent as a result of the convective motions discussed above.

3. Mass must be conserved at all times. This leads to another equation which governs the motion - that is, the equation of continuity.

4. Mass flow changes system properties such as moment of inertia, momentum.

Implementation of these considerations is simplified if, rather than a general continuum, one analyzes a one-dimensional beam-like substructure, for example. If it is uniform and if only linear vibrations are involved, then deployment velocity is identically equal to rate of change in length as measured along the undeformed axial direction.

5.2.3 Incorporating Flex Foreshortening

To include foreshortening ($\underline{u}_{fs}$) in a CONTOPS type of algorithm, it can be added explicitly to the displacement field [21] so that Equation (9) becomes

$$\underline{R}^k = \underline{R}^{hk} + \underline{r}^k + \underline{u}^k + \underline{u}_{fs}^k \qquad (13)$$

To date $\underline{u}_{fs}$ has been approximated only for beam-like, plate-like structures. If we take,

$$\underline{u}^k = u_1^k \underline{a}_1 + u_2^k \underline{a}_2 + u_3^k \underline{a}_3 \; ; \qquad (14)$$

where unit vector $\underline{a}_1$ lies along the undeformed axial or 'long' direction of the structure, then

$$\underline{u}_{fs}^k \simeq [1/2 \int_0^{x^k} [(u_2^{k'})^2 + (u_3^{k'})^2] \, dx \ldots] \underline{a}_1 \qquad (15)$$
$$+ \text{ higher order terms}$$

where $(\,)' = \partial(\,)/\partial x^k$

When differentiating to arrive at velocity, acceleration it must be kept in mind that the approximated foreshortening displacement is time dependent. Alternatively, since the TREETOPS approach makes use of the partial velocity description in the kinematics, one can adopt the procedure given by Kane et. al [27]. In this case the foreshortening is embedded in the velocity field.

5.3 A Model Reduction Approach for CONTOPS

The CONTOPS formulation is based on the method of substructures. The elastic deformations are expanded in terms of the natural modes of individual flexible substructures. The number and type of substructure modes required to accurately describe the elastic behavior has not always been addressed properly. Unwise choice of mode shapes will result in modeling error producing poor quality solutions. Also, retaining the substructure modes with insignificant contribution to the solution results in unnecessary computational penalty.

In what follows we outline an approach based on the modal cost analysis (MCA) approach, a well proven technique for model reduction. The MCA approach to model reduction is widely used in the control science discipline.

This approach was first proposed by Dr. R.E. Skelton of Purdue University (See Ref. 38, for example). In this approach a system is defined to be comprised of many components. The central idea of the modal reduction method of the MCA is to exploit the precise statement of the optimal control problem in order to predict which system components will make the largest contributions in the total quadratic performance criterion. These components are retained and the balance are discarded. Mathematically, the concepts of this model reduction process are explained as follows. Let a state space description of a time invariant system be given as

$$\dot{x}(t) = Ax(t) + Dw(t)$$

$$y(t) = Cx(t) \tag{16}$$

where the n states are represented by x(t), the k outputs are y(t) and the q process noise are w(t). w(t) is assumed to be zero white noise with unit intensity. It is assumed that the model is controllable, observable and asymptotically stable. It is also assumed that there are k independent outputs, i.e., rank (C) = k and q independent inputs, i.e., rank (D) = q.

Let a quadratic cost function associated with (16) be given as

$$V = \lim_{t \to \infty} E[V(t)], \quad V(t) = \frac{1}{t} \int_0^t y(\tau)^T Q y(\tau) d\tau \tag{17}$$

where E is the expectation operator and Q is a symmetric positive definite weighting matrix.

Now, partitioning the state vector $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$, the component costs V_i associated with each component x_i is defined by

$$V_i = \frac{1}{2} \lim_{t \to \infty} E[(\frac{\partial V(t)}{\partial x_i}) x_i], \quad i = 1, 2, \ldots, n \tag{18}$$

and are calculated according to the following component cost formula

$$V_i = [C^T Q C X]_{ii}, \quad i = 1, 2, \ldots, n \tag{19}$$

where $(.)_{ii}$ denotes the (i,i) element of (.) and where X, the steady state covariance of the states, is defined by

$$X = \lim_{t \to \infty} E[x(t)x(t)^T] \tag{20}$$

satisfies

$$XA^T + AX + DD^T = 0 \tag{21}$$

Clearly, since $V = Tr(C^TQCX)$ the component costs satisfy the cost decomposition property

$$V = \sum_{i=1}^{n} V_i \tag{22}$$

The component costs are ranked in the following manner

$$V_1 \geq V_2 \geq \ldots \geq V_n \tag{23}$$

and the state vector is reordered correspondingly. The most critical component of the system is x_1 having component cost V_1 and the least critical component of the system is x_n having component cost V_n.

