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Optimal Design of Piping Systems for District Heating

Gary Phetteplace

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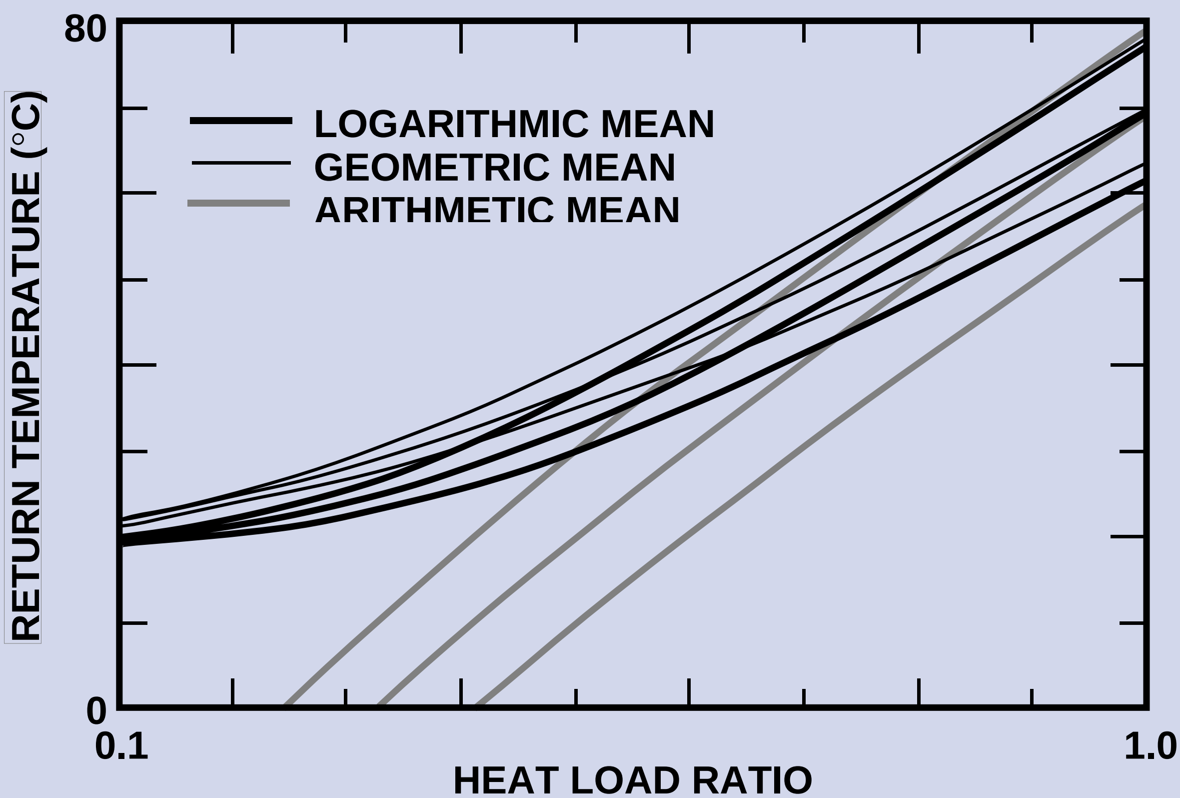

Abstract

First, a method for determining the optimal size for a single pipe segment in a district heating system is developed. The method is general enough to allow for any set of economic or physical parameter values. In addition, any form of load management, i.e., temperature or flow modulation, or both, can be accommodated by the integral form of the coefficients in the cost equation. An example is presented that shows a 17% savings in life cycle costs over a design based on a common rule of thumb. Next the heat consumer and his effects on the piping system are studied. A new model is developed for the consumer's heat exchanger that uses the geometric mean temperature difference as an approximation for the logarithmic mean temperature difference. The new consumer model is integrated into the previous single pipe model and, for a sample case, its effect is determined. For systems having multiple pipes and consumers, the constraints are first developed and then the general solution strategy. The method makes use of the solution to the unconstrained problem as a starting point for the constrained solution. Monotonicity analysis is then used to prove activity of some of the constraints, and thus simplify the problem. Finally, the branch-and-bound technique is shown to be suitable for finding a design with discrete values for all the pipe diameters. A simple example is provided. In addition, a method is also demonstrated for further refinement of the pipe network to eliminate excessive throttling losses in the consumer's control valves. The method developed here should be feasible for designing the piping networks for district heating systems of moderate size, and its major advantage is its flexibility.

Cover: Results from three return temperature models.

For conversion of SI units to non-SI units of measurement consult ASTM Standard E380-93a, *Standard Practice for Use of the International System of Units,* published by the American Society for Testing and Materials, 1916 Race St., Philadelphia, Pa. 19103.

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U.S. Army Corps of Engineers
Cold Regions Research & Engineering Laboratory

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PREFACE

This report was prepared by Dr. Gary E. Phetteplace, Mechanical Engineer, Applied Research Division, Research and Engineering Directorate, U.S. Army Cold Regions Research and Engineering Laboratory. The work reported here was funded in part by DA Project 4A762784AT42, Work Unit CO-M06, *Low Temperature Heat Distribution Systems*.

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NOMENCLATURE

a	coefficient in friction factor equation determined by curve fitting (dimensionless)
A_1	empirical parameter related to pump costs (\$/pump)
A_2	empirical parameter related to pump costs (\$/W)
A_3	parameter related to pipe costs (\$/m)
A_4	parameter related to pipe costs (\$/m^2)
A_5	defined parameter related to pumping energy at the design conditions (\$ m^{5+b+c})
A_6	defined parameter related to the density and dynamic viscosity of the supply and return fluids at the design conditions (m^{3+c} s^c/kg^{1+c})
A_7	defined parameter related to the density and dynamic viscosity of the supply and return fluids (m^{6+c} s^c/kg^{2+c})
$A_{7,\mathrm{d}}$	A_7 evaluated at the design condition (m^{6+c} s^c/kg^{2+c})
A_8	defined parameter related to the fluid density (m^6/kg^2)
A_9	defined parameter related to maintenance and repair costs (\$/m)
A_{10}	defined parameter related to heat losses (m)
A_{11}	defined parameter related to pipe costs (m)
A_{12}	defined parameter related to pumping energy (m^{b+1})
A_{13}	over-design factor for the consumer's radiators (dimensionless)
A_{14}	empirical coefficient related to the annual load curve (dimensionless)
A_{15}	empirical coefficient related to the annual load curve (dimensionless)
A_{16}	defined parameter related to heat losses (\$/[°C hr])
A_η	empirical coefficient related to pumping (dimensionless)
A_{he}	expression that relates the fluid properties and physical properties of the heat exchanger to the pressure drop at a given flow rate ($kg^{1-\beta}$/m–$s^{2-\beta}$])
$A_{\mathrm{m\&r}}$	annual maintenance and repair rate as a fraction of initial capital costs (dimensionless)
A_{t}	number of hours per year (8760)
AF	approach factor for the heat exchanger (dimensionless)
b	exponent in friction factor equation determined by curve fitting (dimensionless)
c	exponent in friction factor equation determined by curve fitting (dimensionless)
C_{cpe}	cost of pumping energy dissipated in the consumer's heat exchanger and control valve (\$)
C_{e}	cost of electricity (\$/Wh)
Ch	cost of heat (\$/Wh)
Ch_{l}	cost of heat losses (\$)
$C_{\mathrm{m\&r}}$	cost of maintenance and repair (\$)
C_{p}	cost of the pipes (\$)
c_{p}	specific heat of water at constant pressure (kJ/kg °C)
C_{pe}	cost of pumping energy (\$)
C_{pp}	capital costs of pipes and pumps (\$)

C_{fixed} fixed cost of pipes and pumps, and the maintenance and repair on this portion of their costs ($)
C_{pv} diameter variable cost of pipes and the maintenance and repair on that portion of pipe cost ($)
C_{pev} diameter variable cost of pumps and pumping energy attributable to piping pressure losses, and the maintenance and repair on that portion of pump costs ($)
C_{pvc} variable cost of pumps attributable to the pressure losses at the consumer ($)
C_{pumps} cost of the pumps ($)
C'_{pt} total diameter variable pipe costs for the system ($)
C_{st} total system cost ($)
C_{t} total system owning and operating cost ($)
C'_{t} portion of total system cost that is dependent on the pipe diameter ($)
d inside diameter of pipe (m)
D_{o} outer diameter of insulation (m)
f friction factor (dimensionless)
g acceleration of gravity (9.8 m/s^2)
g_{l} equality constant numbers l (dimensions vary)
h_{l} inequality constant numbers l (dimensions vary)
H_{p} burial depth to pipe centerline (m)
I_1 defined integral parameter related to heat loss costs ($)
I_2 defined integral parameter related to pumping energy ($ m^{5+b+c})
I_3 defined integral parameter related to pumping energy and maintenance and repair costs ($ m^{5+b+c})
k_{i} insulation thermal conductivity (W/m °C)
k_{s} soil thermal conductivity (W/m °C)
L pipe length (m)
$\dot{m}$ mass flow rate (kg/s)
$\dot{m}_{\text{d}}$ maximum (design) mass flow rate (kg/s)
n total number of heat consumers
n_1 empirically determined exponent in radiator equation (dimensionless)
n_2 empirically determined exponent in the radiator equation (dimensionless)
n_{p} number of pumps
n_{pi} total number of pipe segments (measured in supply and return pipe pairs)
P_{a} atmospheric pressure ($\approx 10^5$ N/m^2)
P_{asa} minimum safety margin above atmospheric pressure (N/m^2)
$P_{\text{hp,s}}$ absolute pressure in supply pipe at heating plant (N/m^2)
$P_{\text{hp,r}}$ pressure in the return line at the inlet to pump (N/m^2)
P_{I} pressure at the inlet to the pipe segment (x=0) (N/m^2)
P_{max} maximum absolute pressure for the piping system being used (N/m^2)
P_{NPSH} minimum allowable pressure at the pump inlet due to NPSH requirements (N/m^2)

PP_a actual pumping power required, including pump and pump driver inefficiencies (W)
PP_f frictional pumping power, exclusive of pump and pump driver inefficiencies (W)
P_s absolute pressure in supply pipe at point in question (N/m^2)
P_{saf} minimum allowable safety margin on saturation pressure requirements (N/m^2)
PVF_e present value factor for electrical energy (dimensionless)
PVF_h present value factor for heat (dimensionless)
$PVF_{m\&r}$ present value factor for maintenance and repair costs (dimensionless)
$P_{x,sat}$ saturation pressure of the liquid at point x within the pipe segment (N/m^2)
P_x pressure at point x (N/m^2)
q heat output from the radiator (W)
Q_{hl} rate of heat loss (W)
Re Reynolds number for the pipe flow (dimensionless)
R_o overall resistance to heat transfer (W/m °C)
RR relative roughness of pipe (dimensionless)
t time of year (hr)
t_u equivalent full load utilization time (hr)
T water temperature (°C)
T_a indoor air temperature (°C)
T_{ao} air temperature at radiator outlet (°C)
T_{avg} average of supply and return temperature (°C)
T_g soil temperature (°C)
T_m mean soil temperature (°C)
T_{ma} arithmetic mean temperature difference (°C)
T_{mg} geometric mean temperature difference (°C)
T_{ml} logarithmic mean temperature difference (°C)
T_p pipe outer surface temperature (°C)
T_r return temperature (°C)
T_s supply temperature (°C)
T_{smin} minimum supply temperature required by next consumer (°C)
v flow velocity (m/s)
w_1 weight of term 1 (dimensionless)
w_2 weight of term 2 (dimensionless)
x position along the pipe in the direction of flow with $x = 0$ being defined as the inlet end to the pipe segment in question (m)
z elevation at point in question relative to heating plant (m)

Greek

β exponent yielding the appropriate mass flow rate dependency for the heat exchanger (dimensionless)
ε absolute roughness of the piping (m)
ε_a relative approximation error for the arithmetic mean temperature difference (dimensionless)

ε_g relative approximation error for the geometric mean temperature difference (dimensionless)
γ k_i/k_s (dimensionless)
η_p pump efficiency (dimensionless)
η_{pm} efficiency of the motor driving the pump (dimensionless)
μ dynamic viscosity (Pa s)
ρ fluid density (kg/m^3)
ρ_d fluid density at design conditions (kg/m^3)
ΔP_{cv} pressure drop in the control valves (N/m^2)
ΔP_{cvm} minimum pressure drop in the control valve (N/m^2)
$\Delta P_{cvs,i}$ slack variable for consumer control valve pressure losses (N/m^2)
ΔP_d total pressure drop (supply and return) at design flow rate (N/m^2)
ΔP_{he} pressure drop in the heat exchangers (N/m^2)
ΔP_{hp} pressure increase across the pump (N/m^2)
$\Delta P_{ncv,i}$ non-control-valve pressure losses (N/m^2)
$\Delta P_{r,j}$ pressure loss in the servicing return line j (N/m^2)
ΔP_r pressure drop in the return piping (N/m)
$\Delta P_{s,j}$ pressure loss in supply pipe j (N/m^2)
ΔP_s pressure drop in the supply piping (N/m^2)
$\Delta P_{s\&r}$ combined pressure loss of supply and return (N/m^2)
ΔT_{ra} smallest temperature difference between fluids in the consumer's heat exchanger (°C)
ΔT_{sa} greatest temperature difference between fluids in the consumer's heat exchanger (°C)
Δx_i insulation thickness (m)
$(\partial z/\partial x)$ partial derivative of the elevation of the pipe with respect to its position (dimensionless)
$(dP/dx)_d$ hydrodynamic pressure gradient (N/m^3)
$(dP/dx)_h$ hydrostatic pressure gradient (N/m^3)

Subscripts

d design maximum load condition
i consumer index
j pipe segment index
sp straight pipe heat exchanger
he conditions within the heat exchanger or physical parameters of the heat exchanger
0 "design" condition for the consumer's heat exchanger, usually the maximum load condition at maximum supply temperature
1 condition for the consumer's heat exchanger of actual supply temperature with the flow rate as determined under the design condition
2 any actual operating condition in the consumer's heat exchanger

Optimal Design of Piping Systems for District Heating

GARY PHETTEPLACE

CHAPTER 1: INTRODUCTION

District heating is the practice of heating multiple buildings from a single heating plant. Heat is conveyed to the buildings by means of hot water or steam. District heating systems offer enormous potential for energy conservation, in addition to the advantages of fuel flexibility and reduced environmental impact. For these reasons district heating has been used extensively in Europe with favorable results. For example, in Denmark district heating serves 42% of the demand for space and hot tap water heating (NRC 1985).

In the United States district heating is much less widespread, accounting for about 4% of the space and hot tap water heating (NRC 1985). A few cities have systems, as well as a number of college campuses and other large institutions. With approximately 6000 miles (10,000 km) of district heating piping in place (Segan and Chen 1984), the Department of Defense is the single largest user of the technology within the United States. A major barrier to more widespread use of district heating in the United States is the high capital cost of the piping required to convey the heat to the buildings. The piping system is most often the major cost of district heating. However, the lack of development of district heating in the United States is often attributed to "institutional barriers." Such barriers, where they truly exist, would be significantly weakened if not removed should the economics become more favorable.

CURRENT DESIGN PRACTICE

Because the hot water or steam piping networks represent such a major portion of the capital costs, they also represent an opportunity for significant cost savings by optimal design. Despite this, in practice little effort is expended either here or in Europe to ensure that proposed designs reduce costs as much as possible. Currently, most designs are based on previous experience and often may be far from optimal. Rules of thumb are commonly used, as are design guides developed for other purposes, such as for plumbing within buildings.

To achieve an optimal design with minimum life cycle costs, all major costs associated with constructing and operating the system must be considered. Capital costs for piping and installation vary widely and must be determined for each case. Operating costs strongly depend on the nature of the load and the load management strategy adopted. For these reasons, it is impossible to develop guidance that can be applied universally to obtain designs that are sufficiently close to the lowest life cycle cost.

As with most areas in the practice of engineering, computer-aided design methods are becoming more widespread in district heating system design. The use of such methods allows the rapid evaluation of many alternate designs, a formidable task if carried out without such methods. A number of computer-aided design methods are available for thermal piping networks (Reisman 1985, Rasmussen and

Lund, undated, Cowiconsult 1985, Hart and Ponsness 1992). Most of the computer-aided design methods that have been developed are proprietary and thus any optimization that they profess to make is not open to inspection or modification. An optimal design model that is open to inspection and modification is therefore needed. The objective of this work is the development of such a model.

OPTIMIZATION IN DISTRICT HEATING SYSTEM DESIGN

Determination of pipe sizes is one of the major decisions that the designer of a district heating system faces. Other critical decisions that must be made are heat source, distribution media, distribution temperatures and load management strategy. Many of these factors have been addressed by previous studies. The emphasis of this work will be on determining pipe size.

Currently, pipe sizes are usually determined on the basis of simple criteria, such as maximum pressure loss or maximum flow velocity. A number of investigators have addressed the issue of pipe size determination, trying to improve on these simple criteria. Aamot and Phetteplace (1976) presented a method that relies on establishing the ratio between the heat losses and the pumping cost and then finds the lowest cost pipe diameter by minimizing the sum of capital, heat loss and pumping costs. Their work only addressed a single pipe segment and did not include the effect of varying load over the yearly cycle. Szepe and Calm (1979) presented a model for single pipe segments that neglected heat losses and time varying loads, but used geometric programming theory to achieve additional insight into their simplified problem. In later work, Phetteplace (1981) included the effect of annual load variations, but only single pipe segments were addressed. Frederiksen (1982) provided a detailed analysis of the heat generating station and the consumer's systems, but simplified the transmission network to a single supply and return pipe.

A number of investigators have addressed multiple pipe networks. Of course, a great deal of work has been done for water distribution systems where the problem is much simpler owing to the lack of heat losses and load variation with temperature as well as mass flow rate. Marconcini and Neri (1979) described a model that calculates the flow rate, pressure and temperature in networks of steam pipes. They discussed the effect that pipe diameter has on operation, but did not offer any methodology for selecting diameter values.

Stoner (1974) discussed models that are capable of modeling either steam or water networks. Although the models do not determine optimum diameters, he gave a procedure for achieving an optimal design by sensitivity analysis, but did not discuss how this process would be accomplished for networks of more than one pipe.

Zinger et al. (1976) described a computer program for calculating flows and pressure levels in branched networks of hot water pipes. Their program accounts for pressure drops in the consumers' equipment and throttling devices placed in the network. Diameters are assumed to be known and they did not discuss how to determine them.

Morofsky and Verma (1979) developed a feasibility analysis and costing tool for district energy systems, not intended for detailed design. They found the appropriate pipe sizes by finding those that absorbed all of the available pressure difference. They started the search for pipe size at the smallest available discrete pipe diameter and then calculated pressure losses. If the pressure losses were more than the available pressure difference, they increased the pipe size to the next discrete size and repeated the calculation. They proceeded in this fashion until they reached a discrete pipe diameter that did not result in pressure losses greater than the available pressure difference.

McDonald and Bloomster (1977) discussed a model for laying out and sizing the piping network for a city heated with geothermal water. Pipe diameter is determined using a "simple search" of feasible pipe sizes by minimizing the sum of the annual capital cost, heat loss cost and pumping cost. They provided no information on how to handle network constraints or consider annual load variations.

Bøhm (1986) noted that, in the case of consumers directly connected to the network, the "classical" approach of determining the optimal diameter by finding the minimum of the sum of the capital, heat loss and pumping costs results in pressures that are too high at the heating plant. He suggested the use of Munser's (1980) method, which proportions the total available pressure loss in a network using the equation

$$\Delta P_1 / \Delta P_0 = \frac{L_1}{\sum_{i=1}^{n} L_i \left(\dot{m}_i / \dot{m}_1 \right)^{1/3}} \tag{1-1}$$

where ΔP_1 = pressure loss in pipe number 1 (N/m^2)
ΔP_0 = total pressure loss in the pipe network (N/m^2)
L_1 = length of pipe number 1 (m)
L_i = length of pipe i (m)
$\dot{m}_1$ = mass flow rate in pipe number 1(kg/s)
$\dot{m}_i$ = mass flow rate in pipe i (kg/s)
n = total number of pipes
i = pipe index.

Equation 1-1 is intended for use on "linear networks" that do not have branches.

Koskelainen (1980) developed a method that is able to solve for optimal diameters in a branched network. His method consists of successively assuming that the objective function and constraints locally are linear and repeatedly solving the problem with a linear programming algorithm. He gives an example where his "optimal" network has a cost that is 16.4% less than one sized using a head loss design rule.

In this work we develop a rational design method that yields the optimal pipe sizes for an application based on case-specific parameter values. This method allows for the inclusion of all major costs and can account for such factors as escalation of energy prices, seasonal energy costs, increases in heat losses over system life, variation in seasonal heat demand, load management strategy, the effect of the heat consumer, etc. Each of the major constraints on the design of a realistic district heating network is derived and considered. This method is felt to be practical for sizing much of the piping of a district heating system.

We begin our study in Chapter 2 by first finding a suitable method for determining the optimal size for a single pipe, independent of any others. In developing this method, we endeavor to keep the formulation as simple as possible, yet complete and accurate enough for design calculations. We make use of geometric programming theory to identify a lower bounding problem that can be used to guide us to our solution. At the end of Chapter 2 is an example that shows a 17% saving in life cycle cost.

In Chapter 3 we study the heat consumer and the effect he has on the piping system. We develop a new model for the consumer's heat exchanger, which uses the geometric mean temperature difference as an approximation for the logarithmic mean temperature difference, thus allowing for an explicit expression for return temperature. We integrate this consumer model with our single pipe model of Chapter 2 and show what effect the consumer has on the system.

In Chapter 4 we develop the constraints for systems with multiple pipes and consumers. Both absolute and differential pressure constraints are derived and where possible strategies are given to allow for constraint satisfaction at all points implicitly without considering every point in the system.

After a brief review of general methods for constrained nonlinear optimization techniques at the beginning of Chapter 5, our general solution strategy is developed for systems with multiple pipes and consumers. The method makes use of the solution to the problem, unconfined by the network constraint requirements. Monotonicity analysis is used to prove activity of some of the constraints and thus simplify the problem somewhat. The result is used as a starting point for two methods proposed to find a solution to the constrained problem with continuous values for some of the pipe diameters. Finally, the branch-and-bound technique is used to find a design with discrete values for all the pipe diameters.

In Chapter 6 we work a simple example with only four consumers and seven pipe segments. The example illustrates the use of our method and also shows how the branch-and-bound technique can be used to quickly eliminate candidate designs.

In Chapter 7 is a summary of our results and offers some conclusions and suggestions for further study.

Because of the inordinate number of variables and parameters involved in the analysis that follows, in choosing symbols for them, an attempt has been made to make their meaning as intuitive as possible. Where accepted symbols exist they have been used to the maximum extent possible. Where it has been mathematically convenient to represent quantities that may have no particular physical significance by a symbol, subscripted A's have been used for sums, products and quotients and I's have been used for integrals.

CHAPTER 2: OPTIMAL PIPE DIAMETER FOR A SINGLE PIPE SEGMENT

To find the optimal diameter for a single pair of supply and return pipes, we need to consider the costs involved and minimize their sum with respect to the pipe diameter. The cost minimization is done for the life cycle of the system using a net present value approach. Some types of heat distribution systems may have a salvage value, while others will, in fact, have a disposal cost associated with the end of their useful lifetime. Since these will in general be mild functions of the pipe diameter, they will not significantly affect the optimal pipe diameter and thus will not be treated here. With these limitations in mind, our objective function, the total life cycle cost, becomes

$$\text{min. } C_{\mathrm{t}} = C_{\mathrm{hl}} + C_{\mathrm{pe}} + C_{\mathrm{pp}} + C_{\mathrm{m\&r}} \tag{2-1}$$

where C_{t} = total system owning and operating cost ($)
C_{hl} = cost of heat losses ($)
C_{pe} = cost of pumping energy ($)
C_{pp} = capital costs of pipes and pumps ($)
$C_{\mathrm{m\&r}}$ = cost of maintenance and repair ($).

Now let's look at each of the costs in eq 2-1 in detail, starting with the cost of heat losses.

COST OF HEAT LOSS

The basic form of the heat loss cost is

$$C_{\mathrm{hl}} = PVF_{\mathrm{h}} \int_{yr} C_{\mathrm{h}} Q_{\mathrm{hl}} \mathrm{d}t \tag{2-2}$$

where C_{h} = cost of heat ($/Wh)
PVF_{h} = present value factor for heat (dimensionless)
Q_{hl} = rate of heat loss (W)
t = time of year (hr $[0 \le t \le 8760]$).

In the most general case, the cost of heat C_{h} can be a function of time because of seasonal usage rates. The rate of heat loss Q_{hl} will also be a function of time over the yearly cycle. In fact, deterioration of the thermal insulation will result in increasing heat losses as the system ages. This can not be incorporated directly into the formulation as given above, but could be allowed for by using an appropriate escalation factor in the present value factor for heat costs PVF_{h}.

The only variable defined above that is dependent on our decision variable, the pipe diameter d, is the heat loss rate itself Q_{hl}. For a single buried pipe, the relationship is

$$Q_{\mathrm{hl}} = L(T_{\mathrm{p}} - T_{\mathrm{g}})/R_{\mathrm{o}} \tag{2-3}$$

where T_{p} = pipe outer surface temperature (°C)
T_{g} = soil temperature (°C)
R_{o} = overall resistance to heat transfer (W/m °C)
L = pipe length (m).

The dependence on d is from the overall thermal resistance R_{o}. This resistance is found by adding the resistance attributable to the soil to that resulting from the pipe insulation (Phetteplace and Meyer 1990). After simplification the following result is obtained

$$R_o = \ln[\,(D_o/d)\,(4H_p/D_o)^{\gamma}\,]/2k_i \tag{2-4}$$

where $\gamma = k_i/k_s$ (dimensionless)
k_i = insulation thermal conductivity (W/m °C)
k_s = soil thermal conductivity (W/m °C)
D_o = outer diameter of insulation (m)
H_p = burial depth to pipe centerline (m).

In this form it becomes easy to see how each factor affects this parameter. The $(4H_p/D_o)^{\gamma}$ factor represents the contribution of the soil to the overall thermal resistance. If $\gamma << 1$, that is, if the soil conductivity is much greater than the insulation thermal conductivity, then this factor will be close to unity and the overall thermal resistance reduces to the thermal resistance of the insulation alone.

To obtain a simpler form for the cost of heat loss, we make the following assumptions:

1. That the soil temperature at the pipe depth varies sinusoidally over the yearly cycle about a mean temperature.

2. That the cost of heat is constant over the yearly cycle. This does not limit us to fixed heat cost over the life of the system, since escalation factors may be used.

3. That the outer surface temperature of the carrier pipe is equal to the temperature of the carrier medium.

The result of these assumptions is the following form for the cost of heat loss

$$C_{hl} = I_1/\ln(A_{10}/d) \tag{2-5}$$

where $I_1 = PVF_h\, L\, 4\pi k_i\, (\int C_h\, T_{avg}\, dt - A_t\, C_h\, T_m)$ (\$)
$A_{10} = D_o(4H_p/D_o)^{\gamma}$ (m)
$T_{avg} = (T_s + T_r)/2$ (°C)
T_m = mean soil temperature (°C)
T_r = return temperature (°C)
T_s = supply temperature (°C)
A_t = number of hours per year (8760).

COST OF PUMPING

Now let's consider the pumping costs. A cost is associated with the electrical energy input to drive the pumps. The portion of this energy that results in frictional heating of the fluid in the pipes is recovered as heat. In general the value of the heat recovered will, of course, be less than the value of the electrical energy input to drive the pumps. It can be significant, however, and therefore it has been included here. Thus, we have the following for the pumping cost

$$C_{pe} = PVF_e \int_{yr} C_e PP_a\, dt - PVF_h \int_{yr} C_h PP_f\, dt \tag{2-6}$$

where PVF_e = present value factor for electrical energy (dimensionless)
C_e = cost of electricity (\$/Wh)
PP_a = actual pumping power required, including pump and pump driver inefficiencies (W)

PP_f = frictional pumping power, exclusive of pump and pump driver inefficiencies (W).

The first integral term represents the total cost of electrical energy input to drive the pumps. The second integral term is the value of heat recovered in frictional heating of the fluid. The actual pumping power and the fluid frictional portion are related as follows

$$PP_a = PP_f / \eta_p \eta_{pm} \tag{2-7}$$

where η_p = pump efficiency (dimensionless)
η_{pm} = efficiency of the motor driving the pump (dimensionless).

The pumping power of a closed system with return lines will not be affected by elevation differences within a network and therefore they need not be considered here. Elevation differences will, however, become a factor in determining the absolute pressure level within a network. A constraint will arise owing to absolute pressure limitations of the piping. This will be addressed later.

Now we assume that the product of the pump and motor efficiency can be expressed as a function of the fraction of maximum volumetric flow. A similar approach was used by Phetteplace (1981) based on data from Gartman (1970). This gives an expression of the form

$$\eta_p \eta_{pm} = A_\eta \, (\dot{m} / \rho) \, (\rho_d / \dot{m}_d) \tag{2-8}$$

where A_η = empirical coefficient (dimensionless)
$\dot{m}$ = mass flow rate (kg/s)
$\dot{m}_d$ = maximum (design) mass flow rate (kg/s)
ρ = fluid density (kg/m^3)
ρ_d = fluid density at design conditions (kg/m^3).

The frictional pumping in the supply or return line is given by

$$PP_f = 2\,(2/\pi)^2 f \, L \, \rho^{-2} \, \dot{m}^3 \, d^{-5} \tag{2-9}$$

where f is a friction factor (dimensionless).

Using the above expression for both the supply and return pipes, we substitute the results, along with our earlier result for PP_a, and our expression for PP_f, into our original expression for the pumping energy cost and simplify to obtain

$$C_{pe} = d^{-5} A_{11} \int_{yr} \left(\frac{C_e \rho \, \dot{m}_d}{A_\eta \dot{m} \rho_d} - \frac{PVF_h}{PVF_e} C_h \right) A_8 \dot{m}^3 f \mathrm{d}t \tag{2-10}$$

where $A_{11} = (4/\pi)^2 \, PVF_e \, L$ (m)

$A_8 = \left(\rho_s^{-2} + \rho_r^{-2}\right)/2$ (s and r subscripts denote supply and return conditions respectively) (m^6/kg^2).

Now we would like to find a simple function to approximate the friction factor f over a range of interest. A simple power function relationship would be desirable to keep the number of terms to a minimum and thus not complicate the above expression further. The form of such a function is suggested by the dimensionless groups of the Moody diagram (Jeppson 1976). A method of finding an approximating function by converting to logarithmic variables and using a least-squares curve fit was developed (Appendix A) to fit an equation of the form

$$f = a\,(\varepsilon/d)^{b}\,Re^{c} \tag{2-11}$$

where a, b and c = coefficients determined by curve fitting (dimensionless)
ε = absolute roughness of the piping (m)
Re = Reynolds number for the pipe flow (dimensionless).

As an example, the following coefficients are obtained over the parameter range given

$a = 0.119$
$b = 0.152$
$c = -0.0568$

for

$50 \le T \le 130$
$0.5 \le v \le 4.5$
$0.050 \le d \le 0.770$

where T = water temperature (°C)
v = flow velocity (m/s).

When compared to the Colebrook and White equation (Jeppson 1976), the maximum error of this approximation is 6.9%, with the average error over the range given being only 1.1%. If more accuracy is required, a much better result could be obtained by narrowing the parameter ranges. Some examples of results for other parameter sets are given in Appendix A. The coefficients will be carried for the general case in the derivations following to allow for values obtained with other parameter sets.

By expressing the Reynolds number as a function of the quantities previously used in the formulation, our equation for the friction factor becomes

$$f = a\,(4/\mu\pi)^{c}\,\varepsilon^{b}\,d^{-(b+c)}\;\dot{m}^{c} \tag{2-12}$$

where μ is dynamic viscosity (Pa s).

Now if we substitute this result into our expression for the cost of pumping energy and simplify, we obtain

$$C_{\mathrm{pe}} = I_2 d^{\,-(5+b+c)} \tag{2-13}$$

where

$$I_2 = A_{12}\int_{yr}\left(\frac{C_{\mathrm{e}}\dot{m}_{\mathrm{d}}\rho}{A_{\eta}\dot{m}\rho_{\mathrm{d}}} - \frac{PVF_{\mathrm{h}}}{PVF_{\mathrm{e}}}\,C_{\mathrm{h}}\right)A_7\,\dot{m}^{3+c}\,\mathrm{d}t\left(\$\ \ \mathrm{m}^{5+b+c}\right)$$

$$A_7 = [(\rho^{\,-2}\mu^{\,-c})_{\mathrm{s}} + (\rho^{-2}\mu^{-c})_{\mathrm{r}}]/2\ \ (\mathrm{m}^{6+c}\ \mathrm{s}^{c}/\mathrm{kg}^{2+c})$$

$$A_{12} = a\,(4/\pi)^{2+c}\,\varepsilon^{b}\,PVF_{\mathrm{e}}\,L\ \ (\mathrm{m}^{b+1}).$$

COST OF PIPES AND PUMPS

Now we need to find expressions for the capital cost of the pipes and pumps. In general, for the entire system the pump capital costs will be assumed to be of the following form

$$C_{\text{pumps}} = A_1 n_p + A_2(\dot{m}_d / \rho_d)\,\Delta P_d \tag{2-14}$$

where A_1 = empirical constant (\$/pump)
A_2 = empirical constant (\$/W)
n_p = number of pumps
ΔP_d = total pressure drop (supply and return) at design flow rate (N/m^2).

The total pressure drop at maximum flow conditions is given by

$$\Delta P_d = a\,\varepsilon^b\,(4/\pi)^{2+c}\,A_6\,\dot{m}_d^{\,2+c}\,L\,d^{-(5+b+c)} \tag{2-15}$$

where $A_6 = [(\rho^{-1}\,\mu^{-c})_{d,s} + (\rho^{-1}\,\mu^{-c})_{d,r}]/2$ (m^{3+c} s^c/kg^{1+c}).

So, our pump cost becomes

$$C_{\text{pumps}} = A_1\,n_p + A_5\,d^{\,-(5+b+c)} \tag{2-16}$$

where $A_5 = A_2 a\,\varepsilon^b (4/\pi)^{2+c}\,A_{7,d}\,\dot{m}_d^{\,3+c}\,L$ (\$ m^{5+b+c})

$A_{7,d} = A_7$ evaluated at the design condition (m^{6+c} s^c/kg^{2+c}).

For the capital cost of the supply and return piping, including installation, we assume the following form

$$C_{\text{pipes}} = (A_3 + A_4 d)L \tag{2-17}$$

where A_3 = empirical constant (\$/m)
A_4 = empirical constant (\$/m^2).

COST OF MAINTENANCE AND REPAIR

The cost of maintenance and repair is assumed to be of the following form

$$C_{\text{m\&r}} = PVF_{\text{m\&r}}\,A_{\text{m\&r}}\,C_{\text{pp}} \tag{2-18}$$

where $A_{\text{m\&r}}$ = annual maintenance and repair rate as a fraction of initial capital cost (dimensionless)
$PVF_{\text{m\&r}}$ = present value factor for maintenance and repair costs (dimensionless).

TOTAL COST

With each of the component costs defined, our expression for the objective function, the total cost C_t, becomes

$$C_t' = I_1/[\ln(A_{10}/d)] + I_3\, d^{-(5+b+c)} + A_9\, d \tag{2-19}$$

where $C_t' = C_t - [(1+ PVF_{\mathrm{m\&r}}A_{\mathrm{m\&r}})(A_1 n_{\mathrm{p}} + A_3 L)]$

$I_3 = I_2 + (1+PVF_{\mathrm{m\&r}}A_{\mathrm{m\&r}})\, A_5$

$A_9 = (1+PVF_{\mathrm{m\&r}}A_{\mathrm{m\&r}})\, A_4 L.$

Minimizing C_t' with respect to d is, of course, equivalent to minimizing the original total cost function C_t. Therefore, we can work with C_t' for convenience. If we neglect the first term, which represents the cost of heat losses, we have a geometric programming problem (Papalambros and Wilde 1988) with zero degrees of difficulty. Without specifying the parameter values, we see from inspection that the weights of the two remaining terms will be

$$w_1 = 1/(6 + b + c) \text{ and } w_2 = (5 + b + c)/(6 + b + c).$$

With heat losses neglected, at the optimum pipe diameter the variable costs associated with pumping are $1/(6 + b + c)$ of the total variable costs. The variable costs attributable to pipe capital and maintenance costs are the remaining portion. Here, the variable costs represent that portion which is a function of our decision variable, the pipe diameter. Also note that the pumping costs include the variable portion of the capital cost of the pumps and the maintenance associated with that portion, as well as the pumping energy costs.

Considering a more specific case, if the values of parameters b and c found in the example given for eq 2-11 ($b = 0.152$, $c = -0.0568$) are used, we find the following values for the weights

$$w_1 = 16.4\%$$
$$w_2 = 83.6\%.$$

These results vary very little over the range of values found for b and c in Appendix A. Thus, when heat losses are neglected, we find this very simple solution is applicable in most cases.

Once values for the remaining parameters are known, the pipe diameter is found by using the equations given above and the two terms of the objective function remaining. The resulting expression is

$$d = [(5 + b + c)(I_3/A_9)]^{[1/(6+b+c)]}. \tag{2-20}$$

It should be noted that this solution obtained using geometric programming theory also could have been easily obtained using classical differential methods, as used later. The advantage of the geometric programming method is that it ensures that a *global* rather than a *local* minimum has been found. Differential methods only ensure a local extremum and require the evaluation of second order terms to determine the nature of the extremum, i.e., maximum or minimum.

To arrive at this simple expression for the pipe diameter, we have neglected the heat losses. Because the cost of heat losses will always be greater than zero, we have constructed a lower bounding problem for our original problem by neglecting this cost, i.e.

$$I_3\, d^{-(5+b+c)} + A_9\, d \le I_1/[\ln(A_{10}/d)] + I_3\, d^{-(5+b+c)} + A_9\, d.$$

The cost of any design that includes heat losses can never be less than the same design excluding the cost of heat losses. And, since we have found the optimum design (lowest cost) neglecting heat losses, we now know that no design can achieve a lower cost when heat losses are included. This simple result can be very useful. It may be possible to find a design, not necessarily known to be optimal, whose cost including heat losses is acceptably close to that of the optimal design for the lower bounding problem.

The solution to the complete problem including heat losses is slightly more complicated, but is easily obtained. To find the extremum of the total variable cost function C_t', we simply take its partial derivative with respect to d and set the result to zero. Before proceeding to do so, however, we must take note of the value of A_{10} in the heat loss term being a function of the pipe diameter. This results from the outer diameter of the pipe being a function of pipe diameter and insulation thickness. The appropriate insulation thickness is determined by a separate optimization procedure that would consider the insulation and jacket material costs and the cost of heat loss. As a result of this separate "sub-optimization," the insulation thickness becomes a function of the pipe diameter.

For a given set of operating conditions and economic data, the optimal insulation thickness can be found as a function of the pipe diameter. Here, for the sake of simplicity, we will assume that the insulation thickness is fixed. We then find an expression for A_{10} as follows

$$A_{10} = (d + 2\Delta x_i)^{1-\gamma}\,(4H_p)^{\gamma} \tag{2-21}$$

where Δx_i is the insulation thickness (m). In turn we approximate this expression by one of the following form

$$A_{10} = (d^{1-\gamma} + (2\Delta x_i)^{1-\gamma})\,(4H_p)^{\gamma}. \tag{2-22}$$

For a typical set of parameter values

$$k_i = 0.030 \text{ W/m °C}$$
$$k_s = 1.3 \text{ W/m °C}$$
$$\Delta x_i = 0.050 \text{ m}.$$

This approximation is within 2% for values of d from 0.025 to 1.0 m. Using this approximation for A_{10}, we obtain the following equation for C_t'

$$C_t' = \left[I_1/\ln\left((4H_p)^{\gamma}\,(d^{-\gamma} + (2\Delta x_i)^{1-\gamma}\,d^{-1})\right)\right] + I_3 d^{-(5+b+c)} + A_9 d. \tag{2-23}$$

If we take the partial derivative of C_t' and set the result to zero, we have

$$0 = \left\{I_1/d\,\left[\ln\left((4H_p)^{\gamma}(d^{-\gamma} + (2\Delta x_i)^{1-\gamma}\,d^{-1})\right)\right]^2\right\}\left\{1-\left((1-\gamma)/(1+(2\Delta x_i)^{1-\gamma}\,d^{\,\gamma-1})\right)\right\}$$
$$- (5+b+c)I_3 d^{-(6+b+c)} + A_9. \tag{2-24}$$

This equation can not be solved explicitly for d. A solution can be obtained by using a root-finder technique. An initial estimate needed for the solution can be found by using the value of d obtained from the solution to the lower bounding problem, which neglects heat losses. The cost associated with this optimal design that neglects heat losses also provides us with a global lower bound on the actual cost. In the next section, we consider the solution to a representative case.

A SIMPLE EXAMPLE

To illustrate the application of the method developed in the previous section, an example using realistic parameter values is presented in this section. In addition, the results obtained are compared to those obtained by using a common rule of thumb. Before we can proceed with the calculation of the parameters, we need to define the economic and technical assumptions upon which our solution is based.

Economic assumptions

The assumptions concerning economic conditions of an application are the most controversial in an analysis of this type. Here, we will endeavor to select conditions that are felt to be representative of the majority of applications rather than any specific application. The analysis is quite sensitive to these economic assumptions, so the reader is cautioned that specific information must be obtained before applying these results. This can not be overemphasized.

The most significant cost to be considered is the capital cost of the piping system. This cost is also highly variable, depending on the piping system used and, above all, the site conditions. Our pipe cost equation is from Phetteplace (1981) and has been adjusted for inflation from 1980 to 1988 using factors from R.S. Means Co. (1987). The following expression results

$$C_p = 218\, L + 2180\, Ld. \tag{2-25}$$

The capital costs of pumps is also taken from Phetteplace (1981), with cost adjustments made as indicated above to arrive at

$$C_{pumps} = 1060\, n_p + 0.242\, \Delta P_d\ \dot{m}_d/\rho_d. \tag{2-26}$$

The present value factors (PVF's) are assumed to be equal for electrical cost, heat cost, and maintenance and repair costs. As noted earlier, these PVF's could be modified to allow for escalation of energy, labor or material costs and could even be modified to allow for increasing heat loss over the life of the system. For simplicity, however, such modifications are not made here. We assume an interest rate of 10% per annum and a system lifetime of 25 years in calculating the PVF's using the following expression (CRC Press 1987)

$$PVF = [1 - (1+i)^{-n}]/i = [1 - (1.10)^{-25}]/0.10 = 9.08. \tag{2-27}$$

We assume that both heat and electricity costs are constant over the year, although the formulation allows for varying rates over the yearly cycle. Again, these costs are highly variable, dependent mainly on the sources of the energy and the values given are not for a specific application, although it is felt that they are representative. The assumed costs are

$$C_e = 7.0 \times 10^{-5}\ \$/\text{Wh}\ (\$0.07/\text{kWh})$$

$$C_h = 3.4 \times 10^{-5}\ \$/\text{Wh}\ (\$10/10^6\text{Btu}).$$

The rate of maintenance and repair on the system is also taken from Phetteplace (1981) as 2% of the capital cost per year. This factor is assumed to apply to both the piping system and the pumps. Again, note that maintenance and repair increasing with component age could be easily accounted for using escalation factors to adjust $PVF_{m\&r}$, although this is not done here.

Technical assumptions

The major technical assumptions that we make are related to the heat load characteristics and the method by which the heat load is met. In district heating systems, the amount of heat supplied can be varied to accommodate varying demand by adjusting either the supply temperature or flow rate. However, certain constraints imposed by consumer equipment and minimum temperature requirements must be observed. In larger systems, both the supply temperature and flow rate are varied over the course of the year. In small systems, which must adopt simpler control strategies, often only the flow rate is varied. For the sake of simplicity, we assume the latter here and assume that the supply and return temperatures remain fixed over the yearly cycle. This is never actually the case, but for system design, it is felt that this is an appropriate simplifying assumption for a first analysis. Ideally, for example, the supply temperature, return temperature and flow rate at the consumer would be determined by the heat transfer characteristics of his heat exchanger under the prevailing load. A model that simulates the consumer's heat exchanger is developed in Chapter 3, but here it is not considered.

The actual heat load in district heating systems has several major components. A detailed treatment of the heat load would be difficult and is not warranted for design purposes. For an excellent treatment of the actual heat loads in operating district heating systems, see Werner (1984). The assumption we make here for design purposes, that the heat load can be approximated as sinusoidal, is supported by the data presented by Werner (1984) as well as by the data of Phetteplace et al. (1981). We assume here that the heat load varies sinusoidally from a minimum of 15% of its maximum value to its maximum value. The assumed minimum load of 15% would result primarily from hot tap water use and heat losses from the pipelines. Thus, with our assumption of constant supply and return temperatures, the mass flow rate as a function of time is

$$\dot{m} / \dot{m}_d = 0.575 + [0.425 \cos(2\pi t/8760)]. \tag{2-28}$$

The ratio of the mass flow rate to its maximum value as determined by eq 2-28 is shown in Figure 1. With this simple function for the load curve, it is easy to determine the shape of what is normally referred to as the "load duration curve." Since eq 2-

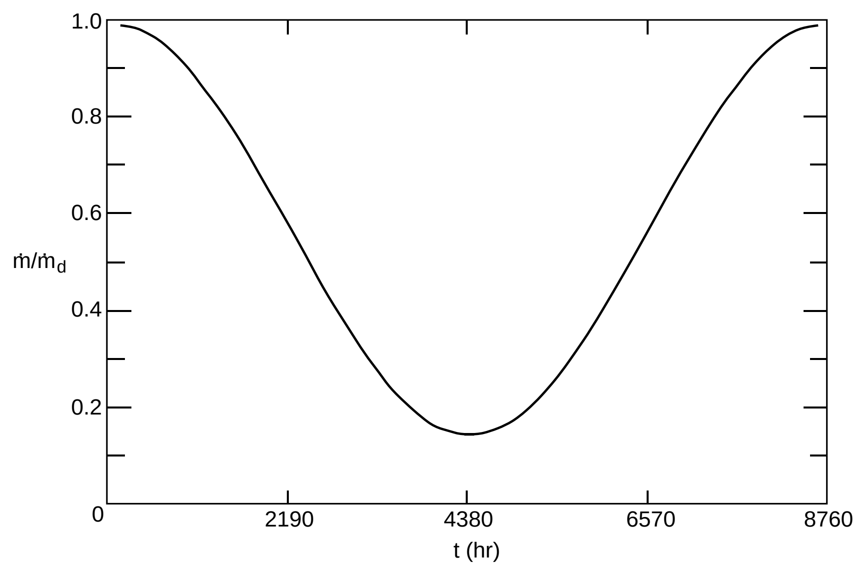

Figure 1. Assumed annual load curve.

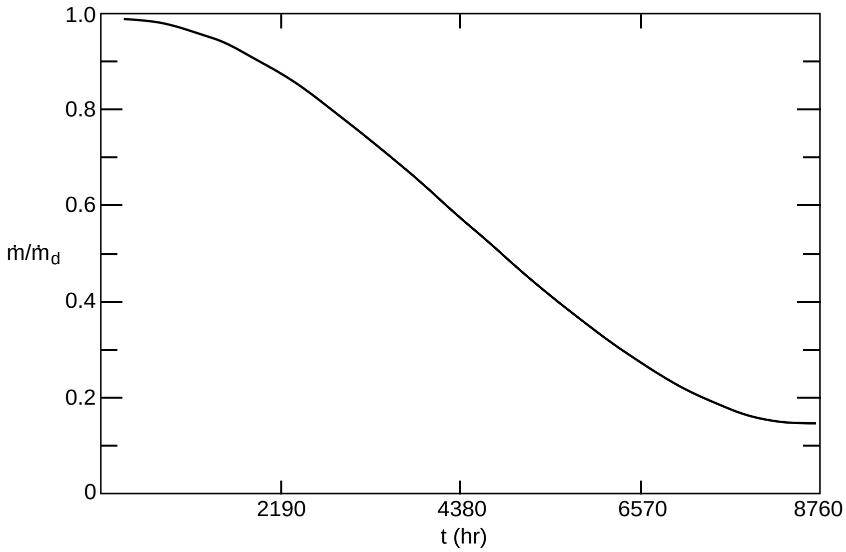

Figure 2. Load duration curve.

28 is an even function about its midpoint of 8760/2 = 4380 hours, the number of hours for which the load will exceed any given level is simply twice the number of hours from the time of maximum load ($t = 0$) until the time that load would have occurred. This yields an equation of the same form as eq 2-28, except that the period of the function is now twice as long, i.e.,

$$\dot{m} / \dot{m}_d = [0.425 \cos(2\pi t/17{,}520) + 0.575]. \tag{2-29}$$

The form of the resulting load duration curve is shown as Figure 2. This result is similar to the shape of empirical load curves determined by other investigators, such as Werner (1984), except that it over-predicts the number of hours at which high loads occur. Since this will result in a conservative design, this is a suitable approximation for design purposes. The equivalent full load utilization time is another factor used to evaluate the load in district heating systems. The equivalent full load utilization time is the amount of time at which the load would have to be at its maximum value to result in the total heat supplied being equal to that supplied over the actual yearly cycle. This time can be easily found by integrating eq 2-28 over the yearly cycle

$$t_u = \int_0^{8760} (0.425 \cos(2\pi t/8760) + 0.575)\, dt \tag{2-30}$$

where t_u is the equivalent full load utilization time (hr). Carrying out this integration yields a value of t_u = 5037 hours. This value is near the upper end of the range of measured values reported by Bøhm (1988) for a number of Danish district heating systems.

A number of other technical parameters need to have values assigned to them for us to proceed with this sample calculation. The following values selected are felt to be well within the range of what could reasonably be expected in an actual design:

A_η = 0.90 (dimensionless)
T_m = 6.4°C
k_i = 0.030 W/m °C

$k_s = 1.3$ W/m °C
$H_p = 1.0$ m
$\Delta x_i = 0.050$ m
$\varepsilon = 5 \times 10^{-5}$ m
$a = 0.119$ (dimensionless)
$b = 0.152$ (dimensionless)
$c = -0.0568$ (dimensionless)
$T_s = 120$°C
$T_r = 60$°C.

Application

For the application, we consider a main portion of a distribution system that would serve a large number of consumers. We assume a maximum heating load, including pipeline heat losses, of 25 MW. At the temperature difference specified above, this would require a "design" or maximum flow of $\dot{m}_d = 100$ kg/s. We also assume that the length of the pipeline is 1000 m. Note, however, that the length used will not affect the diameter determined, as the length could be factored out of each of the variable terms in the objective function eq 2-23, and the calculation would then be done on a unit length basis. To arrive at a realistic total cost, which includes the cost fixed with respect to d, the calculations here are for the system length specified above. We also assume that only one pump is associated with the system.

Solution

For the problem described above, we arrive at the following values for the parameters in the objective function:

$\gamma = 0.0231$ (dimensionless)
$A_9 = \$2.58 \times 10^6$/m
$I_1 = \$8.56 \times 10^4$
$I_3 = 44.1$ \$ $m^{5.095}$
$A_1 = \$1060$/pump
$A_3 = \$2.18 \times 10^5$.

The calculation of the above parameters is straightforward with the exception of I_3. The integral in the I_3 parameter was evaluated numerically by a FORTRAN program adapted from Ferziger (1981), which uses Romberg integration. The program is included in Appendix B.

Before solving eq 2-24 to determine the optimum diameter, we first find an approximate solution using eq 2-20, which neglects the heat losses. From eq 2-20 we solve for the diameter directly, obtaining $d = 0.216$ m. Using this value of d as an initial estimate, we can proceed to solve eq 2-24. We know that the solution to eq 2-24, which includes heat losses, will be a smaller diameter than the solution to eq 2-20, which does not include heat losses, since heat losses are an increasing function of the diameter. Various "root finder" methods can be used to find the solution to eq 2-24. Guided by the value obtained above, a simple trial-and-error method was used here, which yielded a solution to three significant digits with several function evaluations. The optimal diameter d was found to be 0.208 m. The total cost for this design is $C_t = \$1.11 \times 10^6$. In the following section, this result will be compared to one obtained using a common design rule of thumb.

Comparison with a design based on a rule of thumb

Ideally, an analysis similar to the one above would be used to size all major district heating pipes. In reality, however, most systems are designed on the basis of rules of thumb that have evolved from practice. Although such rules of thumb may prove

Table 1. Pressure drops and costs for discrete pipe sizes under maximum flow conditions (pipe data from Marks 1978).

Nominal pipe size (in.)	*Inside diameter schedule 40 (in.)*	*(m)*	ΔP_d *(Pa/m)*	C_t *($\$ \times 10^6$)*
—	8.187	0.208	340	1.111
8	7.981	0.203	384	1.112
10	10.020	0.255	120	1.178
12	11.938	0.303	50	1.305

adequate in some cases, they lack the flexibility to account for varying conditions, most notably economic. Because these rules of thumb are based on designs proven only to be functional, they cannot profess to yield least life cycle cost designs. To see how the results of the above example would compare with a rule of thumb based design, we consider a very common design rule of thumb used in Europe for systems in this temperature range: that the pressure loss in the piping not exceed 100 Pa/m. For this example, standard schedule 40 pipe sizes are used.

To apply the above rule of thumb, we simply calculate the pressure loss that would result at maximum flow conditions using increasing pipe size until we find a size that satisfies the rule. This calculation is done using eq 2-15 given earlier. The results are shown in Table 1. We see from Table 1 that a 12-in. (300-mm) pipe would be necessary to satisfy the rule of thumb. The pressure loss for the 10-in. (250-mm) pipe exceeds the 100-Pa/m level by over 20% and therefore would probably be considered unacceptable.

Now we need to determine what discrete pipe diameter would be recommended by the procedure outlined in the previous section. The optimal nondiscrete diameter was found to be 0.208 m or 8.187 in. We see from Table 1 that this lies between the inside diameters of the 8- and 10-in. (200- and 250-mm) nominal pipe sizes.

To determine which to use, we simply calculate the cost of each alternative using eq 2-19. These results are also included in Table 1. We see from these figures that the total life cycle cost of the 8-in. pipe is about 6% less than the 10-in. pipe and thus the 8-in. pipe should be selected. We also note that the life cycle cost of the 8-in. pipe is only 0.1% greater than that of an optimal 8.187-in. inside diameter pipe, if such a pipe were available.

If we compare the cost of the 8-in. pipe, which our method recommends, to the 12-in. pipe required by the maximum pressure drop rule of thumb, we find that the life cycle cost of the rule based design is 17% greater. This great saving in life cycle cost is also accompanied by an even greater 30% reduction in capital costs (sum of eq 2-16 and 2-17). As the financing of a new district heating system is often a barrier to implementation, such large reductions in capital costs could make a system feasible where it might not be otherwise.

We have arrived at an optimal pipe size that promises to save 17% in life cycle cost over a rule of thumb based design. This result is consistent with the results of others (Bøhm 1986, Koskelainen 1980) who have compared optimized designs with rule of thumb based designs. In determining this pipe size, we have not considered any constraints on the selection, other than it be a commercially available size. Of course, in reality, other constraints exist. Before this method could be used to design an entire system, the constraints that arise from interconnection of the pipes need to be considered. Constraints also arise because of the consumer's equipment and minimum temperature requirements. Other constraints are associated with the limitations of the piping system and the plant that supplies the heat. These constraints will be considered in the following chapters.

CHAPTER 3: THE CONSUMER'S HEAT LOAD

In the design and subsequent operation of a district heating system, the characteristics of the load can be very significant. The load will not only dictate the combination of supply temperature and mass flow rate necessary for its satisfaction, but the heat exchanger equipment used at the consumer will also determine the return temperature of the water. Lowering return temperature is desirable because it results in larger temperature differences and thus lower mass flow rates, pumping energy expenditure, and possibly smaller pipes. The importance of this issue is evidenced by many district heating utilities in Europe having taken significant actions to achieve large temperature differences. Thus, it is essential that our design methodology for the distribution piping system account for the characteristics of the consumer's load and the constraints that result.

The primary type of heat load for most district heating systems is space heating. In some cases industrial process loads can also be significant. In most cases where buildings rely on a district heating system for space heat, they also use the system to heat hot water. Here we will develop simple models for space heating loads only.

SIMPLE MODEL FOR THE CONSUMER'S SPACE HEATING EQUIPMENT

As noted above, in addition to the maximum magnitude of the load placed on the district heating system by the consumer, several other characteristics of the load are important. The way in which the load varies is of primary importance. This was discussed in Chapter 1 and will be addressed in more detail later. The other major way in which the load affects heat distribution systems is through the response of the consumer's heat exchanger to changes in supply temperature. To address this issue, we need a model for the consumer's heat exchanger equipment. We will develop such a model in this section.

In district heating systems using hot water, the water-to-air heat exchangers of the consumers can either be directly connected to the network or indirectly coupled by a heat exchanger. Each type of connection has its advantages and limitations. For the sake of simplicity, we will assume that the buildings are directly connected in this work. To address indirect systems, it would be necessary to either develop alternate models or attempt to modify the model for a direct system developed below.

The normal radiator common on many residential and light commercial hydronic heating systems can be classified as a cross flow heat exchanger with one of the fluids mixed (water) and the other fluid (air) unmixed, as described by Kays and London (1964). Although the term "radiator" is commonly used for these heat exchangers, they function via both convective and radiative heat transfer within the temperature ranges normally encountered in practice. A schematic representation of this cross flow heat exchanger is shown in Figure 3.

Because the water is considered to be ideally mixed, its temperature is assumed to be uniform in the direction of air flow at any point along the heat exchanger. As the water moves through the heat exchanger, it varies from the supply temperature T_s at the water inlet to the return temperature T_r at the outlet. The incoming air temperature T_a is assumed to be constant along the length of the heat exchanger. However, the outgoing air temperature T_{ao} will vary along the length of the radiator owing to the decline in water temperature.

Although it would be possible to describe the performance of a radiator using traditional approaches, such as those described by Kays and London (1964), simpler equations have been proposed. These are based on experimental results for such heat exchangers, an example being the equation given by Bøhm (1988)

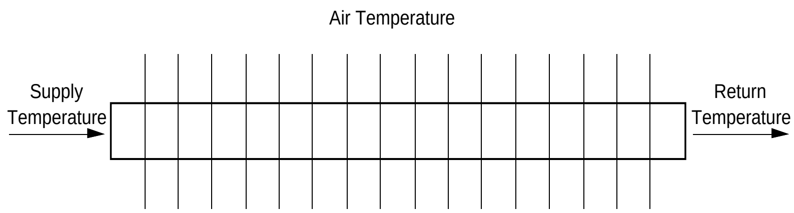

Figure 3. Schematic of a hydronic heating system radiator.

$$q_2/q_0 = [(T_{ml})_1/(T_{ml})_0]^{n_1}\,[(T_{ml})_2/\,(T_{ml})_1]^{n_2} \tag{3-1}$$

where q = heat output from the radiator (W)
T_{ml} = logarithmic mean temperature difference (°C)
n_1, n_2 = empirically determined coefficients (dimensionless).

and the subscripts denote the following operating conditions

0 = "design" condition for the radiators, usually the maximum load condition at maximum supply temperature
1 = condition of actual supply temperature with the flow rate as determined under the design condition
2 = any actual operating condition.

Equation 3-1 uses the logarithmic mean temperature difference T_{ml} and experimentally determined constants to predict heat exchanger performance. The logarithmic mean temperature difference is defined as

$$T_{m1} = \frac{(T_s - T_a) - (T_r - T_a)}{\ln(T_s - T_a) - \ln(T_r - T_a)} = \frac{T_s - T_r}{\ln\left[(T_s - T_a)/(T_r - T_a)\right]}\ . \tag{3-2}$$

One problem that results from using the logarithmic mean temperature difference is that an explicit expression for either the supply temperature T_s, or the return temperature T_r, cannot be obtained from the expression for the logarithmic mean temperature. This limits the extent of closed form analysis and ultimately, when calculations are required, it forces solution by iterative numerical methods. As a solution to these problems, the use of the arithmetic mean as an approximation for the logarithmic mean was proposed by Soumerai (1987). The arithmetic mean temperature difference for this case is defined as

$$T_{ma} = [(T_s - T_a) + (T_r - T_a)]/2 = (T_s + T_r - 2T_a)/2. \tag{3-3}$$

The arithmetic mean temperature difference has the advantage that it can be used to find a very simple explicit expression for either the supply temperature T_s or the return temperature T_r given the value of the arithmetic mean temperature difference. The disadvantage of using the arithmetic mean temperature difference as an approximation for the logarithmic mean temperature difference is the error induced by this approximation. As Soumerai (1987) points out, within certain ranges of the temperatures involved, the resultant errors are usually acceptable, given the other uncertainties in heat transfer engineering. Soumerai (1987) recommends the use of the arithmetic mean as an approximation for the logarithmic mean in cases where the approach factor *AF* is equal to or greater than 0.5. The approach factor for this type of heat exchanger is given by

$$AF = (T_r - T_a)/(T_s - T_a)\ . \tag{3-4}$$

In the case where the above criterion for the approach factor is met, the error of approximation is always less than 4%. The arithmetic mean always overestimates the logarithmic mean and thus any estimates of the heat transfer based on the arithmetic mean will overestimate the actual heat transfer that will be achieved. This could result in undersized heat exchangers, assuming that no other margin of safety is included, which is of course seldom the case.

As an alternative approximation to the logarithmic mean temperature difference, the use of the geometric mean temperature difference is proposed. The geometric mean temperature difference for this type of heat exchanger is defined as

$$T_{mg} = (T_s - T_a)^{1/2}\,(T_r - T_a)^{1/2} = (T_a^2 + T_s T_r - T_a T_s - T_a T_r)^{1/2}. \tag{3-5}$$

The geometric mean temperature difference, like the arithmetic mean temperature difference, has the advantage that an explicit expression for the supply or return temperature can be obtained from it. The geometric mean temperature difference, however, is a much better approximation of the logarithmic mean temperature difference than is the arithmetic mean temperature difference, as will be shown below.

To simplify the analysis, we introduce the following expressions

$$\Delta T_{sa} = T_s - T_a \tag{3-6}$$

$$\Delta T_{ra} = T_r - T_a \tag{3-7}$$

$$AF = \Delta T_{ra}/\Delta T_{sa} \tag{3-8}$$

where ΔT_{sa} = greatest temperature difference between fluids (°C)
ΔT_{ra} = smallest temperature difference between fluids (°C)
AF = approach factor for the heat exchanger (dimensionless).

Two limiting cases of heat transfer set the range of values possible for the approach factor *AF*. The first case is the case where no heat transfer takes place in the heat exchanger. In this case the temperature of the water flowing through the radiator will not decrease, and thus the supply and return water temperatures will be equal and the approach factor becomes unity. The other limiting case occurs when the maximum amount of heat transfer occurs in the heat exchanger, in which case the return temperature equals the air temperature and approach factor becomes zero. Thus, we have the following range of values for the approach factor *AF*

$$0 \le AF \le 1\ . \tag{3-9}$$

Now we can examine the errors that can result from each of the approximations presented above over the entire range of possible approach factors. First, we define the relative error of each of the approximations

$$\varepsilon_a = (T_{ma}/T_{ml}) - 1 \tag{3-10}$$

$$\varepsilon_g = (T_{mg}/T_{ml}) - 1 \tag{3-11}$$

where ε_a is a relative approximation error for the arithmetic mean temperature difference (dimensionless) and ε_g is a relative approximation error for the geometric mean temperature difference (dimensionless). Then, by combining eq 3-2, 3-3, 3-6, 3-7, 3-8 and 3-10, we can arrive at the following expression for the relative error of the arithmetic mean temperature difference ε_a in terms of the approach factor *AF*

$$\varepsilon_a = \{ [(AF + 1) \ln(AF)]/[2(AF - 1)] \} - 1. \tag{3-12}$$

Similarly, we combine eq 3-2, 3-5, 3-6, 3-7, 3-8 and 3-11 to arrive at an expression for the relative error of the geometric mean temperature difference ε_g in terms of the approach factor *AF*

$$\varepsilon_g = \{[(AF)^{1/2} \ln(AF)]/[AF - 1]\} - 1. \tag{3-13}$$

Now we can study the approximation errors for the arithmetic and geometric mean temperature differences over the range of possible approach factors by examining eq 3-12 and 3-13 respectively. It is immediately obvious that the error from the arithmetic mean temperature difference approximation ε_a becomes infinite as *AF* approaches zero. However, it is not clear what the error from the geometric mean temperature difference approximation becomes as *AF* approaches zero. To determine what value ε_g approaches as *AF* approaches zero, we use l'Hôpital's rule. It states that

$$\underset{x \to \lambda}{\text{limit}} \left(f(x)/g(x)\right) = \underset{x \to \lambda}{\text{limit}} \left(f'(x)/g'(x)\right) \tag{3-14}$$

where *f(x)* and *g(x)* are some functions of *x* that both approach either zero or infinity when *x* approaches the value λ. To apply this to the error expression for the geometric mean temperature difference, we let

$$\begin{aligned} x &= AF \\ f(x) &= f(AF) = \ln(AF) \\ g(x) &= g(AF) = (AF-1)/AF^{1/2} = AF^{1/2} - AF^{-1/2} \\ \varepsilon_g &= f(AF)/g(AF) - 1 . \end{aligned}$$

Taking the first derivatives of *f(AF)* and *g(AF),* we have

$$\begin{aligned} f'(AF) &= 1/AF \\ g'(AF) &= 0.5\, AF^{-1/2} + 0.5\, AF^{-3/2} . \end{aligned}$$

Now we can determine the value that ε_g approaches as $AF \to 0$ from

$$\underset{AF \to 0}{\text{limit}} \left(\varepsilon_g + 1\right) = \underset{AF \to 0}{\text{limit}} \left(f(AF)/g(AF)\right) = \underset{AF \to 0}{\text{limit}} \left(f'(AF)/g'(AF)\right) =$$

$$\underset{AF \to 0}{\text{limit}} \quad (AF^{1/2}/[0.5(AF+1)]) = 0$$

Thus, we find that the error from approximating the logarithmic mean temperature difference with the geometric mean temperature difference ε_g reaches –100% as the approach factor *AF* goes to zero. Although this is a very high relative error, it is still much better than that of the arithmetic mean temperature difference approximation, which becomes infinite at the same condition. Of course, in reality this limiting case, where heat transfer is at its maximum value and the approach factor becomes zero, is never achieved. As we will now show, the errors attributable to using the geometric and arithmetic mean temperature difference approximations for the logarithmic mean temperature difference are always less than the values for an approach factor of zero.