Based on this component cost ordering, components having lower component cost are truncated.

The MCA approach outlined above may be applied to CONTOPS for selecting the appropriate modes from a given set of modes for individual substructures.

For CONTOPS application the state vector x(t) may be arranged as follows

$$x(t) = [\ldots, \underset{\alpha\text{th, entry}}{\eta_1^k(t)}, \underset{(\alpha+1)\text{th entry}}{\dot{\eta}_1^k(t)}, \ldots] \tag{24}$$

where $\eta_1^k(t)$ is the 1th modal coordinate of the kth substructure (body B_k). The component cost corresponding to the 1th mode of B_k is

$$V(\eta_1^k, \dot{\eta}_1^k) = [C^TQCX]_{\alpha,\alpha} + [C^TQCX]_{(\alpha+1),(\alpha+1)} \tag{25}$$

where X satisfies equation (21). In the technical discussion above the component cost computation is based on a scalar, y^TQy. This concept may be expanded to account for multiple objective problem. This can be accomplished by computing the component costs for each of

$$y_i^2, \quad i = 1, \ldots, k \tag{26}$$

The component costs for each $y^2{}_1$ should be ranked and the resulting dominant modes based on each $y^2{}_1$ should be retained. The MCA algorithm can easily be integrated into CONTOPS.

6.0 DISCUSSION AND CONCLUSIONS

A wide range of issues related to the modeling of the dynamics of multi-bodied systems has been reviewed In this report. While it has not been possible to evaluate all areas fully, it is hoped that bringing so many of these areas together in one report has served to demonstrate the scope of the problem.

Casting a multibody dynamic system into an understandable, manageable framework is a challenging task because of the large numbers of bodies involved and because of the high degree of coupling between them. Nevertheless, the exact procedure adopted for derivation of the equation set does not appear to be a concern since all aproaches will, if competent, produce equivalent mathematical representations. For instance, equations can be put into a 1^{st} order velocity format using either a Newton-Euler approach or using the procedure favoured by Kane and Levinson [10]. The degree of effort and preference for a method will, in large measure, always be dictated by the analysts experience. Choice of coordinates and frames of reference may leave more room for debate for specialized applications but for a very general model this is not so much the case (6DOF is 6DOF!). What does stand out as a predominant issue facing multibody experts is the choice of a configuration topology and of methods for dealing with it. In this regard it would appear that the direct path method (which is conducive to use of recursive interbody equations and makes it straight forward to eliminate nonworking constrained forces) is gaining acceptance over the barycentric approach. One of the more versatile topologies is the tree analogy with open or closed branch loops. The bookkeeping aspects of the kinematics might also be helped through vector-networking or through bond-graph models.

Deriving equations need not be done manually. According to the literature, a digital computer can be used as a symbol manipulator to generate equations based on whatever rules of mechanics one chooses and, if desired, to translate these equations directly into computer code (e.g. FORTRAN) for solution. This approach can be expected to yield more and more efficient computational algorithms and to be less error prone as a problem grows in complexity. A decided advantage here is the capacity to generate equations for the specific problem at hand rather than to generate one algorithm which attempts to cover all possible situations. Numerical efficiency can be further enhanced by carrying out computations in parallel whenever possible rather than in a serial manner as has been more the custom to date. This strategy would appear to complement the order n recursive modeling which avoids the need to directly invert a system order mass matrix and hence should augment further the speed at which the simulation can be executed.

Obviously the order of a model has a strong influence on computational performance. As alluded to already, the order n approach combined with symbolic manipulation can provide a much reduced model for the system. Two other factors are important in determining system size - flexibility and constraints. Equivalent continuum modeling together with modal cost analysis or simple truncation can be used to minimize the flex contribution. Singular Value Decomposition can be applied to minimize degrees of freedom for constrained systems. Model reduction can then be applied to the resulting system. This combined reduction may yield the most meaningful and efficient definition of the system dynamics depending on the computational penalty paid to achieve each stage of the reduction.

The algorithm adopted to carry out a numerical integration also plays an important role in how accurately and how quickly a particular piece of software can respond. While this has not been addressed in great detail here, it is implicit throughout. Evaluations should be made to etermine relative merits of Runge-Kutta versus Predictor-Corrector methods. The Gear approach can be vital to the success of a 'stiff' multibody situation in which characteritic times associated with the motions are highly divergent (this can occur easily for cases involving extension of a substructure from a very small length say out to a very large value).

It has been pointed out that it is not the fundamental principles of mechanics which stand in the way of achieving a high level of modeling fidelity. From this review it is concluded that, in a very general sense, what does restrict the modeling, simulation process is:

(i) level of geometric detail used to describe a finite dimensioned physical system;

(II) adequacy of the phenomenological models. {This ranges from uncertainties present in the basic structural model (including, in particular, energy dissipation) through to questions related to fluid behaviour and including effectiveness in accurately capturing the effect of environmental influences.}

(iii) efficiency of the solution algorithm; and

(iv) numerical techniques (integrators, parallel vs serial processing, ...)