The other limiting value for the approach factor is unity. In this case no heat transfer occurs. First, let's examine what happens to the error from the arithmetic mean approximation ε_a. We must again use l'Hôpital's rule, proceeding as before

$$f(AF) = (AF - 1)\ln(AF)$$
$$g(AF) = 2(AF - 1)$$
$$f'(AF) = 1 + \ln(AF) + 1/AF$$
$$g'(AF) = 2\ .$$

In the limit as AF approaches unity, we have

$$\lim_{AF \to 1} \left(\varepsilon_g + 1\right) = \lim_{AF \to 1} \left(f(AF)/g(AF)\right) = \lim_{AF \to 1} \left(f'(AF)/g'(AF)\right) =$$

$$\lim_{AF \to 1} \left((1 + \ln(AF) + 1/AF)/2\right) = 1\ .$$

So, we see that the error induced by using the arithmetic mean temperature difference as an approximation for the logarithmic mean temperature difference approaches zero as AF approaches unity. Now let's look at what happens to the error for the geometric mean temperature difference as AF approaches unity. Again we see that l'Hôpital's rule is needed and we proceed as follows

$$f(AF) = AF^{1/2}\ln(AF)$$
$$g(AF) = AF - 1$$
$$f'(AF) = AF^{-1/2}\,[(\ln(AF)/2) + 1]$$
$$g'(AF) = 1\ .$$

In the limit as AF approaches unity we have

$$\lim_{AF \to 1} \left(\varepsilon_g + 1\right) = \lim_{AF \to 1} \left(f(AF)/g(AF)\right) = \lim_{AF \to 1} \left(f'(AF)/g'(AF)\right) =$$

$$\lim_{AF \to 1} \left(AF^{-1/2}[(\ln(AF))/2) + 1]\right) = 1\ .$$

Thus, we find that the error for the geometric mean temperature difference approximation to the logarithmic mean temperature difference also approaches zero as AF approaches unity. The errors resulting from using the arithmetic and geometric mean temperature differences as approximations for the logarithmic mean temperature difference are shown in Figure 4. Some numerical values for these errors are also given in Table 2.

Several important observations can be made by studying Table 2. First, we note that the error from approximating the logarithmic mean temperature difference with the arithmetic mean temperature difference is always positive. Since the heat transfer is proportional to the logarithmic mean temperature raised to some positive power, using the arithmetic mean temperature difference as an approximation will always over-predict the actual heat transfer. Also note that, as we have shown analytically, the arithmetic mean temperature difference approaches infinity as the AF goes to zero and approaches zero as AF goes to unity. For the geometric mean temperature difference, the error resulting from using it as an approximation for the

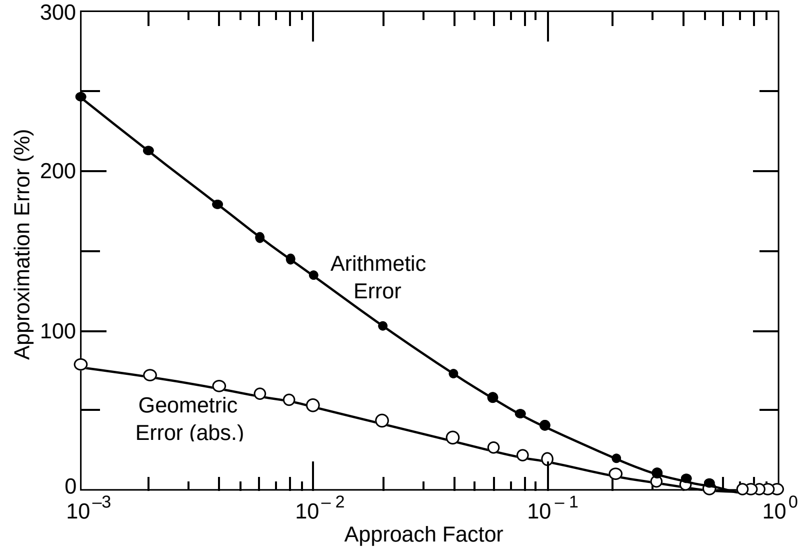

Figure 4. Errors from approximating the logarithmic mean temperature difference with arithmetic and geometric mean temperature differences.

logarithmic mean temperature difference is always negative. Thus, the predicted heat transfer using this approximation would always be conservative; i.e., it would under-predict the actual heat transfer. We also note from Table 2 that, as we had shown analytically, the error resulting from the use of the geometric mean approximation approaches 100% in magnitude as *AF* goes to zero and approaches zero as *AF* approaches unity.

The ratio of the error from using the arithmetic mean and geometric mean approximations is also given in Table 2. Because the error from the arithmetic mean approximation becomes infinite and the error from the geometric mean approximation approaches –100% as the approach factor *AF* goes to zero, their ratio approaches zero at that point. Thus, the geometric mean approximation is infinitely better than the arithmetic mean approximation at that point. Since neither approximation is acceptable near that point, this observation is of little use. However, it is of interest to note that the ratio of errors approaches 1/2 as *AF* approaches unity. Although this

Table 2. Errors from approximating the logarithmic mean temperature difference.

Approach factor AF	*Arithmetic mean error* $\varepsilon_a(\%)$	*Geometric mean error* $\varepsilon_g(\%)$	*Ratio of errors* $\varepsilon_g/\varepsilon_a$
0.0	+ ∞	–100.0	– 0.00
0.0001	361.0	–90.8	– 0.25
0.001	246.0	–78.1	– 0.32
0.01	135.0	–53.4	– 0.40
0.1	40.7	–19.1	– 0.47
0.2	20.7	–10.0	– 0.48
0.3	11.8	–5.79	– 0.49
0.4	6.90	–3.41	– 0.495
0.5	3.97	–1.97	– 0.497
0.6	2.17	–1.08	– 0.498
0.7	1.058	– 0.528	– 0.4992
0.8	0.415	– 0.207	– 0.4997
0.9	0.0925	– 0.0462	– 0.4999
1.0	0.0	0.0	→ – 0.500...

is seemingly apparent from Table 2, we can prove this analytically by using l'Hôpital's rule. In this case it becomes necessary to take successive derivatives up to the third derivatives in order to arrive at an expression that is not indeterminate. This is an acceptable application of l'Hôpital's rule, however. The analysis is presented briefly below.

$$\frac{\varepsilon_g}{\varepsilon_a} = \frac{AF^{1/2}\ln(AF) - AF + 1}{[(AF+1)\ln(AF)/2] - AF + 1}$$

Let

$$\varepsilon_g = f(AF) = AF^{1/2}\ln(AF) - AF + 1$$

$$\varepsilon_a = g(AF) = [(AF + 1)\ln(AF)/2] - \mathrm{AF} + 1$$

$$f'(AF) = [AF^{1/2}\ln(AF)/2] + AF^{-1/2} - 1$$

$$g'(AF) = \ln(AF)/2 + 1/2AF - 1/2 .$$

In the limit as $AF \to 1$, we see that both $f'(AF)$ and $g'(AF)$ approach zero; thus, we still have an indeterminate expression. Applying l'Hôpital's rule to that expression

$$f''(AF) = -[AF^{-3/2}\ln(AF)]/4$$

$$g''(AF) = (AF^{-1} - AF^{-2})/2 .$$

Again, we see that both $f''(AF)$ and $g''(AF)$ approach zero as AF approaches unity and we are left with another indeterminate expression. Once again we take derivatives so that we can apply l'Hôpital's rule

$$f'''(AF) = [(3AF^{-5/2}\ln(AF))/8] - AF^{-5/2}/4$$

$$g'''(AF) = AF^{-3} - AF^{-2}/2$$

$$\lim_{AF \to 1} \left(\varepsilon_g + \varepsilon_a\right) = \lim_{AF \to 1} \left(f'''(AF)/g'''(AF)\right) = (-1/4)/(1/2) = -1/2$$

And we now have our desired result.

The significance of this result is that we now know that the error from using the geometric mean temperature difference as an approximation to the logarithmic mean temperature difference is always 50% or less of the error that would result from using the arithmetic mean temperature difference. For applications where the use of the logarithmic mean temperature difference is undesirable, use of the geometric mean temperature difference will result in errors of less than 5% for values of AF greater than 0.33. As Soumerai (1987) points out, given the other uncertainties in heat exchanger design calculations, errors of this magnitude are certainly acceptable. Most heat exchanger designs will have approach factors greater than 0.33 and thus our findings here should be applicable in the majority of cases.

Now that we have this approximation for the logarithmic mean temperature difference, we can construct a simple model for the consumer's heat exchanger using it. Equation 3-1 will be used to construct our model. For our model two possible cases exist, dependent on the values of the empirical parameters n_1 and n_2. Here, we will only address the simpler case where $n_1 = n_2$. This is the result that occurs for "high radiators" according to Bøhm (1988), in which case $n_1 = n_2 = 1.3$. In that case eq 3-1 becomes

$$q_2 / q_0 = \left[(T_{m1})_2 \big/ (T_{m1})_0\right]^{n_1} . \qquad (3\text{-}15)$$

Our model of the consumer's heat exchanger should accept as input the supply temperature to the radiators and the heat load q_2. The values of the operating parameters at the design "0" condition will also be needed. The output from the model will be the return temperature. We would like this model to be as accurate as possible while being in a simple form, thus allowing ease in both analytical and numerical procedures involving it. As stated earlier, it is not possible to obtain an explicit model if the log mean temperature difference is used. Thus, we will proceed using our geometric mean approximation for the log mean temperature difference. Making this substitution, we have

$$q_2 / q_0 = \left[\left(T_{\mathrm{mg}}\right)_2 / \left(T_{\mathrm{mg}}\right)_0\right]^{n_1}. \tag{3-16}$$

Note that the log mean temperature difference at the design 0 condition is also being approximated with the geometric mean temperature difference. Since all of the temperatures are known at the design condition, we could have evaluated the log mean temperature difference and used the result here and still achieved an explicit result. However, to ensure that no error occurs in the resultant model at the design condition, we have used the geometric mean approximation. This will also reduce the errors at the "off-design" (2) condition. The same procedure has been adopted for the model using the arithmetic mean temperature difference.

To obtain our model for the return temperature as a function of the load and supply temperature, we simply solve eq 3-16 for the return temperature at the 2 condition (actual load). The result is

$$(T_{\mathrm{r}})_2 = T_{\mathrm{a}} + \left[(T_{\mathrm{s}} - T_{\mathrm{a}})_2^{-1} (T_{\mathrm{mg}})_0^2 (q_2 / q_0)^{2/n_1}\right]. \tag{3-17}$$

We can also obtain a model for the return temperature using the arithmetic mean approximation to the log mean temperature difference. It is

$$(T_{\mathrm{r}})_2 = 2\left\{T_{\mathrm{a}} + \left[(T_{\mathrm{ma}})_0 (q_2 / q_0)^{1/n_1}\right]\right\} - (T_{\mathrm{s}})_2. \tag{3-18}$$

To evaluate the performance of our models that use approximations to the log mean temperature difference, we need a model that uses the log mean temperature difference. As noted earlier, this model will be implicit and thus will require solution by an iterative numerical method of some type. The model can be arranged in several forms for numerical solution, one being

$$(T_{\mathrm{r}})_2 = T_{\mathrm{a}} + \left\{(T_{\mathrm{s}} - T_{\mathrm{a}})_2 / \exp\left[(q_2 / q_0)^{-1/n_1} (T_{\mathrm{s}} - T_{\mathrm{r}})_2 / (T_{\mathrm{m1}})_0\right]\right\}. \tag{3-19}$$

A number of iterative methods can be used to solve this implicit equation for the return temperature $(T_r)_2$. Most iterative methods are very sensitive to the quality of the initial estimate. Here, we are fortunate to have the geometric mean temperature difference approximation that can be used to obtain the initial estimate. Figure 5 below shows some of the results for the three models developed. In addition more detail as well as numerical values are given in Table 3. It is clear from Figure 5 that the model using the arithmetic mean temperature difference is unacceptable for most values of the load ratio q/q_0, while the model using the geometric mean temperature difference is acceptable over the entire range of values given for q/q_0. The results for the model using the log mean temperature difference were obtained

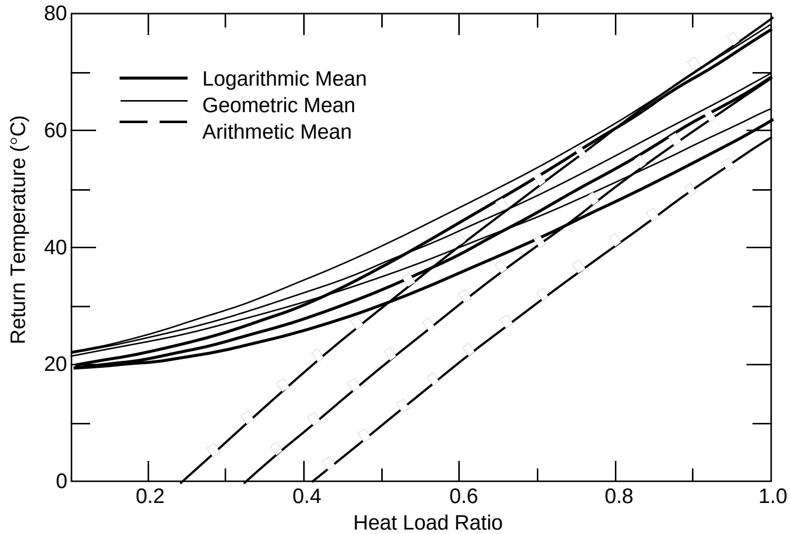

Figure 5. Results for the return temperature models. Top family is for T_s = *80°C; middle family is for* T_s = *90°C; bottom family is for* T_s = *100°C.*

by using the geometric mean approximation as an initial estimate, then proceeding with a simple "iterative improvement" method where the previous estimate of $(T_r)_2$ was substituted into the right-hand side of eq 3-19, which yielded the next estimate. This procedure is repeated until successive estimates of $(T_r)_2$ varied by no more than the prescribed tolerance.

For each of the consumer models, we can easily develop an expression to calculate the flow rate for any given load condition by starting with the heat balance for the radiator. We assume that the mass of the radiator is negligible so that with the gradual temperature changes typical of these systems, conditions are very close to steady state. Treating the radiator as a control volume, we have

$$q = \dot{m}\, c_p (T_s - T_r) \tag{3-20}$$

where c_p is the specific heat of water at constant pressure (kJ/kg °C).

Thus, our mass flow rate relative to the mass flow rate at the design condition is given by

$$\dot{m} / \dot{m}_d = (q/q_d)(T_s - T_r)_d / (T_s - T_r)_2. \tag{3-21}$$

Notice that we have used the *d* subscript to denote the design condition for the piping system rather than the 0 subscript used to denote the design condition for the consumer's radiators. If both the network piping and the consumer's radiators are designed for the same maximum load condition, then it would not be necessary to distinguish between these two conditions. However, in most cases this will not be the case. For the piping network, little or no over-design is desirable in order to keep costs at a minimum. In fact, diversity of demand between consumers will allow the network to be designed for a total maximum demand of less than the sum of the individual demands, as will be discussed later. The consumer's radiators, on the other hand, will always be somewhat oversized. In addition to the normal conservatism in design, quick recovery from night setback and other off periods also favors significant over-design. Relative mass flow rates calculated using eq 3-21 for each of the consumer models are given in Table 3. The results in Table 3 assume the same design condition for the piping network and the consumer's radiators. Some examples with differing design conditions will be given later.

Table 3. Return temperatures and flow rates calculated with the consumer models for $n_1 = n_2 = 1.3$.

Supply temp. (°C)	*Heat loss ratio* (q/q_o)	*GMTD rtn. temp. (°C)*	*AMTD rtn. temp. (°C)*	*LMTD rtn. temp. (°C)*	*GMTD AF*	*AMTD AF*	*LMTD AF*	*GMTD error*	*AMTD error*	*LMTD* m/m_d	*GMTD* m/m_d	*AMTD* m/m_d	*GMTD* m/m_d *error*	*AMTD* m/m_d *error*
100	1	63.75	60.00	62.75	0.55	0.50	0.53	–0.0160	0.0438	0.5369	0.5517	0.5000	–0.0277	0.0687
100	0.9	57.20	50.66	55.56	0.47	0.38	0.44	–0.0295	0.0883	0.4051	0.4206	0.3648	–0.0383	0.0994
100	0.8	51.04	41.07	48.73	0.39	0.26	0.36	–0.0473	0.1572	0.3121	0.3268	0.2715	–0.0471	0.1300
100	0.7	45.27	31.21	42.31	0.32	0.14	0.28	–0.0700	0.2625	0.2427	0.2558	0.2035	–0.0541	0.1614
100	0.6	39.94	21.01	36.39	0.25	0.01	0.20	–0.0974	0.4227	0.1887	0.1998	0.1519	–0.0590	0.1948
100	0.5	35.06	10.41	31.09	0.19	–0.12	0.14	–0.1277	0.6653	0.1451	0.1540	0.1116	–0.0611	0.2309
100	0.4	30.68	–0.70	26.57	0.13	–0.26	0.08	–0.1550	1.0263	0.1089	0.1154	0.0794	–0.0594	0.2708
100	0.3	26.86	–12.47	23.04	0.09	–0.41	0.04	–0.1657	1.5411	0.0780	0.0820	0.0533	–0.0522	0.3158
100	0.2	23.68	–25.21	20.81	0.05	–0.57	0.01	–0.1379	2.2113	0.0505	0.0524	0.0319	–0.0376	0.3675
100	0.1	21.27	–39.58	20.03	0.02	–0.74	0.00	–0.0618	2.9763	0.0250	0.0254	0.0143	–0.0157	0.4271
95	1	66.67	65.00	66.20	0.62	0.60	0.62	–0.0071	0.0181	0.6944	0.7059	0.6667	–0.0165	0.0400
95	0.9	59.68	55.66	58.62	0.53	0.48	0.51	–0.0181	0.0505	0.4948	0.5097	0.4575	–0.0301	0.0753
95	0.8	53.11	46.07	51.38	0.44	0.35	0.42	–0.0335	0.1034	0.3668	0.3819	0.3270	–0.0411	0.1086
95	0.7	46.96	36.21	44.55	0.36	0.22	0.33	–0.0540	0.1874	0.2775	0.2914	0.2381	–0.0500	0.1420
95	0.6	41.27	26.01	38.22	0.28	0.08	0.24	–0.0798	0.3195	0.2113	0.2233	0.1739	–0.0567	0.1770
95	0.5	36.07	15.41	32.49	0.21	–0.06	0.17	–0.1099	0.5258	0.1600	0.1697	0.1256	–0.0606	0.2147
95	0.4	31.40	4.30	27.55	0.15	–0.21	0.10	–0.1397	0.8438	0.1186	0.1258	0.0882	–0.0605	0.2563
95	0.3	27.32	–7.47	23.62	0.10	–0.37	0.05	–0.1569	1.3163	0.0841	0.0887	0.0586	–0.0547	0.3034
95	0.2	23.92	–20.21	21.03	0.05	–0.54	0.01	–0.1378	1.9610	0.0541	0.0563	0.0347	–0.0408	0.3579
95	0.1	21.35	–34.58	20.05	0.02	–0.73	0.00	–0.0651	2.7253	0.0267	0.0272	0.0154	–0.0177	0.4216
90	1	70.00	70.00	70.00	0.71	0.71	0.71	0.0000	0.0000	1.0000	1.0000	1.0000	0.0000	0.0000
90	0.9	62.52	60.66	62.00	0.61	0.58	0.60	–0.0084	0.0216	0.6429	0.6550	0.6135	–0.0188	0.0457
90	0.8	55.47	51.07	54.33	0.51	0.44	0.49	–0.0210	0.0600	0.4486	0.4634	0.4110	–0.0330	0.0837
90	0.7	48.88	41.21	47.06	0.41	0.30	0.39	–0.0388	0.1243	0.3260	0.3405	0.2869	–0.0444	0.1199
90	0.6	42.79	31.01	40.27	0.33	0.16	0.29	–0.0625	0.2300	0.2413	0.2542	0.2034	–0.0533	0.1570
90	0.5	37.21	20.41	34.09	0.25	0.01	0.20	–0.0917	0.4013	0.1789	0.1894	0.1437	–0.0592	0.1966
90	0.4	32.21	9.30	28.68	0.17	–0.15	0.12	–0.1231	0.6756	0.1305	0.1384	0.0991	–0.0611	0.2401
90	0.3	27.84	–2.47	24.30	0.11	–0.32	0.06	–0.1460	1.1017	0.0913	0.0965	0.0649	–0.0571	0.2895
90	0.2	24.20	–15.21	21.30	0.06	–0.50	0.02	–0.1363	1.7139	0.0582	0.0608	0.0380	–0.0441	0.3470
90	0.1	21.45	–29.58	20.07	0.02	–0.71	0.00	–0.0686	2.4741	0.0286	0.0292	0.0167	–0.0201	0.4152
85	1	73.85	75.00	74.17	0.83	0.85	0.83	0.0044	–0.0112	1.8471	1.7931	2.0000	0.0292	–0.0828
85	0.9	65.79	65.66	65.75	0.70	0.70	0.70	–0.0006	0.0014	0.9351	0.9370	0.9306	–0.0020	0.0048
85	0.8	58.20	56.07	57.61	0.59	0.55	0.58	–0.0102	0.0268	0.5843	0.5970	0.5531	–0.0218	0.0533
85	0.7	51.11	46.21	49.86	0.48	0.40	0.46	–0.0249	0.0734	0.3985	0.4131	0.3609	–0.0366	0.0943
85	0.6	44.54	36.01	42.59	0.38	0.25	0.35	–0.0458	0.1545	0.2829	0.2966	0.2449	–0.0482	0.1343
85	0.5	38.54	25.41	35.91	0.29	0.08	0.24	–0.0733	0.2924	0.2037	0.2152	0.1678	–0.0566	0.1762
85	0.4	33.15	14.30	29.99	0.20	–0.09	0.15	–0.1053	0.5231	0.1454	0.1543	0.1132	–0.0609	0.2219
85	0.3	28.45	2.53	25.11	0.13	–0.27	0.08	–0.1331	0.8992	0.1002	0.1061	0.0728	–0.0591	0.2737
85	0.2	24.53	–10.21	21.65	0.07	–0.46	0.03	–0.1331	1.4715	0.0631	0.0661	0.0420	–0.0476	0.3346
85	0.1	21.56	–24.58	20.11	0.02	–0.69	0.00	–0.0722	2.2228	0.0308	0.0315	0.0183	–0.0229	0.4078
80	1	78.33	80.00	78.49	0.97	1.00	0.97	0.0020	–0.0192	13.2420	12.0000	∞	0.0938	∞
80	0.9	69.60	70.66	69.90	0.83	0.84	0.83	0.0043	–0.0108	1.7827	1.7315	1.9268	0.0287	–0.0809
80	0.8	61.38	61.07	61.29	0.69	0.68	0.69	–0.0015	0.0036	0.8553	0.8594	0.8454	–0.0048	0.0116
80	0.7	53.70	51.21	53.02	0.56	0.52	0.55	–0.0127	0.0343	0.5190	0.5323	0.4862	–0.0256	0.0631
80	0.6	46.58	41.01	45.22	0.44	0.35	0.42	–0.0302	0.0931	0.3450	0.3591	0.3078	–0.0409	0.1079
80	0.5	40.08	30.41	37.99	0.33	0.17	0.30	–0.0551	0.1996	0.2380	0.2505	0.2016	–0.0524	0.1529
80	0.4	34.25	19.30	31.52	0.24	–0.01	0.19	–0.0866	0.3876	0.1650	0.1748	0.1318	–0.0596	0.2012
80	0.3	29.15	7.53	26.07	0.15	–0.21	0.10	–0.1181	0.7112	0.1113	0.1180	0.0828	–0.0606	0.2559
80	0.2	24.90	–5.21	22.08	0.08	–0.42	0.03	–0.1277	1.2357	0.0691	0.0726	0.0469	–0.0512	0.3203
80	0.1	21.69	–19.58	20.16	0.03	–0.66	0.00	–0.0757	1.9714	0.0334	0.0343	0.0201	–0.0262	0.3991
Averages								–0.0700	0.6902				0.0360	0.1899

Table Nomenclature: LMTD = log mean temperature difference (°C); GMTD = geometric mean temperature difference (°C); AMTD = arithmetic mean temperature difference (°C); m/m_d = ratio of mass flow rate to design condition mass flow rate.
Assumed conditions: at "0" condition T_s = 90°C, T_r = 70°C; for all calculations T_a= 20°C, $n_1 = n_2 = 1.3$.
Tolerance for iterative calculation of T_r with LMTD model was < 0.01°C.

As expected, the results for the normalized mass flow rates in Table 3, which use the geometric mean approximation for the log mean temperature difference, are much better than those obtained using the arithmetic mean approximation. Clearly, the errors induced by the arithmetic mean approximation are unacceptable for any load–supply temperature condition that deviated significantly from the design condition for the radiators. Over the range of supply temperatures and heat loads given in the Table 3, the average error in the return temperature obtained using the geometric mean approximation is only 7% compared to 69% for the results obtained using the arithmetic mean approximation. Also, note that the errors in approximation of the mass flow rate average only 3.6% for the model using the geometric mean approximation, while the average error for the model using the arithmetic mean approximation is about 19%.

In addition, it should be noted that the model based on the arithmetic mean approximation in a number of instances at lower loads results in physically impossible return temperatures, i.e., ones lower than the room air temperature of 20°C. Note that at lower loads (lower q/q_0 values), the errors of approximation tend to be larger. This is predicted by our error analysis carried out earlier, since the approach factor is lower in these cases. Also note that, like our basic geometric mean approximation, our model based on it is conservative and under-predicts the heat transfer on the average. An exception sometimes occurs at supply temperatures lower than the design condition of 90°C. Since our model for the radiator predicts the return temperature based on the return temperature at the design 0 condition, the error in approximation of the return temperature at the design condition has an effect on the error in our model at conditions other than the design condition. At lower supply temperatures and high loads, for example T_s = 85°C and q/q_0 = 1.0, the approach factor is 0.83 for the geometric mean approximation and is thus higher than the approach factor of 0.71 encountered at the design condition. Because the error in the geometric mean approximation decreases with increasing approach factor, our model for the prediction of the return temperature actually under-predicts it slightly at that point. This under-prediction is not, however, a cause for concern, since it is so slight and in addition it exists at a load–temperature condition that would not normally be encountered because it would require mass flow rates greater than the design condition.

DESIGN OF A SINGLE PIPE SEGMENT WITH A CONSUMER MODEL

In Chapter 2 we developed a methodology to determine the optimal pipe diameter for a single pipe segment. In the example given, it was assumed that both the supply temperature and return temperature were constant over the entire yearly cycle. This is of course not the case, and now that we have a simple model for the consumer's space heating substation, we can examine what the effect is of coupling this model with our design methodology.

First, let's consider what the effect is of assuming a constant supply temperature, as we had done earlier, but rather than assuming a constant return temperature as well, let this be determined by our consumer model. The varying return temperature will affect the heat losses by altering the A_1 parameter (eq 2-5) to some degree; this will be addressed later. The primary effect, however, will be on the mass flow rate. The heat load will be assumed to vary sinusoidally, as before, except now the variation in the mass flow rate will not be sinusoidal itself, but will be determined by the load and the return temperature from the consumer model. The relationship between the mass flow rate, load, design supply–return temperatures and the actual supply–return temperatures was given by eq 3-21. If we substitute our expression for the return temperature as determined using the geometric mean approximation (eq 3-17) into eq 3-21, we have

$$\dot{m} / \dot{m}_{\mathrm{d}} = (q/q_{\mathrm{d}})\,(T_{\mathrm{s}} - T_{\mathrm{r}})_{\mathrm{d}} / \{T_{\mathrm{s}} - T_{\mathrm{a}} - \left[(T_{\mathrm{s}} - T_{\mathrm{a}})^{-1}(T_{\mathrm{mg}})_0^2 (q/q_0)^{2/n_1}\right]\} . \quad (3\text{-}22)$$

Notice that we have used both q/q_{d} and q/q_0 in this expression. In both cases q represents the actual load on the system. The quantity q_0 represents the maximum load for which the consumers' radiators were designed, while the quantity q_{d} represents the maximum load for which the piping network was designed. These two "design" loads will in most cases not be equal, as discussed earlier. They will, however, be related by some "over-design factor," which will be a constant

$$(q/q_{\mathrm{d}}) = A_{13}\,(q/q_0) \quad (3\text{-}23)$$

where A_{13} is the over-design factor for the consumer's radiators (dimensionless).

In the example of Chapter 2, we assumed a sinusoidal form for the variation of the load over the yearly cycle. In general terms this can be written as

$$q/q_{\mathrm{d}} = A_{14} + [A_{15} \cos(2\pi t / 8760)] \quad (3\text{-}24)$$

where A_{14} is the midpoint of the load curve (dimensionless) and A_{15} is the amplitude of the load curve (dimensionless). The midpoint of the load curve A_{14} is simply the average of the maximum and minimum loads. The amplitude of the load curve A_{15} is the maximum load minus the minimum load divided by two.

Now if we combine eq 3-22–3-24, we have the following expression for the normalized mass flow rate over the yearly cycle

$$\dot{m} / \dot{m}_{\mathrm{d}} = \frac{(T_{\mathrm{s}} - T_{\mathrm{r}})_{\mathrm{d}} \left(A_{14} + A_{15} \cos(2\pi t / 8760)\right)}{T_{\mathrm{s}} - T_{\mathrm{a}} - \left[(T_{\mathrm{s}} - T_{\mathrm{a}})^{-1}(T_{\mathrm{mg}})_0^2 \left(\left[A_{14} + A_{15}\cos(2\pi t / 8760)\right] / A_{13}\right)^{2/n_1}\right]} . \quad (3\text{-}25)$$

If we select the same values for A_{14} and A_{15} as we used in the Chapter 2 example (0.575 and 0.425 respectively), we can compare the resulting mass flow rate function of eq 3-25 to the mass flow rate function without the consumer model (eq 2-28). We

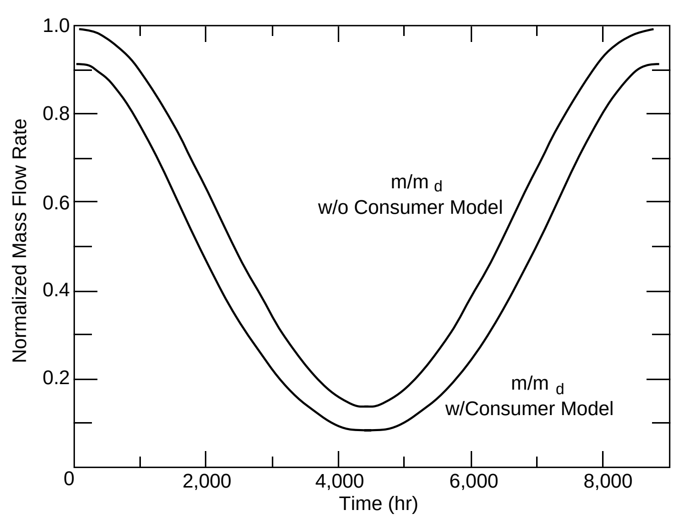

Figure 6. Mass flow rate function with and without consumer models, $T_r = 60°C$.

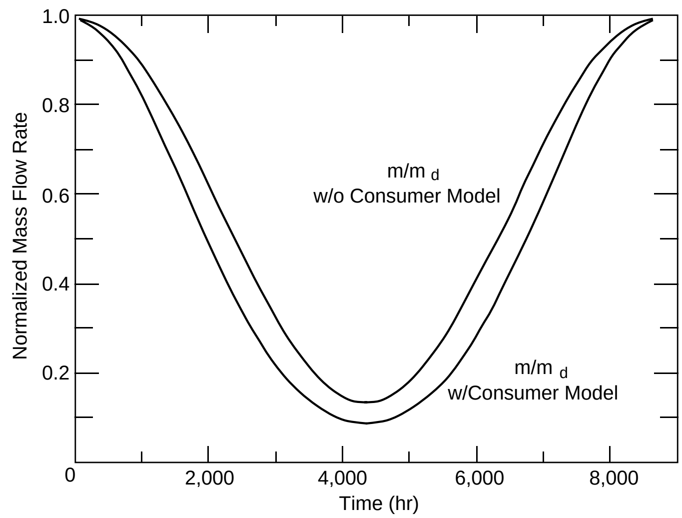

Figure 7. Mass flow rate function with and without consumer models, $T_r = 55°C$.

have done so in Figure 6. Notice, for the case where the consumer model has been included, that the maximum value of the normalized mass flow rate is less than unity. This results from an inconsistency in the design conditions chosen for the original problem of Chapter 2 and the consumer's radiator model. Our model for the consumer's radiators assumes design temperatures of 90 and 70°C for the supply and return respectively. With the supply temperature of 120°C, the radiator model will predict a return temperature of 55°C, rather than the value of 60°C we assumed earlier. In the case where the return temperature is assumed rather than determined by a consumer model, the choice of 60°C is entirely appropriate, as are the normal design temperatures of 90 and 70°C for supply and return radiator temperatures respectively. Normal design practice would be to assign these temperatures independently. Thus, the lower flow rate that results when we include the effect of the consumer model illustrates one of the inaccuracies encountered when normal design practice is followed. If we make the design return temperature for the piping network equal to that which results from the consumer's radiator model, i.e., 55°C, it results in the normalized mass flow rates shown in Figure 7. From Figure 7 it is clear that the effect of including the consumer model on the mass flow rate is still significant once the load condition drops slightly from its maximum value. Averaged over all load conditions, the normalized mass flow rate is 20% less for the case that includes the consumer model.

To determine what effect this change in normalized mass flow rate will have on our optimal pipe diameter determined in the example of Chapter 2, we need to recompute I_3, which is the only parameter affected by the changes in the mass flow rate. We have modified the computer program used in Chapter 2 to calculate I_3 by including eq 3-25 in place of the original normalized mass flow rate as given by eq 2-29. The modified program is included in Appendix B as Program I2-C-GMT. All constants were assumed to have the same values as before, with A_{13} taken as unity. This results in the value of the I_3 parameter decreasing by 15% from 44.1 to 37.5 ($ $m^{5.095}$) because of the effect of the consumer model.

Now we need to determine what other parameters in the solution of the Chapter 2 example would be affected by our consumer model. The only additional effect will be on the I_1 parameter. Since this parameter arises out of heat loss considerations, the varying return temperature caused by the consumer model will change it somewhat. In the example of Chapter 2, since the supply and return temperature were both constant over the yearly cycle, the integration needed to find I_1 (see eq 2-5) is unnecessary. With the varying return temperature produced by our consumer model, however, we will need to carry out this integration. Assuming that the cost of heat C_h is constant, our new equation for I_1 becomes

$$I_1 = A_{16}\left\{\left(\frac{(T_s + T_a)}{2} - T_m\right)A_t + \frac{(T_{mg})_0^2}{2(T_s - T_a)}\int_0^{8760}\left[\left(A_{14} + A_{15}\cos(2\pi t/8760)\right]/A_{13}\right)^{2/n_1} dt\right\} \tag{3-26}$$

where $A_{16} = PVF_h\, L\, C_h\, 4\pi k_i$ (\$/[°C hr]).

Because the cosine function is raised to a non-integer power, it is not possible to carry out the integration in eq 3-26 analytically. Once again we have used the Romberg method of numerical integration to evaluate the integral. The calculation of I_1 was done using the FORTRAN program I1EQ3-26, which is included in Appendix B. Using the parameter values assumed earlier in this section, we obtain $I_1 = \$7.33 \times 10^4$. Thus, we find that including the consumer model reduces the value of the I_1 parameter by 14.4% from 8.56×10^4 to 7.33×10^4.

Now that we have new values for the parameters that are affected by the consumer model, we can recompute the optimal diameter for the sample application given in Chapter 2. We proceed as before, i.e., before solving eq 2-24 to determine the optimum diameter, we first find an approximate solution using eq 2-20, which neglects the heat losses. From eq 2-20 we solve for the diameter directly, obtaining $d = 0.210$ m. Using this value of d as an initial estimate, we can proceed to solve eq 2-24. We know that the solution to eq 2-24, which includes heat losses, will be a smaller diameter than the solution to eq 2-20, which does not include heat losses, since heat losses are an increasing function of the diameter. Guided by the value obtained above, a simple trial-and-error method was once again used here. This method yielded a solution to three significant digits with only four function evaluations. The optimal diameter d was found to be 0.203 m. The total cost for this design is found to be $C_t = \$1.064 \times 10^6$ using eq 2-19. By coincidence, the optimal diameter we have found also is one of the standard discrete diameter pipes available; thus, it is not necessary for us to compute total costs for other discrete diameters as before.

The addition of the consumer model has changed the optimal diameter from 0.208 to 0.203 m, a decrease of only 2.5%. The optimal discrete diameter remains unchanged. While in this particular case, the inclusion of the consumer model had no net effect on the choice of optimal discrete diameter, this obviously will not always be the case.

The cost predicted for any pipe diameter is also changed slightly by the addition of the consumer. The total cost with and without the consumer model is given in

Table 4. Pressure drops and costs for discrete pipe sizes under maximum flow conditions with and without the consumer model (pipe data from Marks 1978).

Nominal pipe size (in.)	*Inside diameter schedule 40 (in.)*	*(m)*	ΔP_d *(Pa/m)*	C_t, *w/o consumer model* ($\$ \times 10^6$)	C_t, *with consumer model* ($\$ \times 10^6$)
—	8.187	0.208	340	1.111	1.065
8	7.981	0.203	384	1.112	1.064
10	10.020	0.255	120	1.178	1.140
12	11.938	0.303	50	1.305	1.267

Table 4 for each optimal diameter and the discrete diameters found when using the rule of thumb based design method. Pressure drops at maximum flow conditions are also given in Table 4. Note that these are unchanged from those in Table 1, since the maximum flow condition remains the same. Thus, the rule of thumb based design would remain the same and a 12-in. nominal diameter pipe would be required. The cost saving of the optimal discrete design increases slightly once the consumer model is added. Now the rule of thumb based design is 19% more costly than the optimal discrete design. Also, note that the total life cycle costs are reduced in all cases when the consumer model is added. Since it is important to have accurate cost predictions when comparing district heating to alternatives, these seemingly minor changes in total life cycle cost can be significant. For instance, the total life cycle cost of our optimal discrete diameter design decreases 4% with the addition of the consumer model. This is a very significant cost reduction. In our example 1-km-long pipe segment with a design capacity of 25 MW, this refinement in predicted life cycle cost amounts to $48,000. Note that since our optimal discrete diameter is unchanged by the addition of the consumer model, the capital cost of this design is unchanged as well. Thus, the optimal discrete design still represents a 30% reduction in capital costs from the rule of thumb based design.

HEAT CONSUMER CONSTRAINTS

Before leaving the topic of the consumers, let's consider the constraints that they place on the design. The consumers of heat place two very basic requirements on the heat supply system:

1. That the delivered temperature of the heat be high enough to meet their requirements.
2. That their heat demand be met at all times.

The first requirement will simply result in the following inequality constraint

$$T_{s,i} \geq T_{smin,i} \tag{3-27}$$

where $T_{s,i}$ is the supply temperature at the heat consumer i (°C) and $T_{smin,i}$ is the minimum supply temperature required by heat consumer i (°C).

Satisfaction of the second requirement will result in an equality constraint that must be obeyed at each heat consumer. This constraint will be based on the model developed in the previous section. The load placed on the system by the consumer will be known, expressed as a fraction of the load under the design condition, i.e., q/q_0. The supply temperature will also be known. The model for the consumer heat exchanger then becomes our constraint on the return temperature. Equation 3-17 is modified slightly by removing the 2 subscript, the unsubscripted values now representing the actual operating condition

$$T_r = T_a\left[(T_s - T_a)^{-1}(T_{mg})_0^2(q/q_0)^{2/n_1}\right]. \tag{3-28}$$

There is also an additional equality constraint on the mass flow rate that results from eq 3-21

$$\dot{m}/\dot{m}_d = (q/q_d)(T_s - T_r)_d/(T_s - T_r). \tag{3-29}$$

In the next chapter, we will examine how these and other constraints interact when multiple consumer designs are considered.

CHAPTER 4: CONSTRAINTS ON SYSTEMS WITH MULTIPLE CONSUMERS AND PIPES

All district heating systems, with the exception of pure transmission systems, will have multiple consumers and pipes. If each pipe were independent of the others, it would be possible to apply the procedure developed earlier to each pipe and create a complete design in that way. Of course each pipe segment does not operate independently of the others and the system can thus not be designed completely in that way for all but the most trivial cases. Many constraints are imposed on the design by the physical process involved in the network, the consumer's requirements and physical limitations of the piping. As we shall see, many of these constraints will be inactive at the optimum system design and thus they can be relaxed. Our task is then to formulate these constraints into mathematical expressions, and then identify those that must be active and use this information to develop a solution methodology. All of this must be done with the minimum amount of computational effort so as not to render the method intractable for large networks, which often have hundreds or thousands of piping segments.

SYSTEM CONSTRAINTS

Constraints on the design of a heat distribution system originate from limitations imposed in several distinct areas. Before we begin to formulate constraints into a form suitable for inclusion in our problem, let's consider where and why these constraints arise. The source of constraints can be grouped into three basic categories:

1. Physical limitations of the piping systems.
2. Fluid dynamic and thermodynamic considerations for the network, consumers, and heat source.
3. Requirements dictated by the consumer's equipment or processes.

In some instances considerations from each of these categories are coupled together into a single constraint or set of constraints. Thus, as we formulate the constraints below, we will address considerations from each of the categories above and their interaction.

DIFFERENTIAL PRESSURE CONSTRAINTS

A very important set of constraints on the system arises from requirements for the pressure difference between supply and return. At the consumer this differential pressure must maintain a minimum level to ensure adequate flow through the consumer's heat exchanger. This pressure differential is consumed in both the heat exchangers and control valves. In the heat exchanger, the pressure losses are caused by fluid dynamic friction. In the control valve, the pressure losses are introduced by a throttling process used to control the flow rate through the heat exchanger and thus control its output. In the supply piping between the heat source and the consumer, pressure losses occur due to friction. Similarly, in the return line from the consumer back to the heat source, pressure losses also occur. There is then, in effect, a requirement at each point in the piping network for a given differential between supply and return pressure necessary to overcome downstream losses, including those in the return system. Ultimately, at the heating plant pumps must be used to provide the total differential pressure needed downstream of that point. In theory it's possible that pumps can be placed anywhere in the system or even dispersed throughout. In practice this is not done very frequently, owing to the practical considerations of monitoring, controlling and maintaining the pumps as well as availability of power for them. Here, we will assume that all pumps are located at one central heating plant, although some very interesting optimal design questions arise if this limitation is removed, as we will see later.

We can write the constraint that arises from all of these differential pressure requirements easily by summing the pressure drops and increases around the system. Since the entire district heating system, consisting of the heating plant, the piping system and the consumer, forms an essentially closed loop, the pressure losses and increases around this loop must sum to zero. Thus, we have the following result

$$\Delta P_{\mathrm{hp}} = \sum_{pipes}\left(\Delta P_{\mathrm{s}} + \Delta P_{\mathrm{r}}\right) + \Delta P_{\mathrm{cv}} + \Delta P_{\mathrm{he}} \tag{4-2}$$

where ΔP_{hp} = pressure increase across the pump (N/m^2)
ΔP_{s} = pressure drop in the supply piping (N/m^2)
ΔP_{r} = pressure drop in the return piping (N/m^2)
ΔP_{cv} = pressure drop in the consumer control valves (N/m^2)
ΔP_{he} = pressure drop in the consumer heat exchangers (N/m^2).

Each consumer will have at least one segment of the piping system that is not shared with any other consumers. In addition, all consumers will have their own control valve and heat exchanger. Thus, we will have one of these equations for each consumer, each one representing a constraint on the design. Therefore, the summation in eq 4-2 must be conducted over only the pipes that serve the consumer in question.

The pressure losses given above will vary with the flow rate in the system. In many cases, flow rates in district heating networks are modulated over the course of the year as a means of meeting varying loads. Flow can be modulated either by using variable speed pumps or using what is called a "shunt" at the heating plant. The shunt simply diverts a fraction of the flow from the pump back to its inlet. The pressure increase across the pump is reduced as is the flow rate into the network. Regardless of how it is done, if flow modulation is used, we must ensure that the constraint of eq 4-2 is not only satisfied for each consumer, but in addition we must also determine that this will be the case for all load (i.e., flow) conditions encountered. However, we will show later, after some other necessary constraints have been introduced, that satisfying this set of constraints for only one load condition will be sufficient, if we assume that all consumers have loads that vary in the same manner. This is a reasonable assumption as long as the primary loads are space and hot water heating, as is the case for most hot water based systems. Steam systems, which are not addressed by this work, often have much larger fractions of industrial and absorption air conditioning loads; thus, this assumption might not be reasonable for them.

In addition to the above equality constraint (eq 4-2), we have both equality and inequality constraints on each of the quantities appearing within that constraint. At this point we will formulate each of these additional constraints.

At the heating plant, the pressure increase by the pump must be related to the pumping power attributable to the consumer in question. This results in the following expression

$$\Delta P_{\mathrm{hp}} = PP_{\mathrm{f,i}}\,\rho / \dot{m}_{\mathrm{i}} \tag{4-3}$$

where the i subscript is the consumer index. The combined pressure loss of the supply and return piping is simply given by

$$\Delta P_{\mathrm{s}} + \Delta P_{\mathrm{r}} = \Delta P_{\mathrm{s\&r}} = a\,\varepsilon_{\mathrm{j}}^{b}\left(4/\pi\right)^{2+c} A_{6,\mathrm{j}}\,\dot{m}_{\mathrm{j}}^{2+c} L_{\mathrm{j}}\, d^{\,-(5+b+c)} \tag{4-4}$$

where the j subscript is the pipe segment index and $\Delta P_{s\&r}$ is the combined pressure loss of supply and return (N/m^2).

The control valve will have varying amounts of pressure drop across it, depending on the consumer's load. The minimum pressure drop for any given flow rate condition will occur when the control valve is completely open and the flow rate through the consumer's heat exchanger is at its maximum value—what we have called the design condition. Thus, we have the following simple constraint for each control valve in the system

$$\Delta P_{cv,i} \geq \Delta P_{cvm,i} \tag{4-5}$$

where ΔP_{cvm} is the minimum pressure drop in the control valve (N/m^2).

And finally, for the heat exchanger the pressure drop will be related to the flow rate in a manner very similar to that for the pipes as given by eq 4-4 above. First, let's assume the following simple form

$$\Delta P_{he,i} = A_{he,i}\, \dot{m}_i^{\beta} \tag{4-6}$$

where A_{he} relates the fluid properties and physical properties of the heat exchanger to the pressure drop and flow rate ($kg^{1-\beta}/m\ s^{2-\beta}$) and β is an exponent yielding the appropriate mass flow rate dependency for the heat exchanger (dimensionless).

In most cases A_{he} and β would probably be empirically determined coefficients and would depend on the type of heat exchanger and its specific design. The basic form of A_{he} would probably be dictated by the heat exchanger geometry. For example, if a straight section of pipe formed the hydraulic passageway for the heat exchanger, the form of A_{he} based on our previous analysis for pressure loss in pipes would be

$$A_{he,sp} = (a/2)(4/\pi)^{2+c}\, \varepsilon_{he}^{b}\, \rho_{he}^{-1} \mu_{he}^{-c}\, L_{he}\, d_{he}^{-(5+b+c)} \tag{4-7}$$

where the *sp* subscript denotes straight pipe heat exchanger and the *he* subscripts denote conditions within the heat exchanger or physical parameters of the heat exchanger.

Equation 4-7 assumes the same form of approximation for the friction factor as was derived for flow in the district heating pipes earlier in Chapter 2. Using this approximation for the friction factor also determines β from eq 4-6 to be

$$\beta_{sp} = 2 + c. \tag{4-8}$$

We can now substitute the results for the pressure losses around the district heating system loop (eq 4-3, 4-4 and 4-6) into our original pressure loss constraint (eq 4-2) to obtain the following constraint

$$PP_{f,i}\, \rho / \dot{m}_i = \sum_{j} \left(a\, \varepsilon_j^{b} (4/\pi)^{2+c} A_{6,j}\, \dot{m}_j^{2+c} L_j\, d_j^{-(5+b+c)} \right) + \Delta P_{cv,i} + A_{he,i}\, \dot{m}_i^{2+c} \tag{4-9}$$

Again, in eq 4-9 the summation over the j pipes only includes those pipes that serve consumer i.

MAXIMUM ABSOLUTE PRESSURE CONSTRAINTS

Several constraints on the absolute pressure of the water within the system must be considered. First, we consider the upper limit on pressure that results from the absolute pressure limits of the piping. This limit will be established by the prevailing piping code. In the case where all points in the distribution system are at or above the level of the heating plant, the maximum absolute pressure will occur in the supply pipe at the heating plant. In the general case, however, this will not always be true and it will be necessary to determine that this constraint is not violated at any point within the system. However, several heuristics will allow us to forgo computation of the absolute pressure level at many of the points. At any point in the supply side of the system the absolute pressure is given by

$$P_s = P_{hp,s} - \sum_j \Delta P_{s,j} - \rho_s g z \tag{4-10}$$

where P_s = absolute pressure in supply pipe at point in question (N/m^2)
$P_{hp,s}$ = absolute pressure in supply pipe at heating plant (N/m^2)
$\Delta P_{s,j}$ = pressure loss in supply pipe j (N/m^2)
z = elevation at point in question relative to heating plant (m).

Again, we have assumed that no intermediate pumping is employed and that the summation over j includes only those pipe segments between the heating plant and the point in question along the supply line. If P_{max} is the maximum absolute pressure for the piping system being used (N/m^2), our constraint arising from it is then

$$P_{max} \geq P_{hp,s} - \sum_j \Delta P_{s,j} - \rho_s g z \tag{4-11}$$

where P_{max} is the maximum absolute pressure for the piping system being used (N/m^2).

We can easily eliminate the need to verify that the upper limit on absolute pressure is not exceeded for the return side of the system. First, we assume that the supply and return line are at the same elevation at any given point, certainly a reasonable assumption. Since there will always be a finite pressure drop across the consumer's heat exchanger and control valve, the absolute pressure in the return line will always be less than that in the supply line at any point, with the difference being the smallest at the consumer. Thus, we need not verify that the maximum absolute pressure constraint (eq 4-11) is satisfied for the return system.

We can also easily show that eq 4-11 only needs to be satisfied at certain points along the supply line. For pipelines that are laid at a constant slope between junction points, the hydrostatic component of the pressure gradient along the pipe will be constant as well. This gradient is given by the following equation

$$(dP/dx)_h = -\rho g(\partial z/\partial x) \tag{4-12}$$

where $(dP/dx)_h$ = hydrostatic pressure gradient (N/m^3)
$(\partial z/\partial x)$ = partial derivative of the elevation of the pipe with respect to its position (dimensionless)
x = position along the pipe in the direction of flow with $x = 0$ being defined as the inlet end to the pipe segment in question (m).

Using our approximation for the friction factor previously determined (eq 2-12), we can find the pressure gradient attributable to frictional losses in the flowing fluid from the following equation

$$(dP/dx)_d = (a/2)(4/\pi)^{2+c}\varepsilon^b\rho^{-1}\mu^{-c}\dot{m}^{2+c}d^{-(5+b+c)} \tag{4-13}$$

where $(dP/dx)_d$ is the hydrodynamic pressure gradient (N/m^3).

The pressure at any point along a segment of the piping system is simply the pressure at the inlet to the pipe segment plus the sum of the hydrostatic and hydrodynamic gradients multiplied by the distance

$$P_x = P_I + [(dP/dx)_h + dP/dx)_d]x \tag{4-14}$$

where P_x is the pressure at point x (N/m^2) and P_I is the pressure at the inlet to the pipe segment ($x = 0$) (N/m^2).

Since $(dP/dx)_h$ and $(dP/dx)_d$ are both independent of x, we only need to know if their sum is positive or negative to determine if the pressure will be higher or lower than the inlet pressure at the outlet of the pipe segment. We can also easily show by using monotonicity analysis (Papalambros and Wilde 1988) that the maximum pressure must occur at either the inlet or the outlet of the pipe section and cannot occur at an intermediate point. To do so we convert the maximization problem to its equivalent minimization problem

$$\text{min. } P_x = -\{P_I + [(dP/dx)_h + (dP/dx)_d]x\} \tag{4-15}$$

subject to

$$g_1 = -x \le 0 \qquad g_1(x\,-) \tag{4-16}$$

$$g_2 = -x - L \le 0 \qquad g_2(x+)\ . \tag{4-17}$$

The conventions for constraints and the labeling of monotonicity are from Papalambros and Wilde (1988). If the sum of the gradients is positive, the objective function is monotonically decreasing in x and must therefore be bounded from above. Constraint g_2 is the only constraint that bounds the objective from above, so it must be critical and thus $x = L$. If the opposite is true, the sum of the gradients is negative, the objective will be monotonically increasing in x and must therefore be bounded below. Constraint g_1 is the only constraint that bounds the objective from below, so it must be critical and $x = 0$. If the sum of the gradients is zero, the pressure will be the same at all points along the pipe segment. Thus, we have shown that the maximum pressure must always be at one end of the pipe segment and it will only be necessary for us to ensure that our absolute pressure constraint (eq 4-11) is satisfied at these points. Remember that in arriving at this result, we assumed that the pipe segment had a constant slope between end points. If in reality this is not the case, the pipe segment in question can be broken up into two or more equivalent pipe segments for applying this constraint.

The number of points at which the maximum absolute pressure constraint must be checked for satisfaction may possibly be reduced even further if we proceed as follows.

1. Starting at the heating plant, we proceed along the supply line checking only the "junction" points as discussed earlier.

2. For any point that is at the same elevation or higher than the upstream point previously identified as having the maximum pressure, we need not compute the pressure.

3. When a point is identified that does not meet the above criteria, we proceed by first computing the sum of the hydrostatic and hydrodynamic gradients. If this quantity is negative, we do not need to compute the pressure.

4. If the sum of the gradients is positive, we will need to compute the pressure at this point. To compute the pressure, we first find the elevation difference and resulting hydrostatic pressure difference between the point in question and the

previously identified point of highest pressure. We then must calculate the pressure losses attributable to the hydrodynamic gradients in each of the pipe segments between the previously identified point of highest pressure and the point in question.

5. If we maintain a running total of pressure losses computed as we complete the above step, we can stop the calculation procedure as soon as this total pressure loss exceeds the hydrostatic pressure difference computed above. Otherwise, we must continue the calculation, in which case we will have identified a new maximum pressure location.

6. Steps 2 through 5 are repeated until we reach the end of the piping network. At branching points we will need to proceed out each branch following the procedure as outlined.

MINIMUM ABSOLUTE PRESSURE CONSTRAINTS

Minimum allowable pressure constraints arise from three distinct considerations.

1. Net Positive Suction Head (NPSH) requirements of the pump.
2. Minimum pressure over atmospheric necessary to preclude the infusion of air into the system.
3. Pressure necessary to prevent flashing of the liquid.

The constraint resulting from minimum NPSH requirements necessary to preclude pump cavitation only needs to be satisfied at the inlet to the pump. This pressure requirement will be a function of the saturation pressure and hence the temperature of the liquid at that point. The NPSH requirement is usually specified by the manufacturer of the pump. Thus, this constraint is simply

$$P_{\mathrm{hp,r}} \geq P_{\mathrm{NPSH}} \tag{4-18}$$

where $P_{\mathrm{hp,r}}$ is the pressure in the return line at the inlet to pump (N/m^2) and PNPSH is the minimum allowable pressure at the pump inlet from NPSH requirements (N/m^2).

The amount of pressure over atmospheric necessary to prevent infusion of air into the system will be another area where engineering judgment will be required. This will be an issue primarily in portions of the system that are operating at temperatures below 100°C, since the saturation pressure constraint (eq 4-20) will dominate it at higher temperatures, given equal safety margins. If, as we assumed earlier, no intermediate pumping is being used, then the minimum pressure level will be at the inlet to the pump for a system that is at or below the level of the heating plant at all points. For other systems, we must check for dominance of this constraint or the saturation pressure constraint derived below, and then constraint satisfaction must be verified at all points within the system. At the heating plant, this constraint can be written as

$$P_{\mathrm{hp,r}} \geq P_{\mathrm{a}} + P_{\mathrm{asa}} \tag{4-19}$$

where P_{a} is atmospheric pressure ($\approx 10^5$ N/m^2) and P_{asa} is the minimum safety margin above atmospheric pressure (N/m^2).

The second constraint on minimum allowable absolute pressure results from the requirement that the fluid must be maintained above its saturation pressure some finite amount to preclude flashing to the vapor phase. The amount of excess pressure above the saturation pressure of the fluid is a matter of engineering judgment. Because localized areas of pressure lower than the "bulk" pressure of the fluid may occur because of hydrodynamic effects, a safety margin above the saturation pressure is prudent. The resulting constraint is

$$P_x \geq P_{x,sat} + P_{saf} \tag{4-20}$$

where $P_{x,sat}$ is the saturation pressure of the liquid at point x within the pipe segment (N/m^2) and P_{saf} is the minimum allowable safety margin on saturation pressure requirements (N/m^2).

The saturation pressure is a function of the fluid temperature, which will vary between supply and return portions of the system, as well as within each portion. Thus, it will be necessary to verify the satisfaction of this constraint at all points within the system. Again, some simple rules will allow us to forgo the calculation at many points, as with the maximum absolute pressure constraint described earlier. As noted earlier, in some cases when the temperature is below 100°C, the air infusion constraint above (eq 4-19) will dominate. The concept of constraint dominance is illustrated later in Chapter 5.

The pressure level at any point in the supply side can be calculated with eq 4-10. For the return side, the absolute pressure is given by

$$P_r = P_{hp,s} - \sum_j \Delta P_{s,j} - \rho_s gz - \Delta P_{cv,i} - \Delta P_{he,i} - \sum_j \Delta P_{r,j} \tag{4-21}$$

where $P_{r,j}$ is the pressure loss in the servicing return line j. The j subscript on the return line summation indicates only those return pipes servicing consumer i between consumer i and the point in question.

The evaluation of pressures in the return pipes using this expression requires some care and forethought to avoid errors and redundant calculations. Errors can result if the summations include pipes other than the appropriate ones, which will be different in the case of supply and return. Equation 4-20, as written, could be evaluated for each consumer at all locations in the piping system. However, all that is required is to find the pressure at each location once for any consumer served through that point. The evaluation of the equation for all remaining consumers served through that point would yield the same result and thus is not required. Some simple rules will allow us to reduce the number of locations where calculation of the pressure will be necessary. For example, consider the case where the entire system is at or below the elevation of the heating plant. In this case, the minimum pressure in the return line would be at the heating plant. In the supply line, however, the lowest pressure could be at any location, dependent on the relative magnitude of the hydrodynamic gradient from friction and the hydrostatic gradient from elevation differences. If the entire system was at the elevation of the heating plant, then the lowest pressure in the return line would be at the consumer, who is, in a hydraulic sense, the most distant from the heating plant.

It is important to note that in the case of this absolute pressure constraint, the supply and return piping must be considered separately, since the temperature, and thus saturation pressure, will usually be quite different in each. Strictly speaking, it would be necessary to determine the actual temperature at each location in the system and compare the saturation pressure requirement with the other applicable low pressure constraints to determine which one is dominant there.

Once the pressure has been calculated at the locations where the minimum pressure constraint could be active, these pressures would be compared to the minimum allowed pressure for that location determined from the dominant constraint of the applicable ones given above. Thus, our minimum pressure constraint for the supply pipe becomes

$$P_{hp,s} - \sum_j \Delta P_{s,j} - \rho_s gz \;\geq\; P_{x,sat} + P_{saf}\,. \tag{4-22}$$

For the return pipe our minimum pressure constraint is

$$P_{\mathrm{hp,s}} - \sum_j \Delta P_{\mathrm{s,j}} - \rho_{\mathrm{s}}\, gz - \Delta P_{\mathrm{he,i}} - \Delta P_{\mathrm{cv,i}} - \sum_j \Delta P_{\mathrm{r,j}} \geq P_{\mathrm{x,sa}t} + P_{\mathrm{saf}}. \tag{4-23}$$

And at the inlet to the pump located at the heating plant, we have two constraints

$$P_{\mathrm{hp,r}} \geq P_{\mathrm{NPSH}} \tag{4-24}$$

$$P_{\mathrm{hp,r}} \geq P_{\mathrm{a}} + P_{\mathrm{asa}}. \tag{4-25}$$

Now we have identified all of the constraints on the multiple consumer–multiple pipe solution and derived mathematical expressions for each one. In the next chapter, we will examine a method of finding an optimal solution that satisfies all of the constraints.

CHAPTER 5: OPTIMAL DESIGN OF SYSTEMS WITH MULTIPLE PIPES AND CONSUMERS

As noted in the Chapter 4, all district heating networks, with the exception of pure transmission systems, will have multiple consumers and pipes. If each pipe were independent of the others, it would be possible to apply the procedure developed earlier in Chapters 2 and 3 to each pipe independently and develop a complete design in that way. Unfortunately, as the constraints introduced in Chapter 4 show, each pipe segment does not operate independently of the others. Thus, the system can not be designed completely in that way for all but the most trivial cases.

However, the "optimal independent design," as we will call it, is very useful, even though we can not guarantee that it would be feasible, for we can use it to form a lower bound on any other designs that we might propose. We know that it is not possible for us to achieve a lower cost design for any one of the pipe segments than the one we have determined independently. Thus, it follows that we also are assured that no design for the entire system of pipe segments can be lower in cost than the sum of the costs for the optimal independent designs. While this may appear to be of little significance to the designer who is subject to system constraints, it's actually a very useful result. It will serve two very important functions for us. First, it will give us a lower bound on total system cost to which we can compare other designs to see if they are sufficiently close to render additional effort at achieving better designs impractical or unnecessary. The second function of this "optimal independent design" will be as a starting point for a solution strategy that will move towards an optimal solution that satisfies all the system constraints. Both of these attributes of the optimal independent design will be exploited in this chapter.

Our objective here will be to develop methodologies that will help us find the optimal discrete pipe diameters for systems with multiple pipes and consumers, while minimizing the computational effort necessary, such that large networks often encountered in practice may be treated with an acceptable degree of effort. Because of excessive computational effort, many of the methods that have been previously applied to problems of this type are felt to be unsuitable. Several of the more common approaches are discussed very briefly below.

SOME POSSIBLE APPROACHES

The classical approach to a constrained optimization problem like this one is to include all the constraints in the problem solution and find a solution that satisfies all the constraints. Here, we have many constraints that would need to be included. For our problem from eq 4-2 and 4-5, we would have two constraints for every consumer. Equations 4-11 and 4-22 would result in two constraints for every node in the supply pipe. Equation 4-23 gives us one constraint for each node in the return pipe, and at the heating plant there are two additional constraints (eq 4-24 and 4-25). If, for example, we considered a moderate sized system, with 100 consumers and 125 nodal points, we would have 577 constraints. Several of the more common methods for handling such problems are discussed below.

The method of linear programming (Wilde and Beightler 1967) is very efficient at solving optimization problems with large numbers of constraints. However, as the name of the method implies, the objective and constraints must be linear. Here, we have a highly nonlinear problem because of the pressure losses being proportional to the pipe diameters raised to approximately the negative five power. Methods have been devised (Reklaitis et al. 1983) to use linear programming algorithms on nonlinear problems by making linear approximations about a point using a Taylor series expansion about that point. The computational effort involved in the use of such methods can be considerable for problems with many variables and constraints, as is the case here. Additionally, if an "optimum" is found by such a method, there is no guarantee that it is a global optimum.