It is clear from this investigation that the state-of-the-art has matured to the point where a contemporary generic multibody code can be assumed to include the following features:

- arbitrary number of point-connected bodies
- allows large angle relative interbody rotations (3DOF)
- allows large relative interbody translations (3DOF)
- allows for linear vibrations (3DOF translation)
- some form of loop closure exists

While DISCOS and TREETOPS/CONTOPS fall into this category, they do not routinely account for deployment, for example, TREETOPS is superior from a numerical viewpoint in that, as much as possible, constraint forces are eliminated. The DISCOS Lagrange multiplier feedback approach, which includes a large number of constraint forces, can be expected to provide for a much more rapid build up of the numerical error. Further, the size of system matrices in TREETOPS is not fixed by the number of hinge points and bodies (as in the case of DISCOS) but is determined only by the number of degrees of freedom input to the program for a specific application. DISCOS, on the other hand, has the advantage of permitting prescribed motion (including orbital) and body spin (or locally stored momentum). This demonstrates that the search for a completely unrestricted 'generic' code is as allusive as ever.

One area neglected in both of the codes discussed above, and in other known general purpose software (except for the development in Ref. 21), is the effect of nonlinearity in the strain displacement relationships of a flexible body which gives rise to the foreshortening in the displacement field as discussed in Section 3.2.3. The significance of this effect is summarized qualitatively here by paraphrasing Ref. 28.: "The generality of a formulation is compromised if one is interested in simulating response for space systems which are large physically, which undergo large attitude excursions (in terms of both displacement and rate) and which have vibrating appendages that engage in significant local translations if the effect of interaction of axial loading with the transverse vibrations is not accounted for. It is well known, for example, that centrifugal loads associated with rotation of a beam-like appendage affect terms which are linear in the vibration variables." In this report, it has been shown quantitatively that the spin-stiffening becomes important as spin rates approach or exceed the fundamental frequency characterizing the nonspinning structure. Note that this is the same effect focused on most recently by Kane et al.[27] Their claim that it has not been included in the major multibody codes is a valid one, however, the phenomenon itself is not new and has already been dealt with extensively in the general literature. Fortunately, existing codes can be modified to accommodate this effect without completely rederiving equations or completely reprogramming the existing codes.

7.0 RECOMMENDATIONS

This investigation has given insight into the level of complexity and the level of uncertainty surrounding multibody simulation dynamics. A series of recommendations is discussed here aimed at providing for improved dynamic models or improved computational efficiency of either present-day or of next-generation software.

The foreshortening effect forms a valid part of the kinematics and should be a part of existing multibody codes and should be allowed for in any future modeling developments. It can require considerable additional calculations even though it is not significant in every application. For this reason, it should be retained in the form of an option. Note that CONTOPS can likely be amended more easily through the generalized speed than can DISCOS which relies on kinetic energy.

The fact that the major codes are amenable to the type of revision alluded to above is encouraging, since it implies that significant enhancements can be achieved without generating an entirely new program. Every effort, therefore, should be made to capitalize fully on existing codes before contemplating construction of any new package. On the other hand, any single piece of software should not be relied on to provide the best solution for all applications. Rather it is more realistic to expect each code to apply to a certain class of problems and configurations. What is important is that the code description spell out clearly the known capabilities as well as the known limitations of a given formulation.

Regardless of which code is being worked with or whether a new model is being generated, in order to arrive at higher fidelity synthesized models it is crucial that the best possible phenomenological models (impedance, standing wave, traveling wave, stochastic, viscoelastic ,...) of substructures and components be first determined and then implemented. Of course, because of the potential for dynamic interactions, all known subsystems should be modeled as well Another very basic factor that should not be overlooked is the level of geometric detail used in describing the physical domains and interbody connections. The more complete the representation, the more realistic the model and the less likely it is that important coupling effects will be lost. This can become particularly important when dealing with very large frame, truss space structures.

An area that will always be of interest is the manner in which the fundamental principles of classical mechanics are interpreted and applied in order to derive the governing equations. At this point in time this represents a fairly mature part of the technology. Consequently it is recommended that the emphasis now be shifted toward topological considerations and to questions relating to the efficiency of the solution algorithm itself. For example, this study indicates that use of a direct path method of formulation which eliminates constraint forces whenever possible is preferable for that based on use of barycenters and augmented bodies. Graph theory should be relied on to provide a method of accounting for interbody connectivity and related kinematics. A newer approach that merits attention for any new development is the recursive, order n algorithm which avoids the need for a direct inversion of the system mass matrix.