Methods have also been developed for general nonlinear problems. Perhaps the method most commonly referred to is Lagrange's method of undetermined multipliers (Wilde and Beightler 1967). This method requires that the solution to a set of nonlinear equations be found. The number of unknowns is equal to the number of variables (pipe diameters and consumer control valve pressure losses) plus the number of constraints. In our case this would result in very large systems of nonlinear equations for all but the most trivial problems. The solution of large systems of nonlinear equations can be a very difficult task, usually done by adapting methods for the solution of linear equations. For this reason, Lagrange's method is felt to be impractical for this problem.

The Generalized Reduced Gradient (GRG) method is a popular one used for nonlinear constrained problems (Reklaitis et al. 1983). It is based on extending methods used for linear problems to nonlinear problems. The basic concept of the GRG method is to follow along the direction of a constraint subset while seeking improvement in the objective function. By requiring some subset of the constraints to be satisfied, the number of degrees of freedom of the problem can be effectively reduced. When inequality constraints are present in the problem, as is the case here, either an active set strategy must be adopted or slack variables must be introduced for each constraint. Gill et al. (1981) indicate that GRG methods can encounter difficulties when highly nonlinear constraints are involved, as is the case here. Because methods developed for linear problems are being used for nonlinear problems, it is necessary to iterate at each step to achieve a feasible design. The Newton-Rapson method is used for this iteration and it becomes the main computation burden of the GRG method (Arora 1989). Other quasi-Newton methods have been proposed, but they can cause other problems with this method (Arora 1989). Vanderplaats (1984) indicates that convergence of the Newton-Rapson method may be a problem when using the GRG method for highly nonlinear problems.

Another class of methods for general nonlinear constrained problems is the penalty function methods (Rao 1984). These methods reduce the constrained problem to an unconstrained problem that can then be solved using any of the various methods suitable for such problems. With many variables, as we have here, the multidimensional optimization problem that results can be quite time consuming to solve. In addition, it's usually necessary to solve the problem repeatedly for different values of the penalty parameter until some convergence criterion has been met. A feasible starting point is required as is an initial value for the penalty parameter and the multiplication factor that is used to adjust the penalty parameter.

In this problem the diameters of the pipe segments must take on discrete values in the final solution, while other variables such as the consumer control valve pressure losses are continuous. Such a problem, which has both discrete and continuous variables, can be formulated as what's called a "mixed integer" problem (Reklaitis et al. 1983). The methods described above can not be applied directly to integer or mixed integer problems. They must be used in combination with another technique, most notably the "branch-and-bound" approach, to find the solution for the discrete variables. The branch-and-bound approach will be discussed later.

In search of a simpler and more efficient method than those described above, we will proceed by starting with our optimal independent (unconstrained) design and identifying methods to move from this design to one that satisfies all the constraints. We will attempt to conduct this process of modifying the solution so that it satisfies the constraints in a manner that will keep us as close as possible to the true globally optimal design.

PIPE AND PIPE JUNCTION LABELING SCHEME

Before proceeding further with the development of our solution technique, we first need to develop a methodology for identifying consumers, pipe junctions and pipe segments. We would like this system to be as simple and intuitive as possible, yet sufficiently general so as to be easily extendible to much larger networks. A method that meets these requirements is a simple identification number for each "node." A node can be any one of the following items within the pipe network.

1. A source node where a net inflow of heat occurs, i.e., the heating plant.
2. A sink node where a net outflow of heat occurs, i.e., a consumer.
3. A pipe junction node where no net inflow or outflow of heat occurs.

Note that there are at least a couple of special cases of the pipe junction node that might be of interest: a storage node where heat could be stored for release at later times, and a "junction" node with only two pipe segments connected. The latter could be simply a transition in pipe size or an intermediate pumping station for instance. These special cases would be of interest for advanced system optimization studies but are beyond the scope here.

The number of a node does not necessarily need to be assigned in any particular fashion. They could be assigned sequentially from the plant or some consumer, or in no particular sequence at all. In fact, alphanumeric characters could be used for identification. The point is that the assigned identification characters have no significance relative to one another, other than being unique to the node in question.

With an identification system established for our nodes, we need to establish the identity of the pipes connecting these. The simple convention we will adopt is to use the node numbers on either end of the pipe segment to identify the pipe segment that connects them. For example, the pressure loss in the pipe segment between nodes 1 and 2 would be written as $\Delta P_{1,2}$. We will establish the convention of letting the first node number in the pair be the upstream node in the supply line, with the second node being the downstream node, again in the supply line. For the return line of the same pipe segment, the convention will be established by the supply line, i.e., the first node number in the pair will be the downstream node in the return line and the second node will be the upstream node. Note that a system segment, as we have currently defined it, can not have any intermediate nodes within it.

Now we are ready to begin the development of our solution method. As always, we start by determining our objective function.

SYSTEM OBJECTIVE FUNCTION

The objective function for an entire system of pipes will include the sum of the individual objective functions for each pipe segment. We must also include the cost of pumping energy dissipated at the consumer and the capital cost of the pumps needed to generate this pumping energy. At first it might seem unnecessary to include costs associated with the consumer in our objective function when in fact there are no decisions to be made about the consumer's equipment. However, constraints that the consumer places on the system will require that these costs be included in order to achieve an optimal design that does not violate these constraints.

Additional costs would also need to be included if we were to expand the objective of our design. For example, if we wished to determine an optimal operational strategy for the system, as well as a design, it would be necessary to include some additional costs in the objective function. These would be the costs of generating the heat ultimately supplied to the consumer. Another example of an expanded objective would be if we also wanted to determine the optimal size for the consumer's heat exchanger equipment, which would require including these costs in the objective as well. Here, however, we will not address these additional issues. With these limitations in scope our objective function becomes

$$\text{min. } C_{\text{st}} = C_{\text{fixed}} + \sum_j \left(C_{\text{hl}} + C_{\text{pev}} + C_{\text{pv}} \right) + C_{\text{pvc}} + \frac{C_{\text{e}}}{\eta_{\text{p}} \eta_{\text{pm}}} \sum_i \int_{yr} \left(\Delta P_{\text{cv,i}} + \Delta P_{\text{he,i}} \right) \left(\dot{m}_{\text{i}} / \rho_{\text{r}} \right) \mathrm{d}t \tag{5-1}$$

where C_{st} = total system cost ($)
C_{fixed} = fixed cost of pipes and pumps, and the maintenance and repair on this portion of their costs ($)
C_{pv} = diameter variable cost of pipes and the maintenance and repair on that portion of pipe cost ($)
C_{pev} = diameter variable cost of pumps and pumping energy attributable to piping pressure losses, and the maintenance and repair on that portion of pump costs ($)
C_{pvc} = variable cost of pumps attributable to the pressure losses at the consumer ($).

Notice that the density used in the last term of this equation is the taken at the return condition. This is done because the pumps are usually located on the return side of the system at the heating plant. The cost of pumps, which was previously lumped with the piping cost, has been broken out as a separate cost since the number of pumps will be discrete for the system.

In general, the mass flow rate for any consumer $\dot{m}_{\text{i}}$ and the pressure losses at the consumer $(\Delta P_{\text{cv}} + \Delta P_{\text{he}})_{\text{i}}$ will be functions of time. Previously, we assumed that the mass flow rate over the yearly cycle was given by eq 3-25. Since the pressure loss in the consumer's heat exchanger $\Delta P_{\text{he,1}}$ is a function of mass flow rate, as given by eq 4-6, it will also be a function of time. As we will show later, the pressure loss in the consumer's control valve $\Delta P_{\text{cv,i}}$ will be used to "balance" the network. Hence, it will become a function of time in most all cases as well. We will have some choices as to the best way to balance the network using the consumer's control valve, as will also be shown later.

In eq 5-1 we have separated the cost of the pumps into the fixed costs, that portion which does not depend on pump capacity, and the variable costs, which are attributable to either pressure losses at the consumer or in the piping network. We have also separated the fixed portion of the pipe cost as well from that portion that depends on pipe diameter. Effectively, this does not change our objective function as far as terms that contain the pipe diameters are concerned, since the fixed costs of the pipes and pumps are not considered in determining the optimum pipe diameter, as can be seen from eq 2-19. For a multiple pipe system, these fixed costs are

$$C_{\text{fixed}} = (1 + PVF_{\text{m\&r}} A_{\text{m\&r}})(A_1 n_{\text{p}} + A_3 \sum_j L_{\text{j}}) . \tag{5-2}$$

The variable cost of pumps attributable to pressure losses at the consumer C_{pvc} will be determined by the pressure losses and flow rate at the design condition. It is this condition for which the pressure difference between the supply and return at the heating plant, as well as the mass flow rate, are greatest. Thus, the pumps must be sized for this condition. This portion of the pump cost will be given by

$$C_{\text{pvc}} = A_2 \, (1 + PVF_{\text{m\&r}} A_{\text{m\&r}}) \sum_i \left[\left(\Delta P_{\text{cv,i}} + \Delta P_{\text{he,i}} \right) \left(\dot{m}_{\text{i}} / \rho_{\text{r}} \right) \right]_{\text{d}} . \tag{5-3}$$

SOLUTION STRATEGY

Now we are ready to formulate our solution strategy. Inspecting the objective function, eq 5-1, we see that the costs have been grouped with respect to their source. All of the costs that are dependent on our decision variables, the pipe diameters, have been included in the first summation over *j*, the pipe segment index. The summations in the third and fourth terms are those that arise from the pumping energy expended at the consumer. The decision variables in the terms of these summations are the $\Delta P_{\mathrm{cv,i}}$ values.

We notice immediately from the form of the objective that it is a separable function with regards to the pipe diameter for each of the system segments *j*. A separable function is a function of more than one variable that may be written as a combination of functions, one independent function for each variable. Thus, from examining the objective function, it appears that we can consider each pipe diameter function independent of the other pipe diameters and find its optimum. We will proceed as if this is the case, although later we will see that the constraints will not allow the diameters to be considered completely independent of one another in all cases.

We begin by inspecting the objective function for monotonicity, since this will help us simplify the solution as much as possible. Looking first at the terms in the summation over the pipe segment index *j*, we look at each term in the summation separately

$$C_{\mathrm{hl,\,j}} = I_1 / \ln(A_{10}/d) \qquad \left(d_{\mathrm{j}}^{+}\right)$$

$$C_{\mathrm{pv,\,j}} = A_9 d \qquad \left(d_{\mathrm{j}}^{+}\right)$$

$$C_{\mathrm{pev,\,j}} = I_3\, d^{-(5+b+c)} \qquad \left(d_{\mathrm{j}}^{-}\right) .$$

The monotonicities with respect to d_{j} of each term are given and we see that we have both increasing and decreasing terms, so we are unable to use monotonicity analysis on these at the outset. This is consistent with our findings in Chapter 2, where we first neglected the C_{hl} term and then used geometric programming theory to find a solution to the lower bounding problem thus formed. This result was used as a starting point for a simple search to find the solution to the problem without neglecting C_{hl}. Since the objective function is separable for each of the d_{j} values, we will proceed with the same methodology and find the "optimal independent" values for each d_{j} in the same way.

The other remaining decisions variables in the objective are the $\Delta P_{\mathrm{cv,i}}$ variables, of which there is one for each consumer. The $\Delta P_{\mathrm{cv,i}}$ variables appear in both of the last two terms of the objective function once eq 5-3 has been substituted for C_{pvc}

$$A_2\left(1 + PVF_{\mathrm{m\&r}}\, A_{\mathrm{m\&r}}\right) \sum_i \left[\left(\Delta P_{\mathrm{cv,i}} + \Delta P_{\mathrm{he,i}}\right)\left(\dot{m}_{\mathrm{i}} / \rho_{\mathrm{r}}\right)\right]_{\mathrm{d}} \qquad (\Delta P_{\mathrm{cv,i}}^{+})$$

$$\frac{C_{\mathrm{e}}}{\eta_{\mathrm{p}}\eta_{\mathrm{pm}}} \sum_i \int_{yr} \left(\Delta P_{\mathrm{cv,i}} + \Delta P_{\mathrm{he,i}}\right)\left(\dot{m}_{\mathrm{i}} / \rho_{\mathrm{r}}\right) \mathrm{d}t \qquad (\Delta P_{\mathrm{cv,i}}^{+}) .$$

Both of these terms are monotonically increasing in each $\Delta P_{\mathrm{cv,i}}$ and thus the objective is monotonically increasing in each $\Delta P_{\mathrm{cv,i}}$. The First Monotonicity Principle, MP1 (see Papalambros and Wilde 1988), therefore tells us that each $\Delta P_{\mathrm{cv,i}}$ must be bounded below by at least one active constraint. We will examine the issue of determining the constraint activity for these decision variables.

Constraint activity for consumer control valve pressure losses

We have two possible constraints for each $\Delta P_{\mathrm{cv,i}}$

$$h_1 = \Delta P_{\mathrm{hp}} - \left(\sum_j \left(\Delta P_{\mathrm{s}} + \Delta P_{\mathrm{r}} \right)_{\mathrm{j}} + \Delta P_{\mathrm{cv}} + \Delta P_{\mathrm{he}} \right)_{\mathrm{i}} = 0 \qquad (\Delta P^{\mp}_{\mathrm{cv,i}}) \qquad (4\text{-}2)$$

$$g_1 = \Delta P_{\mathrm{cvm,i}} - \Delta P_{\mathrm{cv,i}} \le 0 \qquad (\Delta P^{-}_{\mathrm{cv,i}}) . \qquad (4\text{-}5)$$

The inequality constraint g_1 is monotonically decreasing in $\Delta P_{\mathrm{cv,i}}$ so it bounds $\Delta P_{\mathrm{cv,i}}$ in the proper sense. The equality constraint h_1 could also be "directed" (see Papalambros and Wilde [1988] for procedure for directing equality constraints) such that it would bound $\Delta P_{\mathrm{cv,i}}$ in the proper sense. At this point it is not clear which of these two constraints would bound each of the $\Delta P_{\mathrm{cv,i}}$ variables in the proper sense. In fact it's entirely possible that the active constraint may vary depending on the particular consumer in question. If no other decision variables appeared in these two constraints, they would form a conditionally critical set for each $\Delta P_{\mathrm{cv,i}}$ (see Papalambros and Wilde [1988] for definition of conditional criticality).

However, we see that pressure increase across the pump at the heating plant ΔP_{hp} has not yet been fixed. ΔP_{hp} is monotonically increasing in constraint h_1; thus, it becomes a monotonic nonobjective variable in our problem. The Second Monotonicity Principle, MP2 (Papalambros and Wilde 1988) tells us that either ΔP_{hp} is irrelevant and can be deleted from the problem together with all the constraints in which it appears, or it is relevant and bounded by two active constraints, one bounding it from above and one bounding it from below. If for just one consumer i the constraint h_1 is *critical* for $\Delta P_{\mathrm{cv,i}}$, then ΔP_{hp} becomes relevant. A critical constraint is an active constraint whose deletion would cause the problem to become unbounded. This active constraint would have the following monotonicities when directed to bound $\Delta P_{\mathrm{cv,i}}$ in the proper sense

$$h_1 = \Delta P_{\mathrm{hp}} - \left(\sum_j \left(\Delta P_{\mathrm{s}} + \Delta P_{\mathrm{r}} \right)_{\mathrm{j}} + \Delta P_{\mathrm{cv}} + \Delta P_{\mathrm{he}} \right)_{\mathrm{i}} \equiv< \; 0 \qquad \left(\Delta P^{-}_{\mathrm{cv,i}}, \Delta P^{+}_{\mathrm{hp}} \right) .$$

If one constraint is critical for more than one variable in the problem, it is said to be "multiply critical" (Papalambros and Wilde 1988) and this would be the case for h_1 above. Papalambros and Wilde (1988) warn that multiply critical constraints should be eliminated from the problem whenever possible. Here, we have that option since we can combine the h_1 constraints for any two consumers and eliminate ΔP_{hp} from the problem. Before we do so, let's consider briefly what is physically happening in our problem.

First, we note that the pressure increase required across the pump at the heating plant ΔP_{hp} appears in eq 4-2 for each consumer. Since, physically, we know that ΔP_{hp} can only assume one value, it must be the greatest value that results from consideration of all the consumers. For the remaining consumers, $\Delta P_{\mathrm{cv,i}}$ must be greater than the minimum value $\Delta P_{\mathrm{cvm,i}}$. The consumer who requires the greatest ΔP_{hp} will be called the "critical" consumer. Notice that the equality constraint of eq 4-2 can be satisfied for the remaining consumers by letting $\Delta P_{\mathrm{cv,i}} > \Delta P_{\mathrm{cvm,i}}$ as allowed by eq 4-5. This is, in fact, how it is done in practice in most cases; the ultimate balancing of the pipe network flows is done by the consumer's control valves. For the case of the critical consumer, eq 4-5 will be satisfied as a strict equality, i.e., $\Delta P_{\mathrm{cv,i}} > \Delta P_{\mathrm{cvm,i}}$. While these arguments of constraint activity would appear to be completely intuitive, since we would not want to supply any more pumping energy than necessary, they can also be shown analytically as follows.

The pressure difference across the pump at the heating plant ΔP_{hp} must be equal in all of the constraints of eq 4-2 at any instance in time, so we can write this constraint set in the form

$$\left\{\sum_j \left(\Delta P_s + \Delta P_r\right) + \Delta P_{cv} + \Delta P_{he}\right\}_k - \left\{\sum_j \left(\Delta P_s + \Delta P_r\right) + \Delta P_{cv} + \Delta P_{he}\right\}_i = 0 \tag{5-4}$$

where $i \neq k$.

If we have n consumers, there will be $(n-1)$ such equality constraints containing $\Delta P_{cv,i}$ that apparently could be "directed" such that they would bound $\Delta P_{cv,i}$ from below as required. However, these constraints are not all independent. Since we started with n independent equations and then eliminated ΔP_{hp}, we will have at most $(n-1)$ independent equations remaining. Below, we will show that these $(n-1)$ independent equations may bound at most $(n-1)$ of the $\Delta P_{cv,i}$ objective variables in the proper sense. The arguments made will be for an instant in time but must hold for any time during the yearly cycle.

We begin by examining the $\Delta P_{cv,i}$ term for the consumer arbitrarily chosen to be consumer "1." Now, $\Delta P_{cv,1}$ for consumer 1 can be bounded from below as required by any one of the $(n-1)$ constraints in which it appears with another consumer. Suppose we let the constraint with consumer "2" bound $\Delta P_{cv,1}$. Now we have $(n-2)$ constraints remaining that can bound $\Delta P_{cv,2}$ in the proper sense, since the equality constraint with consumer 1 has been directed such that it would bound $\Delta P_{cv,2}$ in the improper sense

Similarly, let $\Delta P_{cv,2}$ be bounded by properly directing its equality constraint with consumer "3." Now, at first it would appear that $\Delta P_{cv,3}$ could be bounded by $(n-2)$ constraints as well, since we have only directed the constraint involving $\Delta P_{cv,2}$ in the improper sense and any one of the remaining $(n-2)$ constraints can be directed as needed. However, since we directed the constraint between $\Delta P_{cv,1}$ and $\Delta P_{cv,2}$ such that it bounded $\Delta P_{cv,1}$ below, we are not free to direct the constraint between $\Delta P_{cv,1}$ and $\Delta P_{cv,3}$ as needed; in fact, it must be directed in the opposite sense of that required, that is, if

$$\left\{\sum_j \left(\Delta P_s + \Delta P_r\right)_j + \Delta P_{cv} + \Delta P_{he}\right\}_2 - \left\{\sum_j \left(\Delta P_s + \Delta P_r\right)_j + \Delta P_{cv} + \Delta P_{he}\right\}_1 \equiv < \; 0 \left(\Delta P^-_{cv,1}, \Delta P^+_{cv,2}\right)$$

and

$$\left\{\sum_j \left(\Delta P_s + \Delta P_r\right)_j + \Delta P_{cv} + \Delta P_{he}\right\}_3 - \left\{\sum_j \left(\Delta P_s + \Delta P_r\right)_j + \Delta P_{cv} + \Delta P_{he}\right\}_2 \equiv < \; 0 \left(\Delta P^-_{cv,2}, \Delta P^+_{cv,3}\right)$$

then

$$\left\{\sum_j \left(\Delta P_s + \Delta P_r\right)_j + \Delta P_{cv} + \Delta P_{he}\right\}_3 - \left\{\sum_j \left(\Delta P_s + \Delta P_r\right)_j + \Delta P_{cv} + \Delta P_{he}\right\}_1 \equiv < \; 0 \left(\Delta P^-_{cv,1}, \Delta P^+_{cv,3}\right).$$

Thus, $\Delta P_{cv,3}$ has only $(n-3)$ $\left\{\left(\Delta P^-_{cv,3}, \Delta P^+_{cv,4}\right), \left(\Delta P^-_{cv,3}, \Delta P^+_{cv,5}\right), \cdots \left(\Delta P^-_{cv,3}, \Delta P^+_{cv,n}\right)\right\}$

constraints that could be directed to bound it in the proper sense.

If we continue to follow this line of reasoning, we find that when we reach consumer $i = n$, we have no constraints remaining that can be directed in the proper sense to bound the $\Delta P_{\mathrm{cv,n}}$ term in the objective. Note that because the assignment of the numerical value for the consumer index i is arbitrary, its assignment will have no impact on what we have shown here and that this result would hold for any set of consumer indices.

Since we have n variables $\Delta P_{\mathrm{cv,i}}$ in the objective, one for each of the n consumers, and the objective is monotonically increasing in each of these variables, we must have n constraints bounding the $\Delta P_{\mathrm{cv,i}}$ from below. Above we have shown that at most $(n - 1)$ of these constraints could result from eq 4-2. The only remaining constraints on the $\Delta P_{\mathrm{cv,i}}$ are the set formed by eq 4-5. Let's assume for the moment that all consumers have the same minimum pressure differential requirement for their control valves, i.e., that $\Delta P_{\mathrm{cvm,i}}$ is the same for all i. Now we see that it must be the consumer with the minimum value of $\Delta P_{\mathrm{cv,i}}$ whose constraint from eq 4-5 is active. This is true since a consumer with any greater value would cause at least one of the other constraints from the set of eq 4-5 to be violated, i.e., if consumer i has the minimum control value pressure loss

$$\Delta P_{\mathrm{cv,k}} > \Delta P_{\mathrm{cv,i}} \qquad \text{for all } k \neq i.$$

And if for all i and k the minimum allowable control valve pressure losses are equal

$$\Delta P_{\mathrm{cvm,i}} = \Delta P_{\mathrm{cvm,k}}.$$

Now, if the constraint for consumer i is active

$$\Delta P_{\mathrm{cv,i}} \geqq \Delta P_{\mathrm{cvm,i}}$$

then the constraint for consumer k can not be active

$$\Delta P_{\mathrm{cv,k}} > \Delta P_{\mathrm{cvm,k}}.$$

We have already shown that $(n - 1)$ of the constraints from the set of eq 4-2 are active and can thus be treated as equalities. These $(n - 1)$ constraints force the pressure loss summations to be equal for all n values of the consumer's index i

$$\left\{\sum_j \left(\Delta P_{\mathrm{s}} + \Delta P_{\mathrm{r}}\right)_{\mathrm{j}} + \Delta P_{\mathrm{cv}} + \Delta P_{\mathrm{he}}\right\}_{\mathrm{i}} = \left\{\sum_j \left(\Delta P_{\mathrm{s}} + \Delta P_{\mathrm{r}}\right)_{\mathrm{j}} + \Delta P_{\mathrm{cv}} + \Delta P_{\mathrm{he}}\right\}_{\mathrm{k}} \qquad \text{for all } i \neq k.$$

This means that we identify the consumer whose value of $\Delta P_{\mathrm{cv,i}}$ is at its minimum allowed $\Delta P_{\mathrm{cvm,i}}$ by finding the consumer with the maximum value for the quantity

$$\left\{\sum_j \left(\Delta P_{\mathrm{s}} + \Delta P_{\mathrm{r}}\right)_{\mathrm{j}} + \Delta P_{\mathrm{cvm,i}} + \Delta P_{\mathrm{he}}\right\}_{\mathrm{i}}.$$

Once the pipe sizes are known, this quantity is easily calculated. This consumer who has the minimum value of $\Delta P_{\mathrm{cv,i}}$ is our so called "critical" consumer.

It follows then that there are only two cases where more than one of the constraints from the set of eq 4-5 may be active:

1. In the case where the $\Delta P_{\mathrm{cvm,i}}$ values are identical for all consumers, if the maximum value of

$$\left\{\sum_j \left(\Delta P_{\mathrm{s}} + \Delta P_{\mathrm{r}}\right)_{\mathrm{j}} + \Delta P_{\mathrm{he}}\right\}_{\mathrm{i}}$$

is identical for more than one consumer.

2. If the consumers have different values of $\Delta P_{\mathrm{cvm,i}}$ such that the maximum value of the quantity

$$\left\{\sum_j \left(\Delta P_{\mathrm{s}} + \Delta P_{\mathrm{r}}\right)_{\mathrm{j}} + \Delta P_{\mathrm{he}}\right\}_{\mathrm{i}} + \Delta P_{\mathrm{cvm,i}}$$

is identical for more than one consumer.

Under either of these two conditions, the corresponding constraints from the eq 4-5 set for the consumers with identical values as described above will be active. However, under neither condition will more than one be critical, since the deletion of any additional constraints from the problem will not make the objective unbounded. This is true because the constraints from the set of eq 4-2 directed as described earlier will bound the $\Delta P_{\mathrm{cv,i}}$ of all the consumers but one in the proper sense.

What is not immediately apparent is why all $(n-1)$ of the constraints from the set of eq 4-2 must be critical for some $\Delta P_{\mathrm{cv,i}}$ objective variable. To illustrate why this is so, consider the case where for an arbitrary consumer k when we let $\Delta P_{\mathrm{cv,k}}$ be bounded below by the constraint from the set of eq 4-5, thus $\Delta P_{\mathrm{cv,k}} = \Delta P_{\mathrm{cvm,k}}$. Now suppose that

$$\left\{\sum_j \left(\Delta P_{\mathrm{s}} + \Delta P_{\mathrm{r}}\right)_{\mathrm{j}} + \Delta P_{\mathrm{he}}\right\}_{\mathrm{k}} = \left\{\sum_j \left(\Delta P_{\mathrm{s}} + \Delta P_{\mathrm{r}}\right)_{\mathrm{j}} + \Delta P_{\mathrm{he}}\right\}_{\mathrm{i}}$$

and that $\Delta P_{\mathrm{cvm,k}} > \Delta P_{\mathrm{cvm,i}}$, where consumer i is the consumer with the minimum value of $\Delta P_{\mathrm{cv,i}}$. The constraint from the set of eq 4-2 that states that

$$\left\{\sum_j \left(\Delta P_{\mathrm{s}} + \Delta P_{\mathrm{r}}\right)_{\mathrm{j}} + \Delta P_{\mathrm{cv}} + \Delta P_{\mathrm{he}}\right\}_{\mathrm{i}} - \left\{\sum_j \left(\Delta P_{\mathrm{s}} + \Delta P_{\mathrm{r}}\right)_{\mathrm{j}} + \Delta P_{\mathrm{cv}} + \Delta P_{\mathrm{he}}\right\}_{\mathrm{k}} = 0$$

would be violated. Notice that this constraint has not been directed and shown to be active, yet it still must not be violated by any feasible solution. Thus, this is not a feasible solution and we see that only one of the constraints from the set of eq 4-5 may be critical, since $(n-1)$ of the critical constraints must come from the set of eq 4-2.

Thus, our general result is that we must have $(n-1)$ of the constraints from the set of eq 4-2 active and critical for $(n-1)$ of the $\Delta P_{\mathrm{cv,i}}$ values. Thus, we have $(n-1)$ critical constraints of the form

$$\left\{\sum_j \left(\Delta P_{\mathrm{s}} + \Delta P_{\mathrm{r}}\right)_{\mathrm{j}} + \Delta P_{\mathrm{cv,k}} + \Delta P_{\mathrm{he,k}}\right\}_{\mathrm{k}} - \left\{\sum_j \left(\Delta P_{\mathrm{s}} + \Delta P_{\mathrm{r}}\right)_{\mathrm{j}} + \Delta P_{\mathrm{cv,i}} + \Delta P_{\mathrm{he,i}}\right\}_{\mathrm{i}} \equiv < \; 0 \qquad (5\text{-}5)$$

Where the consumer index i does not equal the consumer index k and the monotonicities are

$$\left(\Delta P^{-}_{\mathrm{cv,i}}, \Delta P^{+}_{\mathrm{cv,k}}\right)$$

we also have one critical constraint on the remaining $\Delta P_{\mathrm{cv,i}}$ not bounded below by one of the $(n-1)$ constraints of eq 5-5 of the form

$$\Delta P^{-}_{\mathrm{cvm,i}} - \Delta P^{+}_{\mathrm{cv,i}} \leqq 0 \qquad \left(\Delta P^{-}_{\mathrm{cv,i}}\right). \tag{5-6}$$

The assessment of which consumer will be the "critical" consumer having his $\Delta P_{\mathrm{cv,i}}$ bounded below by eq 5-6 was found above to be determined by finding the consumer who has the maximum value of the sum of the non-control-valve pressure losses $\Delta P_{\mathrm{ncv,i}}$ and the minimum control valve pressure loss $\Delta P_{\mathrm{cvm,i}}$. The non-control-valve pressure losses $\Delta P_{\mathrm{ncv,i}}$ are given by

$$\Delta P_{\mathrm{ncv,i}} = \left\{ \sum_{j} \left(\Delta P_{\mathrm{s}} + \Delta P_{\mathrm{r}}\right)_{\mathrm{j}} + \Delta P_{\mathrm{he}} \right\}_{\mathrm{i}} \tag{5-7}$$

where the non-control-valve pressure losses ($\mathrm{N/m^2}$) are easily computed once the pipe sizes are determined using the procedure discussed below.

Initial pipe size determination

As noted earlier, our objective function is separable in each pipe diameter. The pipe diameter function for each pipe segment *j* is increasing in some terms while decreasing in others. Thus, we should be able to find a minimum cost for each diameter by proceeding exactly as we did in Chapter 2 if we at first ignore the constraints. Therefore, we first find the optimal "independent" discrete diameters using the methods developed in Chapter 2.

System constraint satisfaction

Once our pipe sizes are determined, we need to ensure that the constraints are satisfied and, if not, determine a methodology for achieving this. Below is listed the various constraints that were developed in Chapters 3 and 4, categorized by the portion of the system in which they originate.

At each consumer

$$\Delta P_{\mathrm{hp}} = \sum_{j} \left(\Delta P_{\mathrm{s}} + \Delta P_{\mathrm{r}}\right)_{\mathrm{j}} + \Delta P_{\mathrm{cv}} + \Delta P_{\mathrm{he}} \tag{4-2}$$

$$\Delta P_{\mathrm{cv,i}} \geq \Delta P_{\mathrm{cvm,i}}. \tag{4-5}$$

In the supply pipe

$$P_{\mathrm{max}} \geq P_{\mathrm{hp,s}} - \sum_{j} \Delta P_{\mathrm{s,j}} - \rho_{\mathrm{s}}\, gz \tag{4-11}$$

$$P_{\mathrm{hp,s}} - \sum_{j} \Delta P_{\mathrm{s,j}} - \rho_{\mathrm{s}}\, gz \geq P_{\mathrm{x,sat}} + P_{\mathrm{saf}}. \tag{4-22}$$

In the return pipe

$$\Delta P_{\mathrm{hp,s}} - \sum_{j} \Delta P_{\mathrm{s,j}} - \rho_{\mathrm{s}}\, gz - \Delta P_{\mathrm{he,i}} - \Delta P_{\mathrm{cv,i}} - \sum_{j} \Delta P_{\mathrm{r,j}} \geq P_{\mathrm{x,sat}} + P_{\mathrm{saf}}. \tag{4-23}$$

In the return pipe at the heating plant

$$P_{\mathrm{hp,r}} \geq P_{\mathrm{NPSH}} \tag{4-24}$$

$$P_{\mathrm{hp,r}} \geq P_{\mathrm{a}} + P_{\mathrm{asa}}. \tag{4-25}$$

All of these constraints deal with pressure levels at various points within the system. Note that in eq 4-2 we have expressed the total piping pressure loss as its supply and return components because it will be necessary to compute these independently for the constraints given by eq 4-11, 4-22 and 4-23. Since verification of satisfaction for all of these constraints requires either directly or indirectly the calculation of the pressure losses in the supply and return pipes, we begin by doing so for each of the pipe segments. The pressure loss in either the supply or return line is calculated by modifying eq 2-15 slightly so that it applies to each pipe independently. The results are

$$\left(\Delta P_{\mathrm{d,s}}\right)_{\mathrm{j}} = \left(a/2\,\varepsilon^{b} (4/\pi)^{2+c} \left(\rho^{-1}\,\mu^{-c}\right)_{\mathrm{d,s}} \dot{m}_{\mathrm{d}}^{2+c}\, L\, d^{-(5+b+c)} \right)_{\mathrm{j}} \tag{5-8}$$

$$\left(\Delta P_{\mathrm{d,r}}\right)_{\mathrm{j}} = \left(a/2\,\varepsilon^{b} (4/\pi)^{2+c} \left(\rho^{-1}\,\mu^{-c}\right)_{\mathrm{d,r}} \dot{m}_{\mathrm{d}}^{2+c}\, L\, d^{-(5+b+c)} \right)_{\mathrm{j}}. \tag{5-9}$$

Once the piping pressure losses are known, we can calculate the non-control-valve pressure losses $\Delta P_{\mathrm{ncv,i}}$ for each consumer using eq 5-7 and sum this with the minimum control valve pressure loss $\Delta P_{\mathrm{cvm,i}}$ to find the consumer with the highest value of this sum, our critical consumer. The sum of the pressure losses for this will become our pressure increase across the pump at the heating plant ΔP_{hp}, as given by eq 4-2. For this consumer the constraint of eq 4-5 will be active, as shown earlier. Using the value of ΔP_{hp} calculated for the critical consumer, we can then calculate the control valve pressure losses for all of the other consumers using eq 4-2.

With the piping and consumer pressure losses known, we can calculate the absolute pressure level at all nodes in the pipe network with either a maximum absolute pressure assigned to the supply pipe at the heating plant, or a minimum absolute pressure assigned to the return pipe at the heating plant. If we set the minimum pressure level in the return pipe at the heating plant, we can use the constraints of eq 4-23, 4-24 and 4-25 to guide our choice. Note that when eq 4-23 is evaluated at the heating plant, the entire left-hand side of the equation reduces to the value $\Delta P_{\mathrm{hp,r}}$. Dependent on the particular parameter values for the problem at hand, one of these constraints will "dominate" (see Papalambros and Wilde [1988] for concept of constraint dominance). The cases for constraint dominance are simply as follows. If

$$P_{\mathrm{a}} + P_{\mathrm{asa}} \le P_{\mathrm{NPSH}} \le P_{\mathrm{x,sat}} + P_{\mathrm{saf}}$$

eq 4-23 dominates. If

$$P_{\mathrm{a}} + P_{\mathrm{asa}} \le P_{\mathrm{x,sat}} + P_{\mathrm{saf}} \le P_{\mathrm{NPSH}}$$

eq 4-24 dominates. If

$$P_{\mathrm{x,sat}} + P_{\mathrm{saf}} \le P_{\mathrm{NPSH}} \le P_{\mathrm{a}} + P_{\mathrm{asa}}$$

eq 4-25 dominates.

Alternately, as noted above, we can also assign the maximum absolute pressure in the supply pipe at the heating plant and use that value to find the other absolute pressures in the network. The logical choice for the maximum absolute pressure value in the supply pipe at the heating plant would be the maximum absolute pressure allowable for the piping system being used P_{max}. In most cases the maximum absolute pressure in the system will occur at the heating plant in the supply pipe; thus, this is a logical choice. It is possible that this will not be the case, however. Using eq 4-15 we have shown earlier that the maximum pressure must be at a nodal point location. In the discussion after eq 4-15, we also developed a procedure that can be used to minimize the number of nodes at which the absolute pressure must be calculated. If this procedure is used, we can quickly determine if the heating plant will be the location of the maximum absolute pressure.

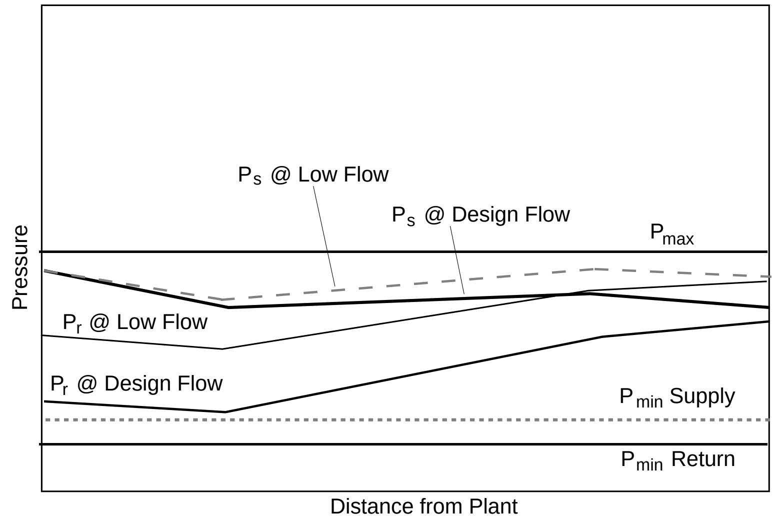

Figure 8. Hypothetical pressure distribution under high and low flow conditions and absolute pressure constraints.

All the calculations made to check for absolute pressure constraint satisfaction should use the maximum (design) mass flow rate. This will assure that constraint satisfaction will be possible at all flow rate conditions. Since the pressure losses in the piping and the consumer's heat exchanger will be greatest under this load condition, the difference between supply and return pressure at the heating plant will also be greatest under this load condition. Thus, under this condition the least flexibility exists to adjust the supply or return pressure at the plant without violating either the maximum pressure constraint in the supply, eq 4-11, or one of the minimum return line pressure constraints at the plant, eq 4-24 and 4-25. If the various maximum and minimum pressure constraints are satisfied for all points in the network at the higher flow rate condition, it will always be possible to satisfy them at the lower flow rates. This is easily shown graphically by considering the pressures in the system along the piping route out to a consumer and back, as shown for a hypothetical consumer in Figure 8.

In Figure 8 the horizontal lines are the constraints on the absolute pressures that must be satisfied at all points along the route to the consumer. The solid lines that have both positive and negative slopes are the supply and return pressures under maximum load conditions. The magnitude and the sign of the slope of these lines are determined by the sum of the hydrodynamic and hydrostatic pressure gradients as given by eq 4-14. The dotted lines that behave in a similar fashion are the supply and return pressures under some mass flow condition that is lower than the maximum. In the extreme case where there is no flow, the pressure losses in the piping and consumer equipment all vanish and the absolute pressure level is identical in the supply and return lines for any point along the route. Also notice that we have shown the pressure drop at the consumer as being lower at the reduced flow condition. This results from lower pressure losses in the consumer's heat exchanger at the reduced flow rate (see eq 4-6) as well as lower losses in the consumer's control valve. If the network were ideally balanced and this consumer were the critical consumer, his control valve would be completely open at all levels of load (i.e., flow rate) and the pressure losses would always be the minimum possible.

By studying Figure 8, we can see that if we are able to "fit" the supply and return absolute pressure lines within the constraints at the maximum flow condition, then we can always do so for any lower flow condition simply by adjusting the absolute pressure of either the supply or return at the heating plant. This results from the hydrodynamic losses in the piping always being lower for the lower flow rates and thus the total pressure increase necessary at the plant is reduced.

A simplified objective function

If all the constraints are shown to be satisfied, then we have found the optimal solution to the multiple-consumer–multiple-pipe problem and we need not do any further calculations. If, however, we find that constraints have been violated, we need to refine our design. The first reaction of the designer when faced with this result should be to closely examine the nature of the constraint that has been violated. Often these constraints are "soft" and may be changed if the optimum design indicates so. For example, in this problem one such constraint would be the maximum allowed pressure. The designer has the option of using a higher pressure class of piping if he or she feels it is warranted. This, of course, will most likely add to the cost and, if this additional cost is significant, the designer may choose to evaluate the design subject to the original constraint and the revised constraint, as well to determine which one yields the lowest cost when the additional cost of the higher pressure class piping is included.

Now that we have shown activity for some of the constraints, let's consider the problem again with a reduced objective and determine if solution is possible. Thus, in our reduced objective, we are only interested in the terms in the objective function that relate to variable piping costs, since we have shown that constraint activity determines the values of the other decision variables. Thus, our problem can be restated as

$$\text{min. } C'_{\text{pt}} = \sum_j \left(C_{\text{hl}} + C_{\text{pev}} + C_{\text{pv}}\right)_{\text{j}} \tag{5-10}$$

where C'_{pt} is the total diameter variable pipe costs for the system ($).

The constraints to which this solution is subject are that the absolute pressure levels not be exceeded. The activity of the constraints of eq 4-2 and 4-5 fixes the pressure increase at the heating plant and therefore the pumping power for the system. Thus, at this point we no longer need to include the pumping power consumed in the piping in our reduced objective function, eq 5-10. If we remove the pumping power from eq 5-10, it becomes a monotonically increasing function of pipe diameter. Thus, for the problem to be bounded, we must have monotonically decreasing constraints on the pipe diameters. The sum of eq 5-8 and 5-9 forms one such equality constraint for each pipe diameter that can be directed to bound it properly. This constraint is

$$\left(a\,\varepsilon^{b}(4/\pi)^{2+c}\left(\left(\rho^{-1}\mu^{-c}\right)_{\text{d,s}} + \left(\rho^{-1}\mu^{-c}\right)_{\text{d,r}}\right)\dot{m}_{\text{d}}^{2+c}\,L\,d^{-(5+b+c)}\right)_{\text{j}} - \left(\Delta P_{\text{s}} + \Delta P_{\text{r}}\right)_{\text{j}} \equiv\, < 0 \tag{5-11}$$

with the monotonicities being $\left(d_{\text{j}}^{-}, \Delta P_{\text{s,j}}^{-}, \Delta P_{\text{r,j}}^{-}\right)$.

Notice that the constraints in the set of eq 5-5 will bound $\Delta P_{\text{s,j}}$ and $\Delta P_{\text{r,j}}$ in the opposite sense to this constraint, so these nonobjective variables are bounded above and below as required by MP2.

Thus, our problem is now to use these active constraints to solve for the diameters. We will have one constraint from eq 5-11 for each pipe segment in the system. In addition, we have already shown that we have $(n-1)$ active constraints from eq 4-2, where n is the total number of consumers. We have one additional active constraint from eq 4-5 for the critical consumer. However, at this point we are unsure as to which consumer is the critical consumer; thus, we must include all n of the constraints from the set of eq 4-2. In addition, we would still need to include all of the constraints from the sets of eq 4-11, 4-22 and 4-23, since at this point we have no way of knowing which of these constraints will be active. For each of these latter sets of constraints, we have one member for each node in the system. We can use the constraint of either eq 4-23, 4-24 or 4-25 coupled with the fact that ΔP_{hp} must be the difference between $P_{hp,s}$ and $P_{hp,r}$ to eliminate $P_{hp,s}$ from eq 4-11, 4-22 and 4-23. Once the parameter values for the problem are known, the choice of which equation to use will be determined by the dominance argument given earlier. Thus, if we have *nn* nodes in the system and *n* consumers, our result would be 3*nn* + 2*n* simultaneous equations.

To introduce the inequality constraints of eq 4-5, 4-11, 4-22 and 4-23 directly into a set of simultaneous equations, we would need to introduce a "slack" variable for each inequality constraint. The slack variable allows us to convert the inequality constraint to an equality constraint. For example, eq 4-5 would be converted into an equality constraint of the form

$$P_{cv,i} = P_{cvm,i} + P_{cvs,i} \tag{5-12}$$

where $\Delta P_{cvs,i}$ is equal to the slack variable for consumer control valve pressure losses (N/m^2).

Equation 5-12 would introduce *n* slack variables into the problem. It could, however, be used immediately to eliminate the $\Delta P_{cv,i}$ unknowns in eq 4-2, thus reducing both the number of unknowns and equations by *n* to 3*nn* + *n*.

The constraints of eq 4-11 and 4-22 would each introduce *nn* slack variables as well. The constraint of eq 4-23 would introduce one less slack variable than these constraints, since there will be no slack variable at the heating plant where the pressure level is determined by the constraint dominance arguments discussed earlier. Thus, eq 4-23 will result in (*nn* – 1) additional slack variables.

In addition to the slack variables, we would still have the diameters of our pipe segments as unknowns as well. The number of pipe segments will always be one less than the number of nodes. This result is easily shown if we consider the process of building the network from one node to the next. The first two nodes introduced into the system will require one pipe. Any subsequent nodes introduced will require one pipe for each node, since one node will already be an existing connected node. The only case where this would not be true is if we had a looped network, rather than the pure branched networks to which we will limit our discussion. We have one additional unknown ΔP_{hp} that appears in all of the constraints of eq 4-2. So, then our total number of unknowns would be (*n* + 4*nn* – 1).

With (*n* + 4*nn* – 1) unknowns and only (*n* + 3*nn*) equations, we have no unique solution. Recall, however, that (*nn* – 1) of the unknowns are the pipe diameters, which must take on discrete values. If the pipe diameters were to be considered as continuous, we would have an infinite number of solutions. It's actually fortunate that they are discrete because this limits the number of possible solutions. The number of possibilities can still be quite large for a system of any significant size. For example, if we were to consider only 3 possible pipe sizes for each pipe segment we would have 3(*nn* – 1) possible solutions. For our system discussed earlier with 125 nodes (124 pipe segments), we would have $3^{124} \cong 1.46 \times 10^{59}$, a combinatorial problem of staggering proportions by any measure.

Notice that by applying monotonicity analysis to this problem we were able reduce it to one of solving for the variables using the constraint set, which has been reduced somewhat. The constraint set is linear in all the variables except the pipe diameters and the pipe diameters only appear in one set of constraints. We could make the problem linear by making the transformation for pipe diameters of

$$\hat{d}_j = d_j^{-(5+b+c)}.$$

We would then substitute this into the constraints of eq 4-2 and solve the resulting problem linearly in the $\hat{d}_j$ values. However, we would still have a significant task in the solution of the system of equations. For this reason we will abandon the possibility of achieving a solution by this approach.

Constraint resolution by pipe size refinement

We have an infeasible solution from the unconstrained problem. For each of the consumers whose absolute pressure distribution of Figure 8 exceeds the constraints, we need to reduce the piping pressure losses by increasing pipe diameters enough to allow for constraint satisfaction. Since the system constraints all deal with pressure levels in the network, we need to find a strategy to resolve these constraint violations. Let's attempt to find a solution by starting with our optimal independent design and identifying methods to move from this design to one that satisfies all the constraints. We will attempt to conduct this process of modifying the solution so that it satisfies the constraints in a manner that will keep us as close as possible to the true globally optimal design. We have the distinct advantage of knowing that our optimal independent design will form an absolute lower bound on system cost. At any point we can compare the cost of our feasible design to the cost of the optimal independent design and determine if further attempts at improvement are warranted.

Examining Figure 8, we see that to bring excessive pressure differences within the bounds of the constraints, we will need to reduce the slope of the pressure vs. distance lines. The slope of these lines is the pressure loss per unit length of pipe. Equations 5-8 and 5-9 tell us that if we are to reduce the slope we must do so by increasing the pipe size. We would like to identify a method of determining which pipe sizes to increase and by how much to satisfy constraints with minimum cost increase.

At first it might seem that the best procedure would be to start by increasing pipe sizes at the consumer's end of the system, where the sizes are smallest and the pipes tend to be shorter. In the smaller pipe sizes, the incremental increases in diameters are in general less than for the larger pipe sizes. Thus, we could make smaller moves away from the lower bounding cost. Starting at the consumer appears to be the most logical way to proceed if the critical consumer is the only consumer who has exceeded the absolute pressure constraint. In the more general case, however, more than one consumer will have violated the absolute pressure constraints; thus, we will examine that case first.

If more than one consumer has violated the absolute pressure constraints, we could achieve constraint satisfaction by increasing pipe sizes that serve each consumer individually until all the constraints are satisfied. Alternately, we could increase pipe diameters in pipes that serve all of the consumers with violated constraints. Because the pipe sizes are discontinuous (discrete) and the incremental differences between adjacent diameters are nonuniform, it's not possible for us to predict a priori which diameters would be the best candidates for increasing. Thus, we need to identify a method that will guide our search for a feasible and acceptable solution expediently. In deciding when to stop our search, we always have the benefit of knowing our lower bounding cost.

If we refer back to Figure 8, we see that satisfaction of the absolute pressure constraints relies on keeping the pressure in the supply and return lines within the bounds prescribed by the maximum absolute pressure constraint and the two minimum absolute pressure constraints. We can adjust pressures at the plant to achieve a state that satisfies all the constraints, as long as the maximum pressure difference within the system does not exceed the absolute pressure constraints discussed above. Since the critical consumer previously identified will be the consumer who requires the largest pressure differential within the system, we will examine this consumer's requirements first and attempt to resolve the constraint

1. For the set of consumers whose constraints are violated, find the pipe segments that they all share in common. Identify those pipe segment within this group that are shared with no other consumers. In the event that there are no pipe segments shared with no other consumers, choose those pipe segments shared with the minimum number of other consumers.

2. Increase the pipe segment diameters within this set enough so that the consumer whose constraint from eq 4-2 was closest to satisfaction in the original solution now has his constraint satisfied. The first pipe segment to have its diameter increased should be the one that this consumer shares with the largest number of other consumers within the set of consumers with violated constraints.

3. Now remove this consumer whose constraint is satisfied from the set and find the new set of common pipe segments for the remaining set of consumers with violated constraints, again including the minimum number of other consumers in this set.

4. Again increase the pipe segment diameters within the remaining set enough so that the consumer whose constraint was closest to satisfaction in the new set now has his constraint satisfied. The choice of the pipe segment diameter to increase first is done in the same way as in step 2 above.

5. Repeat the above steps until all consumers have their constraints satisfied.

Figure 9. Method A.

violations that result. In the process we will consider the other consumers whose constraints have also been violated.

Starting from an infeasible point, which is at the lowest possible cost for any design, we want to move in the direction that will satisfy all of the constraints that are violated by this solution. Since the critical consumer is the consumer whose constraint has been violated by the greatest amount, we will have to travel the "furthest" from our infeasible point to the boundary of his constraint. Thus, it would seem tempting to try to resolve this constraint first and then look and see what other violated constraints remain. However, it's possible that we can plot a course that will take us straight to a point that will resolve all constraints rather than handling them one at a time. To do so we might consider the algorithm given in Figure 9.

Note that the last consumer to have his constraint satisfied is the consumer who was identified as the critical consumer in the original solution. Now, however, all of the consumers whose pressure constraints were violated in the original solution are "critical" consumers as well, having pressure levels just meeting the constraints, within the tolerance achievable with the discrete pipe diameters available.

As an alternate to the above methodology, we could proceed by adjusting diameters of the critical consumer first, but only enough to bring his piping pressure loss to the level of the next highest consumer, i.e., using the method in Figure 10.

Since in many cases consumers will share more than one pipe segment, we still may be left with a number of alternatives that must be evaluated at each of the steps above. If in each case we choose the alternative that produces the minimum amount of increase in cost over the previous design, we should be able to move to an ultimate solution that satisfies all the constraints while reducing the cost as much as practical. Because we may be faced with many possible alternatives when a number of pipe segments are shared by two or more consumers, we may decide to stop the process after finding an alternative whose cost is within some reasonable tolerance of the

1. Proceed by first finding the set of servicing pipe segments unique to the critical consumer; this will be his final service pipe only. Increase the diameter of that pipe segment until it reduces his pressure loss to the same level as the consumer with the next highest pressure losses.

2. Now identify the pipe segments that these two consumers alone share and increase those pipe diameters enough to reduce their pressure losses to the level of the next highest consumer. Note that it may be that there are no shared pipe segments for these two consumers alone. In that event proceed to the next step directly.

3. Again look for pipe segments shared by the three consumers with the highest pressure losses and increase the diameters of those pipe segments enough to bring the pressure losses of these three consumers to the level of the consumer with the fourth highest pressure loss. Once again, in the event that no shared pipe segments exist, proceed directly to the next step.

4. Repeat this procedure until no consumers remain with pressure losses exceeding the constraints.

Figure 10. Method B.

lowest cost up to that point in the process.

To address the instances where more than one alternative is available at a particular step in either of the processes outlined in Figures 9 and 10, we would like a strategy that minimizes cost. Let's investigate the effect of pipe diameter to see if it would be to our advantage to choose smaller or large pipes as candidates for the diameter increase.

First, we note that the capital cost $C_{pv,j}$ is a linearly increasing function of pipe diameter. Thus, an incremental increase in pipe diameter would have the same effect regardless of the absolute value of the pipe diameter.

The cost of heat loss $C_{hl,j}$ is a somewhat complicated function of the pipe diameter. It also includes an approximation introduced in Chapter 2. Within the range of validity of the approximation ($0.025 \text{ m} \le d \le 1.0 \text{ m}$), we can see how the heat loss cost behaves by examining its slope as shown in Figure 11.

The slope of the heat loss cost as plotted below in Figure 11 is essentially the first term of eq 2-24 with the values of the parameters taken from the example of Chapter 3. From Figure 11 we see that the slope of the heat loss curve is always positive within our range of interest. This tells us that whenever we increase the pipe diameter we will increase heat losses, as we would expect. We also see that the slope is a decreasing function of the diameter, except for pipe diameters over about 0.75 m, where it becomes a slightly increasing function. For the portion of the range where the slope is decreasing, we know that an incremental change in pipe diameter will result in less increase in heat loss for larger diameters than for smaller ones.

The pressure loss as a function of pipe diameter is given by the sum of eq 5-8 and 5-9, which is our former eq 4-4

$$\Delta P_{s\&r} = (a\, \varepsilon^b\, (4/\pi)^{2+c}\, A_6\, \dot{m}_d^{\,2+c}\, L\, d^{\,-(5+b+c)})_j. \tag{4-4}$$

If we take the partial derivative of this pressure loss with respect to diameter, we have

$$\partial \Delta P_{s\&r}/\partial d = -(5 + b + c)\, (a\, \varepsilon^b\, (4/\pi)^{2+c}\, A_6\, \dot{m}_d^{\,2+c}\, L\, d^{\,-(6+b+c)})_j.$$

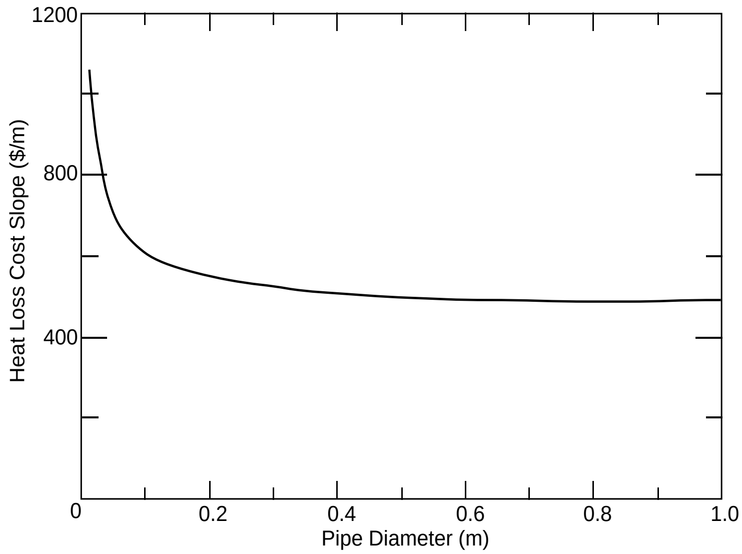

Figure 11. Slope of the heat loss cost term as a function of pipe diameter.

This result is plotted in Figure 12, where we have arbitrarily set the aggregate of all the coefficients of d equal to –1. We see that the slope is everywhere negative and that it is increasingly negative for decreasing values of the pipe diameter. This tells us that an incremental increase in the pipe diameter will have a greater effect on reducing the pressure loss at smaller pipe diameters than at larger ones. Thus, if we have a choice of several possible pipe segments whose diameters we can increase, we can achieve a larger pressure loss reduction for a given increase in pipe size by choosing the pipe segment with the smallest diameter.

Unfortunately, our results tell us we should favor the smaller diameters from a pressure losses reduction standpoint, but from the standpoint of heat loss costs,

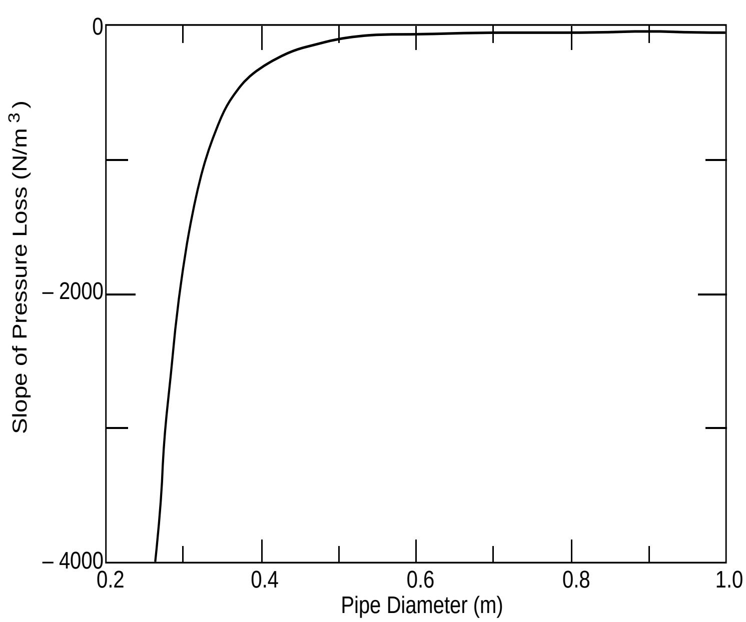

Figure 12. Slope of the pressure loss as a function of pipe diameter.

larger diameters would be better. Thus, it is unclear from this analysis whether we should try to choose smaller or larger pipes for increased diameters to satisfy constraints. We do know that incremental increases in the discrete sizes of pipes will be smaller in general for the smaller pipes. However, this does not necessarily mean that we will be able to get closer to just satisfying the constraint in all cases by increasing the diameter of the smaller pipes. Fortunately, the branch-and-bound method mentioned earlier will provide us with a general solution strategy despite our inability to better characterize the nature of the path to the solution.

Branch-and-bound method

The objective of the branch-and-bound method is to use what is known about designs already explored to reduce the number of remaining ones that must be examined in detail. An additional caveat is that we would like to do so without dismissing any designs superior to the best feasible ones identified. According to Reklaitis et al. (1983), the branch-and-bound algorithm is the most widely used method for mixed integer problems and is the basis for most commercial computer codes for solving such problems. The essence of the branch-and-bound method is that it breaks the problem down into branches, each of which corresponds to some particular choice of a single discrete decision variable.

The first step in the method is to compute the optimal solution to the problem with all the variables assumed to be continuous. This becomes our global lower bound. The branching usually starts by choosing what is felt to be the most fruitful branch (variable) to explore; some criteria are given in Reklaitis et al. (1983). A discrete value for the variable of this branch is chosen; this will be one of the discrete values bracketing the optimum continuous value. The lower bounding cost for this branch is found by finding the optimum continuous values of the remaining decision variables with the branching variable fixed at its discrete value. At this point we can explore the discrete designs within this branch by branching on the other variables or we can go directly to the next branch. If we continue to explore this branch and we find a discrete design sufficiently close to our lower bounding continuous design, we can stop searching and accept this design. Otherwise, we move on to the next branch. If we decide to search this branch further, we do so by comparing costs obtained for designs within the branch by making permutations of the other decision variables in turn to their bracketing discrete values.

This process continues until a feasible design with discrete values for all those variables requiring such values is found. This is our first candidate design and its cost becomes our upper bounding cost. Through the remainder of the process all solutions will be compared in cost to this one until another feasible discrete design with a lower cost is found. Any designs with higher cost will immediately be rejected, and if continuous variables are still included in these designs for variables that must ultimately take on discrete values, then all other designs within that branch will be rejected as well. This can be done since we know that restricting any of the continuous variables to discrete values will only increase the cost. The process of rejecting these other designs for which the cost is never computed is called "implicit fathoming," as opposed to "explicit fathoming" where the cost is computed and the design is rejected because its cost exceeds our upper bounding cost. Any remaining feasible discrete designs are also compared to the lower bounding cost for the branch found with continuous values of the non-branching variables. If any are sufficiently close to this lower bound, the search of the branch is concluded and the design found is accepted as an adequate design representative of what can be expected within the branch.

The next branch to be explored will use the other bracketing value of the first branching variable. Its lower bounding continuous cost design is compared to our current upper bounding cost from the best design of the previous branch. If the lower bounding cost of this branch is less, then we continue the search of this branch, proceeding as we did in the first branch, with one exception. We now have an upper bounding cost and if at any point we find a design, either fully discrete of not, higher in cost than our upper bounding design, we fathom this node and implicitly fathom any branches of this design. However, as long as improvement appears possible, the branch is searched until a discrete design acceptably close to the lower bounding cost is found or all alternatives are exhausted. If a discrete design with a lower cost than the lowest cost discrete design of the previous branch is found, it becomes the new upper bound on cost. If this cost is sufficiently close to the continuous lower bounding cost, then the search is concluded. Otherwise, another variable is chosen to branch on. When exploring alternatives within any main branch, the same basic branch-and-bound approach is applied within the "sub-branch." Below we will show how the branch-and-bound method is applied to our problem.

Solution by the branch-and-bound method

Before we can apply the branch-and-bound method as described above, we must first have a feasible solution point from which to start. To find such a solution, we use one of the methods described earlier in the section entitled *Constraint Resolution by Pipe Size Refinement.* For example, if we use the method A step 2 (Fig. 9), we calculate the *continuous* pipe size necessary to reduce the pressure loss to the next level as described there. This is done by combining eq 4-2 and 4-4 to obtain

$$d_{j_1} = \left\{ \frac{\left[\Delta P_{\mathrm{hp,i}} - \left(\sum_{j \neq j_1} \left(\Delta P_{\mathrm{s}} + \Delta P_{\mathrm{r}} \right)_{\mathrm{j}} + \Delta P_{\mathrm{cv}} + \Delta P_{\mathrm{he}} \right)_{\mathrm{i}} \right]}{\left(a\, \varepsilon^{b}\, (4/\pi)^{2+c}\, A_6\, \dot{m}_{\mathrm{d}}^{2+c}\, L \right)_{\mathrm{j}_1}} \right\}^{(-1/(5+b+c))} . \tag{5-13}$$

In this case the pipe segment index j_1 is the pipe segment whose diameter we have chosen to increase and the consumer index i is for the consumer with the second highest pressure loss at the heating plant as determined by eq 4-2. We continue to use method A, which essentially repeats steps 1 and 2 until all the constraints that were previously violated are now satisfied. When we have finished, we can check our application of the method by taking the pipe segments that were in the sets of those constraints previously violated and decreasing them to the next lowest pipe diameter one at a time. If our application of method A was correct, each pipe diameter that is decreased should result in the violation of at least one constraint.

At the conclusion of the use of method A, we will have a number of pipe segments whose diameters are continuous as a result of the refinement process used by the method. To obtain discrete diameter values for these pipe segments, we use the branch-and-bound method as described above. Note that the *unconstrained discrete* solution, what we have called our "optimal independent design," will form a greater lower bound than the *unconstrained continuous* solution. It may not, however, form a greater lower bound than the *constrained continuous* solution, which we have chosen not to attempt to find owing to the computational effort involved, as discussed earlier. What we have found using method A is a feasible solution whose cost is greater than the *unconstrained discrete* solution. The solution we have must also be higher in cost than the *constrained continuous* solution, since the deletion of the requirement of discrete sizes for those pipe segments that already have discrete sizes would allow us to find a feasible solution at some lower cost. We can not be certain, however, that the solution of method A is lower in cost than the *constrained discrete* solution that we seek. For this reason, when we use the cost found in method

A as our lower bounding cost, we can not be certain that this is the greatest lower bound. If the application of the branch-and-bound method finds feasible solutions with costs lower than our solution found by method A, then we must from that point on use the *unconstrained discrete* solution as the greatest lower bound.

Note that, similar to method A (Fig. 9), method B (Fig. 10) would also provide a number of pipe segments whose diameters are continuous as a result of the refinement process used. Thus, we would proceed in the same manner with the branch-and-bound method regardless of which of the two methods had been used for the pipe size refinement process.

The application of the branch-and-bound method to our feasible solution of method A or B is straightforward and proceeds as described earlier. Many of the branches will have infeasible designs, since we know decreasing any of the continuous pipe diameters will result in the violation of at least one constraint, unless other pipe diameters have also been increased. Thus, any branch that only decreases pipe diameters in any one of the combinations in which they appear in the previously violated constraints will be infeasible and need not be explored.