Another tool that should be developed further to aid the analysis is symbolic manipulation. This technique harnesses the power of the computer to provide more rapid, less error prone sets of equations for complex dynamically coupled systems.

From a numerical viewpoint there are many challenging issues. In order to bring the system dimension down to a tractable order, model reduction should be applied at both the subsystem and at the system level. The order of constrained systems can also be minimized through techniques such as singular value decomposition. As with model reduction, however, there are computational costs which have to be factored in to see if real savings are achieved overall.

The accuracy and efficiency of numerical integrators (Runge-Kutta, predictor-corrector,...) needs further evaluation for space dynamic applications. How do the methods compare in general? What are the real benefits of having a variable step size in the presence of controllers? A major issue is how effectively can the "stiff" dynamics be dealt with. Is accuracy degraded seriously in presence of constraint feedback?

Computational efficiency can be enhanced by carrying out calculations in parallel whenever possible rather than following the step by step serial approach so common in the past. In addition to parallel processing, one should take advantage of symbolic manipulation to write the necessary computer code. This promises to be faster and should definitely cut down on debugging costs. It is likely that this can be used when extending existing codes.

What is clear from this study, is that considerable uncertainty persists in the modeling of multibodied dynamic systems. Ideally one would like to verify the working of simulation software by experimental measurement. This is not yet practicable for the full scale in-orbit environment. Hence, it is recommended here that a set of space-related baseline simulation test cases be established for use in evaluating performance of a given code. The test cases should include simplified, idealized configurations (e.g. rigid dumbell) through to actual mission configurations (representative communications satellite, Orbiter and payload, spinner;...). This process does of course raise questions of its own. Nevertheless, it is viewed here as a very important step forward in providing a framework for validating complex multibody codes and for establishing their range of application.

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APPENDIX

PREVIOUS ASSESSMENTS OF MULTIBODY FORMULATIONS

TABLE 1

Jerkovsky - Reference 15

Table 1 Comparison of 10 *n*-body dynamics formulations

n-body configuration	Dynamics state variables	Comments	Date,Authors
Tree of rigid bodies; no relative translation.	Inertial angular velocities of each body; inertial linear velocity of composite.	Uses barycenters and augmented bodies; constraint torques obtained via Lagrange multipliers.	'65 Hooker, Margulies '67 Roberson, Wittenburg
Tree of rigid bodies; no relative translation (except for point masses).	Relative angular velocities between bodies; inertial linear velocity of composite.	Use of relative angular velocities result sin formation of composites; constraint torques removed by use of symmetric projectors.	'67 Velman
Cluster of rigid bodies; no relative translation.	Relative gimbal angle rates between bodies; inertial linear velocity of composite.	Constraint torques do not appear.	'67 Palmer
Tree of rigid bodies; no relative translation.	Free components of angular momentum of outward composites.	Constraint torques do not appear; mass matrix is not computed explicitly.	'69 Russell
Tree of rigid bodies[23]; terminal bodies may be flexible;[24] hinge points may be time-dependent; no relative translation.	Relative gimbal angle rates between bodies; inertial linear velocity of a material point of Body 1.	Equations are obtained inductively; constraint torques do not appear; uses a nonsymmetric "mass matrix."	'69 Farrenkopf '71 Ness, Farrenkopf
Tree of rigid bodies[25] ; terminal bodies may be flexible[26] no relative translation.	Relative gimbal angle rates between bodies; inertial linear velocity of composite.	Uses barycenters and augmented bodies; constraint torques do not appear.	'70 Hooker '73 Likins
Tree of flexible bodies; closed loops treated via Lagrange multipliers[30] ; relative translation allowed.	Relative gimbal angle rates; relative displacement rates; inertial linear velocity of composite.	Uses barycenters and augmented bodies; constraint torques do not appear except with closed loops.	'72 Roberson '74 Wittenburg '74, '75 Boland, Samin, Willems
Tree of rigid bodies; terminal bodies may be flexible; no relative translation.	Relative gimbal angle rates; inertial linear velocity of a material point of Body 1.	Does not use barycenters and augmented bodies; constraint torques do not appear.	'74, '75 Frisch '74, '77 Ho '75 Hooker
Chain of flexible bodies; no relative translation.	Relative gimbal angle rates; inertial linear velocity of a material pint of Body 1.	Uses quasistatic modes plus vibration modes; constraint forces and torques do not appear.	'74 Ho, Hooker, Margulies, Winarske
Arbitrary configuration of flexible bodies; closed loops allowed; relative translation allowed; prescribed motion allowed.	Inertial angular velocities of each body; inertial linear velocity of material point of each body.	Free body equations are written for each body; all constraint forces and torques are obtained via Lagrange multipliers.	'75 Bodley, Devers, Park