Consider a hypothetical case where we have four pipe segments with continuous diameters after the application of either method A or B. If we limit ourselves to only the two discrete pipe sizes that bracket the continuous values found, we will have $2^4 = 16$ possible discrete solutions. A "tree" diagram of the problem is shown in Figure 13. The symbols within each "node" of the tree represent the particular case being evaluated. The "0" symbol indicates the continuous pipe size as found by either method A or B. A "+" symbol indicates the next larger discrete pipe size and a "–" symbol indicates the next smaller discrete pipe size. The sequence of the symbols represents the order of the four pipe segments under investigation. As noted above, any possibilities that only decrease the size of one or more pipe segments are immediately known to be infeasible. These nodes have been shown with dotted outlines in Figure 13.

Applying the branch-and-bound method, we would proceed initially by choosing the first pipe segment to branch on. Starting at the top of Figure 13, we branch on the first pipe segment by computing the value of the objective function with the branching diameter, rounded both up and down to the adjacent discrete diameter values. However, we notice that one of the options, the (– 000) case, is infeasible. Thus, there is no use in computing the value of the objective at that point since it's of no use as a lower bound on the constrained problem. In this case we then proceed further down this branch in search of a feasible case. Suppose that we find that the (– + + 0) is the first feasible case in this branch. We then branch on the last pipe segment by examining the cases (– + + +) and (– + + –). Assume that we find both of these to be feasible, but the (– + + –) case is lower in cost. This is our first completely discrete and feasible design. It now becomes our upper bounding cost. If this cost is sufficiently close to our current lower bounding cost of the (0000) case, then we can

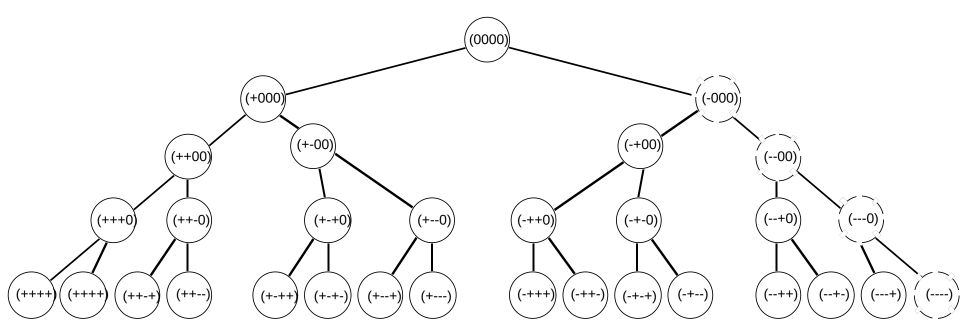

Figure 13. Hypothetical branch-and-bound problem.

accept this solution and terminate the search. Let's assume, however, that this is not the case.

Thus, we now need to start the process of "backtracking" and examining the delayed cases that we passed over. We begin by examining the case (– + – 0). Since we found that the (– + 00) case was infeasible, then we immediately know that the (– + – 0) case must be infeasible as well, since it only decreases diameters from an infeasible case. Consequently, the (– + – –) case must be infeasible as well for the same reason. The (– + – +) case might be feasible and let's assume that we find this to be true. Assume that the cost of this completely discrete design is less than the cost of the (– + + –) design and thus it becomes our new upper bound on cost. Again, we would compare the cost of this upper bound with the lower bounding cost of the (0000) design and decide if further searching is warranted. Let's assume that we are still not satisfied with the gap between the upper and lower bounding costs, so we decide to continue our search.

Backtracking further, we explore the alternatives (– – 0 +), (– – + +), (– – + –) and (– – – +). Assume that we find these to all be infeasible. We then return back to the other side of the first branch explored, the alternatives in the (+ 000) branch. Assume that we find that both the (+ + 00) and the (+ – 00) cases are feasible, but the (+ – 00) case has a lower cost. Thus, we defer any further exploration of the (+ + 00) branch. We continue in the (+ – 00) branch by exploring the (+ – + 0) case and the (+ – – 0) case. Suppose that both cases are feasible, but the (+ – – 0) case has a lower cost. Thus, we continue by checking the cost and feasibility of the (+ – – +) and the (+ – – –) cases. Assume that the (+ – – –) case is infeasible, but the (+ – – +) case is not only feasible but lower in cost than our former upper bounding cost of the (– + – +) design. Assume that this cost is indistinguishable from the lower bounding cost of the (0000) case, and thus we accept the (+ – – +) design and terminate the search.

As illustrated here, one of the techniques used by the branch-and-bound method is to continuously move the upper and lower bounding costs closer together until the remaining possible improvement (i.e., reduction in cost) does not justify further effort. At this point, the search can be stopped regardless of how many alternatives have actually been explored. This process is accomplished by moving the upper bound down as low as possible, i.e., finding the "least upper bound." The least upper bounding cost is always the lowest cost feasible discrete design found up to that point in the process. The difference between the lower and upper bounding costs is also refined by finding the "greatest lower bound." The lower bounding cost is initially determined by the continuous feasible design found by either method A or B. As we proceed with the branching process, we will find lower bounding costs within that branch for designs that have some, but not all, of their diameters at discrete values. These greater lower bounding costs allow us to refine the difference between the upper and lower bounding costs for that branch only. They also, however, may tell us that further exploration of the branch is unwarranted if they exceed our current upper bounding cost.

Once we have reached a solution by the method described above, there is an additional area where we can seek further cost reductions. Note first that those consumers who did not have their constraint of eq 4-2 violated will have pressure losses in their control valves greater than the minimum allowed values. We then observe that if these excessive pressure losses were absorbed by decreasing the sizes of the pipes servicing only these consumers, our constraints would still be satisfied, but the cost of the piping network would be reduced by use of the smaller pipe sizes. This possibility is explored in the next chapter.

The procedure we have developed in this chapter to solve for the pipe diameters in a general multiple-pipe–multiple-consumer system may be used with any number of pipes and consumers. In the next chapter we will illustrate the use of the method presented here on a simple example.

CHAPTER 6: A SIMPLE MULTIPLE-PIPE–MULTIPLE-CONSUMER EXAMPLE

To demonstrate our solution strategy, it's illustrative to use an example. The example should be as simple as possible, but must at the same time include the salient features of a realistic system. Thus, our simplistic system must include as a minimum:

1. A heat generating plant.
2. At least one heat consumer.
3. A section of pipe feeding only other sections of pipe but no consumers.
4. A pipe junction feeding at least two consumers with unequal hydraulic characteristics.
5. A pipe junction feeding only one consumer.

With these requirements in mind we will examine the system shown in Figure 14.

First, we define the physical parameters for the network. For each of the nodes, we assign an arbitrary elevation and, in the case of the consumers, an arbitrary maximum heat demand expressed as maximum flow mass rate at the design condition. The maximum demands assigned to the consumers are representative of multiple residential consumers or large commercial loads. The assigned values are given in Table 5.

We will also need to assign lengths to the pipe segments between the nodes. In Table 6 some arbitrary pipe segment lengths have been assigned. In addition, in Table 6 the flows in each pipe segment and the elevation change for that segment have been determined based on the data in Table 5.

Note that the maximum flow in each pipe segment has been assumed to be the sum of all consumer flows downstream (again, in the supply line sense) from it. Although this must be true at any instance during the operation of the system because of simple conservation of mass principles, for design it may not be the most appropriate assumption. Since the demand for heat by each consumer will most likely not be completely coincident in time, the maximum aggregate demand of the consumers will always be somewhat less than the total of all consumers' maximum demands. This concept, called "demand diversity" is recognized by the district heating industry and is sometimes accommodated to some extent in design calculations. If it's not included directly in the design, it has the effect of providing an additional safety factor. While at this point we will make no effort to include the effect of demand diversity in our solution methodology, it is important to make note of it, since it's here that it would be introduced.

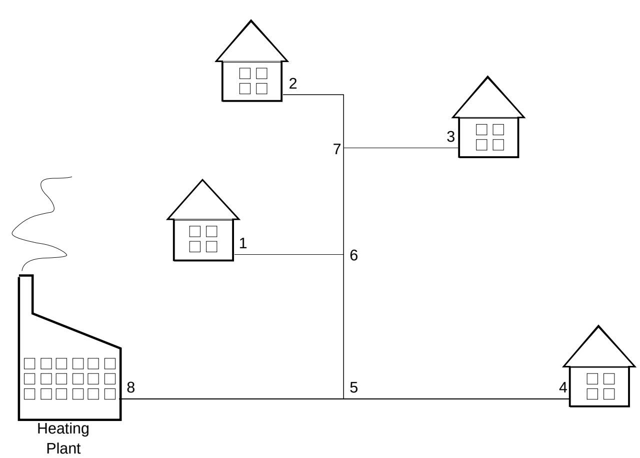

Figure 14. Simple multiple-pipe–multiple-consumer system.

Table 5. Assigned values for nodes of example in Figure 14.

Node number	*Node type*	*Elevation (m)*	*Maximum demand (kg/s)*
1	heat consumer	40	10
2	heat consumer	30	10
3	heat consumer	20	10
4	heat consumer	10	10
5	pipe junction	0	—
6	pipe junction	0	—
7	pipe junction	0	—
8	heating plant	0	40

Now we proceed to find the optimal pipe diameters for our example system of Figure 14. As we found in the last chapter, a convenient starting point is what we will call the "optimal independent design." This is the design that we would arrive at if we use the procedure developed in Chapters 2 and 3 for each pipe segment as if it were independent of the others and its design were unconstrained by the system constraints identified in Chapter 4. We will make use of all of the assumptions for parameter values and operating strategy that we used in the examples of Chapters 2 and 3. For clarity these are repeated below:

$A_{m\&r}$ = 2%/yr
A_{η} = 0.90 (dimensionless)
A_1 = \$1060/pump
A_2 = \$0.242/W
A_3 = \$218
A_4 = \$2180/m
A_{13} = 1.0 (dimensionless)
A_{14} = 0.575 (dimensionless)
A_{15} = 0.425 (dimensionless)
T_m = 6.4°C
k_i = 0.030 W/m °C
k_s = 1.3 W/m °C
H_p = 1.0 m
x_i = 0.050 m

Table 6. Assigned values for the pipe segments in the example of Figure 14.

Pipe segment	*Length (m)*	*Elevation change (m)*	*Maximum flow (kg/s)*
6,1	100	40	10
7,2	25	30	10
7,3	50	20	10
5,4	100	10	10
6,7	50	0	20
5,6	100	0	30
8,5	200	0	40

$\varepsilon = 5 \times 10^{-5}$ m
$a = 0.119$ (dimensionless)
$b = 0.152$ (dimensionless)
$c = -0.0568$ (dimensionless)
$T_s = 120°C$
T_r = determined by consumer model (eq 3-18) (°C)
$C_e = \$7.0 \times 10^{-5}$/Wh
$C_h = \$3.4 \times 1^{-5}$/Wh
$PVF_e = PVF_h = PVF_{m\&r} = 9.08$ (dimensionless)
Load control by flow modulation with consumer model (eq 3-25).

There are a number of additional parameters that were introduced in developing the multiconsumer constraints for which we have not yet assigned any typical values. They are:

$\Delta P_{cvm,i} = 5 \times 10^4$ N/m^2 (for all consumers, i = 1,4)
$\Delta P_{he,i} = 1.0 \times 10^5$ N/m^2 (for all consumers, i = 1,4)
$P_{max} = 1.0 \times 10^6$ N/m^2
$P_{saf} = 1.0 \times 10^5$ N/m^2
$P_{NPSH} = 2.0 \times 10^5$ N/m^2
$P_a = 1.0 \times 10^5$ N/m^2
$P_{asa} = 0.5 \times 10^5$ N/m^2.

Before we can find the optimal independent diameters for the pipe segments, we need to calculate the remaining parameters that are determined by the assumptions above. Because the optimal pipe diameter for a single pipe segment is independent of the pipe length and elevation (see eq 2-23), the optimal independent diameter will be the same for pipe segments (6,1), (7,2), (7,3) and (5,4). Thus, we construct Table 7 with the parameter values needed and the resulting optimal independent diameters. In each case, we have proceeded as before by solving the Lower Bounding Problem (LBP) (eq 2-20), which neglects heat losses first and, subsequently, using that as a starting point for finding the solution to the complete problem including heat losses (eq 2-24). Also, as earlier, FORTRAN programs I1EQ3-26 and I2-C-GMT were used to compute I_1 and I_3 respectively.

The optimal diameters found above do not necessarily correspond to actual discrete pipe diameters available, so before we check this solution to see if it satisfies the constraint set, we first need to determine what the optimal discrete diameters would be. Table 8 contains pipe size data for standard metric pipe sizes in the range needed for our example.

To find the optimal discrete diameters, we proceed as before in the example of Chapter 2 by simply examining the total cost of the discrete pipe diameters that

Table 7. Parameter values and optimal independent diameters for example of Figure 14.

Pipe segment	I_1/L (*\$/m*)	I_3/L (*\$ m$^{4.095}$*)	d *by LBP* (*m*)	d *by eq (2-24)* (*m*)
(6,1), (7,2), (7,3), (5,4)	73.3	4.276×10^{-5}	0.0691	0.0666
(6,7)	73.3	3.289×10^{-4}	0.0966	0.0932
(5,6)	73.3	1.085×10^{-3}	0.1175	0.1134
(8,5)	73.3	2.529×10^{-3}	0.1350	0.1304

Table 8. Standard metric steel pipe sizes (data from DFF 1985).

Nominal diameter (mm)	*Outer diameter (mm)*	*Wall thickness (mm)*	*Inner diameter (mm)*
50	60.3	2.9	54.5
65	76.1	2.9	70.3
80	88.9	3.2	82.5
100	114.3	3.6	107.1
125	139.7	3.6	132.5

bound the optimal diameters we have found. The bounding diameter that has the lowest total cost will become our optimal discrete diameter. Obviously, it is only necessary for us to compute the portion of the total cost that is dependent on pipe diameter in making this decision. Table 9 gives the cost data for the bounding discrete diameter for each of the pipe segments of our example. The costs are calculated on a unit length basis using eq 2-23 divided through by L, the pipe length. The portions of the total variable cost ascribable to each of the major component costs are also given in Table 9. The parameter values of I_1/L and I_3/L used for each pipe segment are the same as those given in Table 7.

Now we need to consider the constraints on our multiconsumer system as derived earlier in Chapters 3 and 4. These are summarized in Chapter 5 in the *System Constraint Satisfaction* subsection. All of these constraints deal with pressure levels at various points within the system. Since verification of satisfaction for these constraints requires calculation of the pressure losses in the supply and return line, we begin by doing so for each of the pipe segments. The pressure losses in the supply or return pipes are calculated with eq 5-8 and 5-9 using the optimal discrete diameters we have determined independently. The results are given in Table 10.

Satisfaction of the constraint of eq 4-2 at each of the consumers requires that we

Table 9. Discrete bounding diameters and variable costs for the example (optimal non–discrete diameters shown in boldface, optimal discrete diameters shown in italic).

		Variable costs ($/m)				
Pipe segment	d *(m)*	*Heat loss*	*Capital*	*Pumping*	*Total*	*Discrete cost premium (%)*
	0.0545	64.76	140.61	117.31	322.68	12.44
(6,1), (7,2),	**0.0666**	**72.90**	**171.83**	**42.23**	**286.96**	—
(7,3), (5,4)	*0.0703*	*75.34*	*181.37*	*32.06*	*288.78*	*0.63*
	0.0825	83.24	212.85	109.11	405.20	4.14
(6,7)	**0.0932**	**90.02**	**240.46**	**58.62**	**389.09**	—
	0.1071	*98.65*	*276.32*	*28.87*	*403.84*	*3.79*
	0.1071	*98.65*	*276.32*	*95.21*	*470.18*	*0.85*
(5,6)	**0.1134**	**102.51**	**292.57**	**71.16**	**466.24**	—
	0.1325	114.03	341.85	32.19	488.07	4.68
	0.1071	98.65	276.32	222.03	597.00	12.51
(8,5)	**0.1304**	**112.77**	**336.43**	**81.44**	**530.64**	—
	0.1325	*114.03*	*341.85*	*75.07*	*530.95*	*0.058*

Table 10. Pressure losses for the pipe segments in the example.

Pipe segment	*Discrete diam. (m)*	*Length (m)*	*Elevation diff. (m)*	*Flow rate (kg/s)*	$P_{d,s}$ *(N/m²)*	$P_{d,r}$ *(N/m²)*
6,1	0.0703	100	40	10	91,585	91,652
7,2	0.0703	25	30	10	22,896	22,913
7,3	0.0703	50	20	10	45,793	45,826
5,4	0.0703	100	10	10	91,585	91,652
6,7	0.1071	50	0	20	20,615	20,630
5,6	0.1071	100	0	30	90,655	90,721
8,5	0.1325	200	0	40	107,219	107,297

sum the pressure losses in the portions of the piping system servicing each consumer. Recall that earlier we assigned values for the minimum pressure drop in the consumer's control valve and the pressure drop in the consumer's heat exchanger, $\Delta P_{\mathrm{cvm,i}}$ and $\Delta P_{\mathrm{he,i}}$, respectively, as follows:

$$\Delta P_{\mathrm{cvm,i}} = 5 \times 10^4 \ \mathrm{N/m^2} \qquad \text{(for all consumers, } i = 1{,}4\text{)}$$
$$\Delta P_{\mathrm{he,i}} = 1.0 \times 10^5 \ \mathrm{N/m^2} \qquad \text{(for all consumers, } i = 1{,}4\text{).}$$

We will also assume initially that for each consumer $\Delta P_{\mathrm{cv,i}} = \Delta P_{\mathrm{cvm,i}}$. The process of calculating the ΔP_{hp} for each of the consumers is summarized in Table 11.

The pressure increase required across the pump at the heating plant ΔP_{hp} that we have calculated is different for each consumer. As shown in Chapter 5, since ΔP_{hp} can only assume one value, it must be the greatest value that results from consideration of all the consumers, and for the other consumers $\Delta P_{\mathrm{cv,i}}$ will be greater than $\Delta P_{\mathrm{cvm,i}}$. The consumer who requires the greatest ΔP_{hp} is called the critical consumer.

Referring to Table 11, we see that consumer 1 is our critical consumer and thus $\Delta P_{\mathrm{cv,1}} = P_{\mathrm{cvm,1}}$. This has been determined for the maximum load condition, but will hold for all load conditions since we have assumed that all consumers have loads varying in the same manner over the yearly cycle. For the other consumers, eq 4-2 will require that $\Delta P_{\mathrm{cv,i}} > \Delta P_{\mathrm{cvm,i}}$. By using the ΔP_{hp} calculated for consumer 1, our critical consumer, we find the following values for the $\Delta P_{\mathrm{cv,i}}$ of the other consumers at the maximum load condition:

$$\Delta P_{\mathrm{cv,2}} = 146{,}183 \ \mathrm{N/m^2}$$
$$\Delta P_{\mathrm{cv,3}} = 100{,}374 \ \mathrm{N/m^2}$$
$$\Delta P_{\mathrm{cv,4}} = 231{,}376 \ \mathrm{N/m^2}.$$

Notice that the control valve pressure drops for all these consumers are high compared to the minimum value of $5 \times 10^4 \ \mathrm{N/m^2}$. This situation sometimes makes it difficult for the control valve to function properly. It also represents a wasteful

Table 11. Heating plant pressure increase required by eq 4-2 for each consumer.

Consumer index, i	*Servicing pipes,* j	$\sum_j \Delta P_{s,j}$ *(N/m²)*	$\sum_j \Delta P_{r,j}$ *(N/m²)*	$P_{cvm,i} + P_{he,i}$ *(N/m²)*	ΔP_{hp} *(N/m²)*
1	(6,1), (5,6), (8,5)	289,459	289,670	150,000	729,129
2	(7,2), (6,7), (5,6), (8,5)	241,385	241,561	150,000	632,946
3	(7,3), (6,7), (5,6), (8,5)	264,281	264,474	150,000	678,755
4	(5,4), (8,5)	198,804	198,949	150,000	547,753

practice, since pumping energy, inherently more expensive than heat energy, is being converted into frictional heating of the fluid. As noted at the close of the last chapter, there is an alternative to these high control valve pressure losses: reduce the pipe sizes further such that the pressure differential at the consumer's control valve is reduced. Such a practice was proposed by DFF (1985), where they suggest reducing the size of the "service pipes," those ultimately connecting the consumer to the network. It may also be possible to reduce some of the pipes sizes within the network as well. For our example problem, we see that we have limited options. Consumer 1 is our critical consumer, so we cannot reduce any of the pipe sizes servicing this customer; this rules out the pipe segments (6,1), (5,6) and (8,5). The remaining pipe segments are (7,3), (6,7), (5,4) and (7,2). Thus, we investigate the possibility of reducing the size of these pipes.

First, let's look at pipe segment (5,4). This is the only pipe segment serving consumer 4; thus, this is the only option for reducing the pressure loss in this consumer's control valve. What we would like to do is find the minimum pipe size that will not violate the constraint of eq 4-2 for consumer 4. Effectively, what has happened here is that we have removed the pumping power term from the objective function so it now becomes monotonically increasing in $d_{(5,4)}$. We need to find the constraint that will bound $d_{(5,4)}$ below. While not immediately obvious, eq 4-2 forms such a constraint on $d_{(5,4)}$ when directed as follows

$$h_1 = \sum_j \left(\Delta P_\mathrm{s} + \Delta P_\mathrm{r}\right)_\mathrm{j} + \Delta P_\mathrm{cv} + \Delta P_\mathrm{he} - \Delta P_\mathrm{hp} \equiv < \;\; 0 \qquad \left(\Delta P^+_{\mathrm{s},(5,4)}, \Delta P^+_{\mathrm{r},(5,4)}\right).$$

From eq 4-4, we see that $\Delta P_{\mathrm{s},(5,4)}$ and $\Delta P_{\mathrm{r},(5,4)}$ are related to $d_{(5,4)}$ by

$$h_2 = \{[(\rho^{-1}\,\mu^{-c})_{\mathrm{d,s}} + (\rho^{-1}\,\mu^{-c})_{\mathrm{d,r}}]$$

$$[(a/2)\;\varepsilon^b\;(4/\pi)^{2+c}\;\dot{m}_\mathrm{d}^{2+c}\;L\;d^{-(5+b+c)}]_{(5,4)}\} - (\Delta P_\mathrm{s})_{(5,4)} - (\Delta P_\mathrm{r})_{(5,4)} \equiv< 0$$

with the monotonicities being $h_2 = \left(d^-_{(5,4)}, \Delta P^-_{\mathrm{s},(5,4)}, \Delta P^-_{\mathrm{r},(5,4)}\right)$.

So, we see that $d_{(5,4)}$ is bounded below by h_2 and that the non-objective variables $\Delta P_{\mathrm{s},(5,4)}$ and $\Delta P_{\mathrm{r},(5,4)}$ are bounded below by this constraint and above by h_1, as required by the second monotonicity principle (see Papalambros and Wilde 1988). Now we can use constraints h_1 and h_2 to find the optimal value of $d_{(5,4)}$. To do so we treat h_1 as a strict equality and solve for $(P_\mathrm{s})_{(5,4)} + (P_\mathrm{r})_{(5,4)}$. We then substitute the result into h_2, again treating it as a strict equality, and solve for $d_{(5,4)}$. The result is

$$d_{(5,4)} = 0.0614 \text{ (m)} .$$

The discrete diameters that bracket this value are 0.0545 and 0.0703 m. The lower bracketing discrete diameter will cause constraint h_1 to be violated since a decrease in $d_{(5,4)}$ will increase $\sum_j \left(\Delta P_\mathrm{s} + \Delta P_\mathrm{r}\right)_\mathrm{j}$. The optimal discrete diameter determined previously was 0.0703 m; thus, we are unable to improve on this result.

Let's look at the remaining pipe segments (7,3), (6,7) and (7,2). These pipe segments serve both consumers 2 and 3. Consumer 2 is served by pipe segments (6,7) and (7,2) and consumer 3 is served by pipe segments (7,3) and (6,7). Both consumers are served by pipe segment (6,7); thus, any decisions we make about this pipe segment must be checked to ensure that both consumer constraints (eq 4-2) are obeyed. Also, notice that if we decrease one of the pipe sizes and this violates constraint h_1, we may be able to increase the other pipe size in the pair serving that consumer such that the total costs for the pipes and heat losses are reduced but constraint h_1 is still satisfied. It is also possible that a pipe size could be reduced or

Table 12. Pipe size combinations for the example.

Combination number	*Pipe segment* (6,7)	(7,2)	(7,3)
1	0*	0	0
2	0	0	+
3	0	0	–
4	0	+	0
5	0	+	+
6	0	+	–
7	0	–	0
8	0	–	+
9	0	–	–
10	+	0	0
11	+	0	+
12	+	0	–
13	+	+	0
14	+	+	+
15	+	+	–
16	+	–	0
17	+	–	+
18	+	–	–
19	–	0	0
20	–	0	+
21	–	0	–
22	–	+	0
23	–	+	+
24	–	+	–
25	–	–	0
26	–	–	+
27	–	–	–

*0 = pipe size unchanged; + = pipe size increased; – = pipe size decreased.

increased by more than one discrete size. We will ignore this possibility for the moment and return to it later, since it would result in many more combinations to be checked, most of which would violate h_1.

If we first look at all the possible combinations of increasing or decreasing the three pipe sizes without regard to the constraints, we have $3^3 = 27$ independent possibilities; they are enumerated in Table 12. Combination number 1 is our design as it now stands, the "do nothing" option. A number of these combinations are known not to yield improvement in our design, however, and may be immediately dismissed without further evaluation.

Specifically, any combination that increases any pipe sizes while decreasing none will only result in additional pipe capital and heat loss costs and thus will be worse than our design as is. Thus, the combinations 2, 4, 5, 10, 11, 13 and 14 can be dismissed.

In addition, we know that any combination that increases the diameter of either the final pipe servicing consumer 2 [(7,2)] or consumer 3 [(7,3)], while decreasing the other and leaving pipe segment (6,7) unchanged, would be more costly than doing the same yet not increasing the diameter of the one pipe; thus, we eliminate combinations 6 and 8. As we proceed to explore the various combinations remaining, we will discover that many other possible combinations will immediately be shown to be infeasible by the infeasibility of related combinations.

In Table 13 we have listed the remaining combinations. Table 13 also gives the

Table 13. Constraint satisfaction and costs for the remaining combinations. Our original design (combination no. 1) is shown in bold, and the other feasible designs are shown in italic.

Comb. no. and type	$d_{(6,7)}$ *(m)*	$d_{(7,2)}$ *(m)*	$d_{(7,3)}$ *(m)*	$(h_1)_2$ *(N/m²)*	$(h_1)_3$ *(N/m²)*	*Variable costs ($)*	*Cost premium (%)*
1 (0,0,0)	**0.1071**	**0.0703**	**0.0703**	**–96,183**	**–50,374**	**38,002**	**0**
3 (0,0,–)	0.1071	0.0703	0.0545	–96,183	193,207	35,435	–6.76
7(0,–,0)	0.1071	0.0545	0.0703	25,607	–50,374	36,718	–3.38
9(0,–,–)	0.1071	0.0545	0.0545	25,607	193,207	34,151	–10.13
12(+,0,–)	0.1325	0.0703	0.0545	–123,482	165,908	39,480	3.89
15(+,+,–)	0.1325	0.0825	0.0545	–149,022	165,908	40,464	6.48
16(+,–,0)	*0.1325*	*0.0545*	*0.0703*	*–1,692*	*–77,673*	*40,764*	*7.27*
17(+,–,+)	*0.1325*	*0.0545*	*0.0825*	*–1,692*	*–128,753*	*42,732*	*12.45*
18(+,–,–)	0.1325	0.0545	0.0545	–1,692	165,908	38,196	0.51
19(–,0,0)	0.0825	0.0703	0.0703	18,468	64,278	34,058	–10.38
20(–,0,+)	0.0825	0.0703	0.0825	18,468	13,198	36,027	–5.20
21(–,0,–)	0.0825	0.0703	0.0545	18,468	307,859	31,491	–17.13
22(–,+,0)	0.0825	0.0825	0.0703	–7,071	64,278	35,042	–7.79
23(–,+,+)	0.0825	0.0825	0.0825	–7,071	13,198	37,011	–2.61
24(–,+,–)	0.0825	0.0825	0.0545	–7,071	307,859	32,475	–14.54
25(–,–,0)	0.0825	0.0545	0.0703	140,259	64,278	32,774	–13.76
26(–,–,+)	0.0825	0.0545	0.0825	140,259	13,198	34,743	–8.58
27(–,–,–)	0.0825	0.0545	0.0545	140,259	307,859	30,207	–20.51

status of the two consumer constraints that must be satisfied and the total of the variable portions of the capital costs and heat loss costs for each combination. We see by examining the constraint satisfaction that only two combinations are feasible, i.e., they satisfy the h_1 constraint for both consumers 2 and 3. However, when we calculate the cost of these feasible combinations, we find that both cost more than our original design. Thus, we are left with the result that none of the alternatives investigated so far are better than our original design. There are some additional designs that we have not investigated, however. Recall that earlier we dismissed the possible designs that would increase or decrease pipe sizes by more than one discrete size from the original design. Depending on how many pipe sizes we are willing to deviate from our original design, there are many alternate designs. Of course, there is no guarantee that these designs will be feasible, let alone lower in cost than the original design. To explore these designs without resorting to "exhaustive enumeration," i.e., calculating the constraint satisfaction and cost of each, we can use the branch-and-bound technique described in detail in Chapter 5. Below we apply this technique to our example problem. In the process of doing so, we will not only explore additional designs not considered yet, but we will show how the technique would have allowed us to dismiss some of the alternatives in Table 13 without computing the constraint satisfaction or total variable cost.

As noted in the previous chapter, the objective of the branch-and-bound technique is to use what is known about designs already explored to reduce the number of remaining ones that must be examined in detail. In addition, we would like to do so without dismissing any designs superior to the best feasible ones identified. We have effectively already used the technique above to dismiss nine of the possible combinations of Table 12. In that case, we used the fact that the variable portions of the heat losses and capital pipe costs were monotonically increasing in pipe diameter. This allowed us to dismiss cases that only increased pipe size.

After our initial elimination of nine combinations, as discussed above, we see that half of our remaining combinations involve the case where $d_{(6,7)}$ is reduced; thus, we will explore that "branch" first. With $d_{(6,7)}$ assigned a discrete diameter one size lower than our original design, we can use the constraint h_1 for consumers 2 and 3 to find the lower bounding continuous values for $d_{(7,2)}$ and $d_{(7,3)}$. We obtain

$$d_{(7,2)} = 0.0778$$
$$d_{(7,3)} = 0.0892.$$

Thus, any combinations with discrete pipe diameters less than these need not be considered, since they would violate the h_1 constraint. This rules out combinations 19, 20, 21, 25, 26 and 27 because these would violate the h_1 constraint for both consumers. Also, we see that combinations 22, 23 and 24 would all violate the h_1 constraint for consumer 3, so they are infeasible as well. Thus, we have eliminated all the combinations in this branch as originally proposed. As noted earlier, there are combinations that deviate by more than one pipe size from our original design that were not considered. Before exploring any of these, we compute the cost of the design above with continuous diameters to see if it is an improvement on our original design. When doing so we find that the variable cost portion of the heat loss and pipe capital costs is slightly less than our original design: a 0.77% reduction. At this point we could decide not to further explore this branch, since it offers such a small potential for improvement; however, we will continue since it illustrates the method to be used. From combinations 22, 23 and 24, we know that if we increase the pipe size of $d_{(7,3)}$ to the next discrete pipe size greater than 0.0892, the h_1 constraint for consumer 3 will be satisfied as well. Thus, we propose the discrete design

$$d_{(6,7)} = 0.0825$$
$$d_{(7,2)} = 0.0825$$
$$d_{(7,3)} = 0.1071.$$

We know that this design is feasible, so now we need to compute its cost to see if it's an improvement over our original design. When the variable portion of the heat loss and pipe capital cost is computed, we see that it's 7.77% greater than the original design. Thus, we dismiss this design as well as any other feasible designs in this branch, since all other feasible designs would need to have larger discrete diameters and would thus be more costly yet.

We have two other major branches yet to explore: one where $d_{(6,7)}$ remains the same as in the original design and one where it is increased one discrete pipe size. The latter branch has four combinations remaining, one more than the other branch, so we will explore it first. We proceed as before by using the h_1 constraint for consumers 2 and 3 to find the lower bounding values for the continuous diameters of $d_{(7,2)}$ and $d_{(7,3)}$, obtaining

$$d_{(7,2)} = 0.0544$$
$$d_{(7,3)} = 0.0624.$$

As before, we also compute the total variable cost portion of the heat losses and pipe capital costs for this design. We find that this cost is 3.88% greater than our original design. Thus, we need not look at any discrete designs in this branch, since all will require larger discrete diameters than those continuous diameters found above and thus they will be more costly. Note that the two feasible combinations 16 and 17 in this branch identified in Table 13 do in fact have costs in excess of 3.88% above the original design.

Now we explore the remaining branch, where $d_{(6,7)}$ is the same discrete pipe size as found in our original design. As before, we compute the minimum continuous diameters using the h_1 constraint for consumers 2 and 3 to find the lower bounding values for the continuous diameters of $d_{(7,2)}$ and $d_{(7,3)}$, obtaining

$$d_{(7,2)} = 0.0564$$
$$d_{(7,3)} = 0.0646.$$

We know that this design must have a lower cost than the original since it has the same pipe size for segment (6,7) and smaller pipe sizes for the other two pipe segments. However, we go ahead and compute this cost saving to see if it justifies exploring this branch further. We find that the saving is a significant 5.40%. We have three combinations (3, 7 and 9) from Table 13 that have not been previously eliminated from this branch. We see, however, that each of these will violate at least one of the h_1 constraints, since at least one of the pipe sizes is smaller than the continuous minimums found above. Thus, we can dismiss all of these combinations. In addition, we can dismiss any other designs in this branch as well, since they would have pipe sizes greater than those of our original and would thus be more costly. Note that our original design is actually in this branch, using the first discrete pipe sizes greater than those found above for $d_{(7,2)}$ and $d_{(7,3)}$.

By using the branch-and-bound technique, we have eliminated all of the combinations of Table 13 and have only computed the cost four times. In addition, we have computed diameters using the h_1 constraint six times. These computations compare favorably with those required for total "exhaustive enumeration" of the possibilities (27 cost and 54 constraint calculations) and favorably to the computations of Table 13, which eliminated nine possibilities based on monotonicity considerations. We have also shown that no other discrete designs in the branches explored, i.e., even those deviating by more than one discrete pipe size, could be both feasible and less costly that the original discrete design. What remains to be shown is that other branches that allow $d_{(6,7)}$ to deviate by more than one discrete pipe size are either infeasible or not cost effective, or both.

To explore the branches where $d_{(6,7)}$ is more than one discrete diameter away from our original design, we once again look to the constraint h_1 for consumers 2 and 3. We notice that there is a limit on how much we can decrease either $d_{(7,2)}$ or $d_{(7,3)}$ and still find values of $d_{(6,7)}$ that will satisfy the constraints. Physically, what has occurred is that we have decreased the pipe sizes of $d_{(7,2)}$ or $d_{(7,3)}$ to the point where all of the pressure loss available between the pipe junction node 6 and either consumer 2 or 3 is being absorbed in the pipe segment (7,2) or (7,3). To utilize this condition, we first ignore the pressure loss of pipe segment (6,7) and calculate the minimum continuous values for $d_{(7,2)}$ and $d_{(7,3)}$ that will satisfy h_1 for consumers 2 and 3 respectively. We then find the next largest discrete diameters in each case, since any actual design would be bounded by these. The results are

$$d_{(7,2)} = 0.0536 \text{ (continuous); } 0.0545 \text{ (discrete)}$$
$$d_{(7,3)} = 0.0614 \text{ (continuous); } 0.0703 \text{ (discrete).}$$

Now, with these discrete diameters, we calculate the minimum continuous value of $d_{(6,7)}$ that would satisfy the constraint h_1 for both consumers 2 and 3. This value is determined to be 0.1296. The minimum discrete value of $d_{(6,7)}$ is then 0.1325. We see that these discrete diameters are identical to those of combination 16 in Table 13. It was shown earlier that for this combination the cost exceeded our original design. Thus, any larger discrete diameters would also exceed the cost of our original design. Since this result is for the minimum possible discrete diameters for pipe segments (7,2) and (7,3), regardless of the size of pipe segment (6,7), no lower cost alternatives can exist since their diameters for pipe segments (7,2) and (7,3) would be greater and thus the designs more costly.

Now the remaining branches yet to be explored are those with discrete pipe sizes for pipe segment (6,7) more than one size below our original design. To explore these branches, we neglect the pressure losses of the pipe segments (7,2) and (7,3) and then calculate the minimum value of $d_{(6,7)}$ that will satisfy constraint h_1 for both consumers 2 and 3. We find that the minimum continuous diameter of $d_{(6,7)}$ is 0.0800. The next largest discrete diameter is $d_{(6,7)} = 0.0825$ and we see that this branch has already been searched. Thus, there are no other feasible branches with discrete values of $d_{(6,7)}$ less than that of our original design. We have now exhausted all the alternatives and have found our original discrete design to be the optimal discrete design.

We still have several constraints remaining that must be checked for satisfaction. The remaining constraints are eq 4-11, 4-22, 4-23, 4-24 and 4-25. These constraints all deal with the absolute pressure level in the piping. Before we can compute the absolute pressure at any point, we must first assign an absolute pressure in the supply pipe at the heating plant. Since we suspect that this will be the place of highest pressure in the network, we let the absolute pressure at that point be equal to the maximum allowed, i.e.

$$P_{\text{hp,s}} = P_{\max} = 1 \times 10^6 \text{ N/m}^2 \ .$$

We start with eq 4-11, which is a constraint on the maximum pressure in the supply pipe. The right-hand side of eq 4-11 equals the pressure level in the supply pipe. As we have shown earlier in this chapter (see eq 4-15), the maximum pressure must occur at a pipe node and not at an intermediate point. In Table 14, we have calculated the pressure in the supply pipe at each of the nodes. We see that the constraint of eq 4-11 is satisfied, since the pressure level does not exceed the maximum allowed at any point in the supply piping.

Equation 4-22 requires that the pressure at each point in the supply pipe (the left-hand side of the equation) exceed the sum of the saturation pressure P_{sat} and a safety margin P_{saf}. For the supply pipe temperature of 120°C, the saturation pressure is 1.985×10^5 N/m^2 (Reynolds and Perkins 1977). Thus, the sum of these two becomes 2.985×10^5 N/m^2. We see by examining Table 14 that this constraint is also satisfied at all nodes.

We have a similar constraint for the return pipe, eq 4-23. The left-hand side of this equation equals the pressure in the return pipe, which has also been computed and is given in Table 14. The temperature and thus the saturation pressure in the return pipe are different from those in the supply pipe, of course. The return temperature will vary with load as per our consumer model, eq 3-25. The maximum temperature will occur at the design condition of maximum load, as can be seen from Figure 5, and its value is 55°C as determined in Chapter 3 for our supply temperature of 120°C and our assumptions regarding the radiator design conditions. The saturation pressure will be greatest at the highest temperature, so if our constraint is satisfied

Table 14. Pressure levels in the piping network.

Node number	ΔP_s *(N/m²)*	P_s *(N/m²)*	ΔP_r *(N/m²)*	P_r *(N/m²)*	$P_s - P_r$ *(N/m²)*
1	659,272	340,728	150,000	190,728	150,000
2	518,744	481,256	246,183	235,073	246,183
3	449,187	550,813	200,374	350,439	200,374
4	291,257	708,743	231,376	477,367	231,376
5	107,219	892,781	–385,267	394,554	498,228
6	197,874	802,126	–294,546	485,275	316,852
7	218,488	781,512	–273,916	505,905	275,607
8	—	1,000,000	—	287,256	712,744

at this condition, it will be satisfied at the lower temperature conditions.

For a return temperature of 55°C, the saturation pressure is 1.576×10^4 N/m^2 (Reynolds and Perkins 1977). The sum of this and our safety margin is 1.157×10^5 N/m^2. By examining the return pressures in Table 14, we see the constraint of eq 4-23 is satisfied at all points in the return piping.

Now we look at the two constraints on the pressure in the return line at the heating plant, eq 4-24 and 4-25. We see that, for the parameter values chosen, eq 4-24 will dominate. Equation 4-24 requires that the pressure in the return line at the heating plant be greater than 2.0×10^5 N/m^2. The pressure in the return line at node number 8, which is our heating plant node, is given as 287,256 N/m^2 in Table 14, so we see this constraint is satisfied as well. Thus, our design has satisfied all the constraints specified.

CHAPTER 7: CONCLUSIONS AND RECOMMENDATIONS

SUMMARY

In Chapter 2 we found a suitable method for determining the optimal size for a single pipe, independent of any others. This method was developed to be as simple as possible yet complete and accurate enough for design calculations. The method is general enough to allow for any set of economic or physical parameter values. In addition, any form of load management, i.e., temperature or flow modulation, or both, can be accommodated by the integral form of the coefficients in the cost equation. A new approximation was developed for the friction factor. The form of this expression was a simple power function of the Reynolds number and the relative pipe roughness. This form allowed us to easily incorporate it into the head loss equation without additional complication or rendering the result implicit. We made use of geometric programming theory to identify a lower bounding problem that can be used to provide us with a very good first estimate of our solution and a global lower bound on cost. At the end of Chapter 2, an example is presented that shows a 17% saving in life cycle cost over a design based on a common rule of thumb.

In Chapter 3 we looked at the heat consumers and the effect that they have on the piping system. We developed a new model for the consumer's heat exchanger that uses the geometric mean temperature difference as an approximation for the logarithmic mean temperature difference. This allowed us to develop an explicit expression for return temperature, a result not possible when using the logarithmic mean temperature difference. We conducted a complete error analysis on the geometric mean approximation and our new consumer model based on it. This analysis confirmed that the resulting error from this model was acceptable for design purposes and much less than the error resulting from using the arithmetic mean temperature difference as an approximation of the logarithmic mean temperature difference, as has been suggested by others. We integrated our new consumer model into our single pipe model of Chapter 2 and for a sample case determined what effect the addition of the consumer has on the integral coefficients of the cost equation. At the end of Chapter 3, we reworked the example of Chapter 2, including the effects of the new consumer model.

In Chapter 4 we developed the constraints for systems with multiple pipes and consumers. Both absolute and differential pressure constraints were derived. By using the monotonicity of the hydrodynamic and hydrostatic pressure gradients, we were able to easily show that the maximum pressure within a pipe segment must occur at one of the end points. We then developed a strategy to allow for constraint satisfaction at all points implicitly without considering every point in the system.

In Chapter 5 we briefly reviewed general methods for constrained nonlinear optimization. For various reasons these alternatives are all abandoned in favor of the approach taken. Subsequently, our general solution strategy is developed for systems with multiple pipes and consumers. The method makes use of the solution to the problem, unconstrained by the network constraint requirements, as a starting point for the constrained solution. Monotonicity analysis was then used to prove activity of some of the constraints and thus simplify the problem. In addition, the concept of constraint dominance is used to reduce the number of constraints that must be considered. Before proceeding with the problem solution, a brief graphical analysis verified that we only needed to provide for constraint satisfaction at the maximum load condition to ensure satisfaction at all other load conditions. The resulting reduced problem was then used as a starting point for two methods proposed to find a solution to the constrained problem with continuous values for some of the pipe diameters. Finally, the branch-and-bound technique is explained and then shown to be suitable for finding a design with discrete values for all the pipe diameters.

In Chapter 6 we worked a simple example with only four consumers and seven pipe segments. The example illustrated the use of our method and also showed how the branch-and-bound technique can be used to quickly eliminate candidate designs. A method is also demonstrated for further refinement of the pipe network to eliminate excessive throttling losses in the consumer's control valves.

CONCLUSIONS

The method developed here should be feasible for designing the piping networks for district heating systems of moderate size, and in particular the systems used on military facilities or college campuses, which tend to be smaller and less complex than those of large cities. For very large systems, the branch-and-bound method used for finding the discrete diameters may become cumbersome and computationally too expensive. However, this remains to be shown and it may be that, with the commercially available software and the enormous power of today's computers, this perceived problem is quite manageable.

The major advantage of the method developed here is its flexibility to accommodate any set of economic and physical parameters and operating strategy. In addition, the approximations, where used, are much more suitable than some made in the past: for instance, linearization of the equations, neglecting heat losses, and oversimplification of the effect of varying load. It is felt that a significant contribution has been made by the derivation of mathematical expressions for all of the major constraints. Perhaps the most significant contribution of this work has been the analysis of constraint activity and the development of a method to exploit that knowledge to arrive at a solution. In addition, we have shown what bounds can be put on the solution such that the designer can be reasonably assured of whether or not further significant cost reduction is possible. This not only gives the designer some comfort in knowing what possible improvement remains, but it also avoids excessive calculations that often result when no such knowledge is available.

Another possible use of the methodology developed here is for studies of the relative merits of various operating strategies and what effect they have on the design of the system. The general form of the cost coefficients can be useful for such studies and can not only be used to develop designs based on the methods presented here, but they may also be used to evaluate the effect of economic or physical parameter changes, including operating strategy changes, on existing designs.

With many systems already in use in Europe, the issue of optimal design is of lesser importance there. Currently, however, the interest in optimal operation of district heating systems is significant in Europe, as evidenced by several recent conferences devoted almost entirely to this topic alone (Nordic Council of Ministers 1989, 1994). Most of the efforts seem to be centered on real-time simulations of operation and subsequent forecasting of short-term operating strategy. It seems that a method such as the one developed here would be useful for studies at a higher level to determine optimal overall operating strategies for the yearly load cycle.

RECOMMENDATIONS

It is recommended that the methodology developed here be field tested on the design of a moderately sized system, such as would be found on a military base or college campus. The design should be compared with a completely independent design, as would be achieved by methods normally used by the district heating design profession.

Under the assumption that the results of the field test were positive, it is recommended that the method be coded for computer execution to the maximum extent possible. The resulting CAD program could then be incorporated into one or more of the currently available CAD systems explicitly created for district heating system design and feasibility studies. If the economic benefits are as great as indicated by the examples worked here, the incentive for doing so is significant.

Perhaps the most troubling aspect of using optimal design methods for sizing district heating piping systems is the level of pressure losses resulting. These pressure losses are rather high when compared to those encountered in operating systems and designs based on other methods. This result is apparent when we examine the pressure losses given in Table 1 for both the method presented here and a common rule of thumb based design. This result has also been observed by others (Bøhm 1986, Koskelainen 1980). It seems that current design practice and the systems that result are ill-suited to the application of optimal design techniques.

Several possible solutions to this conflict exist. The first is simply to increase the maximum pressure capability of the piping system used. The logic of this approach can be seen in European practice where small district heating systems use piping rated for only 6-bar (600-kPa) maximum pressure and often the connections to the consumers are direct, i.e., without heat exchangers. For larger systems, piping rated for 15-bar (1500-kPa) or greater maximum pressure is often used and heat exchangers are used to isolate the consumers' equipment from the high system pressures. The designer should always find the unconstrained optimum pressure level for the network first before making a decision of which pressure class of piping to use. It's quite possible that pressures higher than those used in current practice may be justified in some instances.

The advent of friction reducing additives (Nordic Council of Ministers 1991), which are currently being field tested, offers some relief for the problem of excessive system pressures. Such additives promise to reduce friction and thus pressure losses by 50% or more. Such a change in something always assumed to be a basic given in design renders much of what has been learned to date about district heating system design, optimal or otherwise, nearly worthless. The ability of the method developed here to rapidly reevaluate either existing or proposed designs clearly illustrates the value and necessity of such a design tool. For instance, one possibility is that some of the frictional reducing additives will be relatively short lived when compared to the life of the district heating system, or even the annual operating cycle. However, since the maximum flow rate and thus maximum pressure drop are only encountered over a short period of the yearly cycle, it may be that such friction reducing additives can be very effective. The method developed here would allow for rapid evaluation of any possibility to see if they are worthy of further study or consideration for field testing.

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APPENDIX A: APPROXIMATION OF THE FRICTION FACTOR

For incompressible flow in circular conduits (pipes), the head losses can be calculated using the Darcy-Weisbach equation

$$h_f = f\, L\, v^2/2\, g\, d \tag{A-1}$$

where f = friction factor (dimensionless)
L = pipe length (m)
v = flow velocity (m/s)
g = acceleration of gravity (9.8 m/s^2)
d = inside diameter of the pipe (m).

Numerous other expressions have been proposed for calculating frictional head losses. The Darcy-Weisbach equation is, however, the most fundamentally appealing as it can be derived analytically while the other relationships are empirical in nature (Jeppson 1976). For laminar flow it is possible to show that the friction factor f is a function of the Reynolds number alone. Unfortunately, the flow in heat distribution piping is seldom laminar. For turbulent flow the friction factor has been determined empirically to be a function of the Reynolds number and the relative roughness of the pipe. A number of correlations have been proposed for the friction factor. Those correlations that give the best agreement with the experimental data are implicit in the friction factor. This renders them impractical for analyses such as this one. For this analysis, and other applications, it would be desirable to have a simple expression that would provide sufficient accuracy over limited ranges of interest. To keep the expression as simple as possible, while allowing it to be an accurate approximation, a method is developed here that yields a one-term power function.

To approximate friction factor information in the form of implicit equations or empirical data, we can develop our approximation using the least-squares method. First, we assume a desired form for our expression for the friction factor

$$\hat{f} = a\, RR^b Re^c \tag{A-2}$$

where $\hat{f}$ = predicted friction factor (dimensionless)
a, b, and c = coefficients determined by the least-squares method (dimensionless)
$RR = \varepsilon/d$ = relative roughness of the pipe (dimensionless)
ε = absolute roughness of the piping (m)
Re = Reynolds number for the pipe flow (dimensionless).

If we assume that for any set of values for RR and Re we have an observed friction factor f, we would like to minimize the sum of the squares between the f's predicted by our equation and all the observed f's within the range of interest

$$\min \sum \left(f - \hat{f}\right)^2. \tag{A-3}$$

The summation is taken over all the observations available within the range of interest for the parameters RR and Re. In the event that we are trying to approximate an implicit empirical expression, we would choose incremental values of RR and Re over the range of interest and use these to calculate a corresponding f value. This approach will be illustrated later in this appendix. To accomplish the minimization, we first convert to a linear form by making the following substitutions

$$Y = \ln f \tag{A-4}$$

$$\hat{Y} = \ln \hat{f} = \ln (a\, RR^b Re^c) = \ln a + b \ln RR + c \ln Re$$

$$= \beta_0 + \beta_1 X_1 + \beta_2 X_2 \qquad \text{(A-5)}$$

where $\beta_0 = \ln a$
$\beta_1 = b$
$\beta_2 = c$
$X_1 = \ln RR$
$X_2 = \ln Re$.

Now we can restate the problem in a linear form

$$\min \sum_{i=1}^{n} \left(Y_i - \hat{Y}_i\right)^2 = \sum_{i=1}^{n} \left(Y_i - \beta_0 - \beta_1 X_{1,i} - \beta_2 X_{2,i}\right)^2 . \qquad \text{(A-6)}$$

The summation index i has now been added. The summation occurs over the total number of observations n. To find the minimum for this expression with respect to the parameters β_0, β_1 and β_2, we take the partial derivative of the expression with respect to each of the parameters and set the result to zero in each case

$$\partial / \partial \beta_0 = \sum_{i=1}^{n} \left(Y_i - \beta_0 - \beta_1 X_{1,i} - \beta_2 X_{2,i}\right) = 0 \qquad \text{(A-7)}$$

$$\partial / \partial \beta_1 = \sum_{i=1}^{n} X_{1,i} \left(Y_i - \beta_0 - \beta_1 X_{1,i} - \beta_2 X_{2,i}\right) = 0 \qquad \text{(A-8)}$$

$$\partial / \partial \beta_2 = \sum_{i=1}^{n} X_{2,i} \left(Y_i - \beta_0 - \beta_1 X_{1,i} - \beta_2 X_{2,i}\right) = 0 . \qquad \text{(A-9)}$$

We now have three linear equations in the three unknown parameters by rearranging as follows

$$\beta_0 + \beta_1 \sum_{i=1}^{n} X_{1,i} + \beta_2 \sum_{i=1}^{n} X_{2,i} = \sum_{i=1}^{n} Y_i \qquad \text{(A-10)}$$

$$\beta_0 \sum_{i=1}^{n} X_{1,i} + \beta_1 \sum_{i=1}^{n} (X_{1,i})^2 + \beta_2 \sum_{i=1}^{n} X_{2,i}\, X_{1,i} = \sum_{i=1}^{n} Y_i\, X_{1,i} \qquad \text{(A-11)}$$

$$\beta_0 \sum_{i=1}^{n} X_{2,i} + \beta_1 \sum_{i=1}^{n} (X_{1,i}\, X_{2,i}) + \beta_2 \sum_{i=1}^{n} (X_{2,i})^2 = \sum_{i=1}^{n} Y_i X_{2,i} . \qquad \text{(A-12)}$$

These equations can be written in matrix form as

$$\begin{vmatrix} A_{11} A_{12} A_{13} \\ A_{21} A_{22} A_{23} \\ A_{31} A_{32} A_{33} \end{vmatrix} \begin{vmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{vmatrix} = \begin{vmatrix} C_1 \\ C_2 \\ C_3 \end{vmatrix} \qquad \text{(A-13)}$$

where $A_{11} = n$

$$A_{12} = A_{21} = \sum_{i=1}^{n} X_{1,i}$$

$$A_{13} = A_{31} = \sum_{i=1}^{n} X_{2,i}$$

$$A_{22} = \sum_{i=1}^{n} (X_{1,i})^2$$

Table A1. Constants for the friction factor equation.

Water temp. (°C) min/max	*Flow velocity (m/s) min/max*	*Pipe diameter (m) min/max*	*Reynolds number $\times 10^{-6}$ min/max*	a	b	c	*Max. error (%)*	*Avg. error (%)*
50/130	0.5/3.3	0.05/0.77	0.04/11	0.123	0.146	–0.0626	6.2	1.0
50/130	0.5/4.5	0.05/0.77	0.04/15	0.119	0.152	–0.0568	6.9	1.1
70/150	0.3/6.3	0.03/0.93	0.02/29	0.129	0.156	–0.0589	10.8	2.0
50/90	0.5/2.9	0.05/0.41	0.04/3.7	0.140	0.141	–0.0762	4.1	0.8
90/130	0.5/2.9	0.10/0.46	0.16/5.9	0.116	0.150	–0.0563	2.5	0.6
50/90	0.5/2.9	0.10/0.46	0.09/4.1	0.128	0.132	–0.0751	3.4	0.7
50/90	0.5/3.7	0.10/0.46	0.09/5.3	0.125	0.137	–0.0698	3.9	0.8
90/130	0.5/3.7	0.10/0.46	0.16/7.5	0.113	0.154	–0.0520	2.8	0.6

$$A_{23} = A_{32} = \sum_{i=1}^{n} X_{2,i}\, X_{1,i}$$
$$A_{33} = \sum_{i=1}^{n} (X_{2,i})^2$$
$$C_1 = \sum_{i=1}^{n} Y_i$$
$$C_2 = \sum_{i=1}^{n} Y_i\, X_{1,i}$$
$$C_3 = \sum_{i=1}^{n} Y_i\, X_{2,i}\,.$$

This system of linear equations can be solved by forward elimination and subsequent back solution. The resulting expressions for the parameters are

$$\beta_2 = \frac{\left\{C_3 - (C_1 A_{31} / A_{11})\right\} - \left\{\left[C_2 - (C_1 A_{21} / A_{11})\right]\left[\dfrac{\left(A_{32} - (A_{12} A_{31} / A_{11})\right)}{\left(A_{22} - \left(A_{12}^2 / A_{11}\right)\right)}\right]\right\}}{\left\{A_{33} - \left(A_{13}^2 / A_{11}\right)\right\} - \left\{\left[A_{23} - (A_{13} A_{21} / A_{11})\right]\left[\dfrac{\left(A_{32} - (A_{12} A_{31} / A_{11})\right)}{\left(A_{22} - \left(A_{12}^2 / A_{11}\right)\right)}\right]\right\}} \quad \text{(A14)}$$

$$\beta_1 = \frac{\left\{C_2 - (C_1 A_{21} / A_{11})\right\} - \left\{\left[A_{23} - (A_{13} A_{21} / A_{11})\right][\beta_2]\right\}}{\left\{A_{22} - \left(A_{12}^2 / A_{11}\right)\right\}} \quad \text{(A15)}$$

$$\beta_0 = \left\{C_1 - [A_{12}\beta_1] - [A_{13}\beta_2]\right\} / A_{11}. \quad \text{(A16)}$$

A FORTRAN program FFCONST was written to evaluate the A's and C's in the above expressions and then solve eq A14, A15 and A16 to find the parameters β_0, β_1 and β_2. The parameters in our original expression, eq A2, for the predicted friction factor $\hat{f}$ can then be found. For this program, the "observed" friction factor f is found using the Colebrook-White equation (Jeppson 1976)

$$f = [1.14 - 0.869 \ln(RR + 9.35/Re\ \sqrt{f})]^{-2}. \quad \text{(A-17)}$$

The Colebrook-White equation is implicit in the friction factor f and thus it cannot be solved directly. A number of methods can be used to solve implicit equations such as this one. The Newton-Raphson method has been used in SUBROUTINE CWFF. This method uses knowledge of the first derivative of the function to find the solution. A description of the method can be found in nearly any reference on numerical methods, such as Forsythe et al. (1977).

To use the Newton-Raphson method to solve the implicit Colebrook-White equation, an initial estimate of the *f* value is needed. An explicit equation for the friction factor *f* can be used for this. The explicit equation does not need to be extremely accurate to yield a suitable first estimate. The equation given by Wood (1966) is a good explicit relationship for turbulent flow and can be used. Wood's equation is

$$f = 0.094\,RR^{0.225} + 0.53\,RR + \left[\left(88\,RR^{0.44}\right)/\left(Re^{\,1.62 RR^{0.134}}\right)\right]. \tag{A-18}$$

To calculate the friction factor using either eq A17 or A18 requires that we know the Reynolds number *Re* and the relative roughness *RR*. To calculate these parameters, we need to specify the fluid density and dynamic viscosity as well as the pipe diameter and absolute roughness and the flow velocity. The fluid properties are a function of the temperature of the fluid and to a lesser extent its pressure as well. Here, we will assume that the fluid is at its saturation pressure for the temperature specified. Two FORTRAN subroutines were written to determine the fluid properties. The first, SUBROUTINE SATLN, calculates the saturation pressure for water given the temperature. The second, SUBROUTINE WTRTBL, calculates the density and dynamic viscosity given the temperature and pressure. The main program FFCONST and each of the subroutines mentioned above are included in Appendix B.

Using the program FFCONST, a number of the constants *a*, *b* and *c* were determined for several sets of parameters. Table A1 summarizes the results. In each of the examples of Table A1 the absolute roughness of the pipe was taken as 4.6×10^{-5} m.

APPENDIX B: COMPUTER PROGRAM LISTINGS

Program FFCONST

```
      PROGRAM FFCONST
      DIMENSION VI(10),DI(10),FI(10,10,10),FFCALC(10,10,10),
     *ERROR(10,10,10),RNI(10,10,10),T(10)
100 FORMAT(15X,3E12.5)
200 FORMAT(////,15X,4E15.6,///,15X,4E15.6)
300 FORMAT(////,20X,3F18.6,/////)
400 FORMAT(2X,1F10.0,1F10.2, 1F10.3,1E15.4,3F10.4)
500 FORMAT(2X,4E12.4)
600 FORMAT(////,15X,3F18.6)
      R=.046E-3
      ESUM=0.
      A1=0.
      A2=0.
      A3=0.
      A4=0.
      A5=0.
      A6=0.
      A7=0.
      A8=0.
      A9=0.
      A10=0.
      A11=0.
      A12=0.
      A13=0.
      A14=0.
      A15=0.
      A16=0.
      EMAX=0.
      EMIN=1.
      JV=9
      ID=9
      DELTAD=.025
      DL=.025
      VL=.25
      VDELTA=.75
      VI(1)=VL
      KT=9
      TDELTA=10.
      TL=70.
      T(1)=TL
      N=JV*ID*KT
      DO 4 K=1,KT
      CALL SATLN(T(K),P)
      CALL WTRTBL(T(K),P,RHO,XH,DV)
*     PRINT 500,T(K),P,RHO,DV
```

```
      DO 3 J=1,JV
      DI(1)=DL
      DO 2 I=1,ID
      RR=R/DI(I)
      RNI(I,J,K)=DI(I)*VI(J)*RHO/DV
      RN=RNI(I,J,K)
      F=.094*(RR**.225)+(.53*RR)+(88.*(RR**.44))/(RN**(1.62*(RR**
      *.134)))
      FI(I,J,K)=F
      A1=A1+LOG(F)
      A2=A2+LOG(RR)
      A3=A3+LOG(RN)
      A4=A4+LOG(F)*LOG(RR)
      A5=A5+((LOG(RR))*(LOG(RR)))
      A6=A6+LOG(RN)*LOG(RR)
      A7=A7+LOG(RN)*LOG(F)
      A8=A8+((LOG(RN))**2.)
*     PRINT 100,RNI(J),DI(I),FI(I,J)
      DI(I+1)=DI(I)+(DELTAD*I)
    2 CONTINUE
      VI(J+1)=VI(J)+VDELTA
    3 CONTINUE
      T(K+1)=T(K)+TDELTA
    4 CONTINUE
      A9=A6-(A2*A3/N)
      A10=A5-(A2*A2/N)
      A11=A6-(A2*A3/N)
      A12=A4-(A1*A2/N)
      A13=A8-(A3*A3/N)
      A14=A7-(A1*A3/N)
      A15=A13-(A11*A9/A10)
      A16=A14-(A12*A9/A10)
      C=A16/A15
      B=A12/A10-C*A11/A10
      ALN=A1/N-C*A3/N-B*A2/N
      A=EXP(ALN)
*     PRINT 200,A1,A2,A3,A4,A5,A6,A7,A8
      PRINT 300,A,B,C
      RN=RNL
      DO 7 K=1,KT
      DO 6 J=1,JV
      D=DL
      DO 5 I=1,ID
      RN=RNI(I,J,K)
      FFCALC(I,J,K)=A*((R/D)**B)*(RN* *C)
      ERROR(I,J,K)=(FFCALC(I,J,K)-FI(I,J,K))/FI(I,J,K)
*     PRINT 400,T(K),VI(J),D,RN,FI(I,J,K),FFCALC(I,J,K),ERROR(I,J,K)
      D=D+(DELTAD*I)
      EABS=ABS(ERROR(I,J,K))
```

```
      ESUM=ESUM+EABS
      EMAX=MAX(EMAX,EABS)
      EMIN=MIN(EMIN,EABS)
    5   CONTINUE
    6   CONTINUE
    7   CONTINUE
      EAVG=ESUM/N
      PRINT 600,EMIN,EMAX,EAVG
      STOP
      END
```

Subroutine WTRTBL(T,P,RHO,XH,DV)

```
      SUBROUTINE WTRTBL(T,P,RHO,XH,DV)
*     THIS SUBROUTINE CALCULATES THE THERMODYNAMIC AND
*     TRANSPORT PROPERTIES OF WATER GIVEN THE TEMPERATURE AND
*     PRESSURE CONDITIONS.
*     INPUT VALUES ARE T(C) AND P(BARS). THE OUTPUTS ARE DENSITY
*     RHO (KG/M3), THE ENTHALPY XH(KJ/KG), AND THE DYNAMIC
*     VISCOSITY DV IN KG/M-SEC. THE EQUATIONS FOR THE VISCOSITY ARE
*     NOT VALID FOR TEMPERATURES GREATER THAN 300 C.
      T=T+273.16
*     CALL SATLN(T,P)
      DATA RI1,B0,B1,B2,B3,B4,B5,B6,B7,B8,B9,Ul ,Wl/22129.,-37444.8692,
      *466453.368,-2666876.77,9030271.53,- 19769400.2,28949239.9,
      *-28309932.7,17808942.6,-6534676.01,1065198.53,.58620689,
      *.41666667/
      DATA G,H,RK,RL,RM,RN,Fl,Gl ,Hl,RKl,RLl,RMl,RNl,Ql,Rl,Zl/
      *.417,1.139706E-4,9.949927E-5,7.241165E-5,.7676621,1.052358E- 11,
      *3.7E8,3.122199E8,199985.,1.72,1.362926E16,1.500705,.6537154,
    *62.5,13.10268,1.5108E-5/
      DATA A2,A3,A4,A5/.3828209486,.2162830218,.1498693949,.4711880117/
    P1=221.287
      T1=647.3
      T=T/T1
      P=P/P1
      U=F1-(G1*T*T)-(H1*(T**(-6.)))
      W=U+SORT((RK1*U*U)+(RL1*(P-(RM1*T))))
      V1=(G/(W**(1./3.4)))-H+(RK*T)
      V2=((ABS(RN1-T))**2.)*(RL+(((ABS(RN1-T))**8.)*RM))
      V3=(RN*(R1+(R1*P)+(P*P)))/(Z1+(T**11.))
      RHO=1./(V1+V2-V3)
      H0=B0+(B1*T)+(B2*T*T)+(B3*T**3.)+(B4*T**4.)+(B5*T**5.)+
      *(B6*T**6.)+(B7*T**7.)+(B8*T**8.)+(B9*T**9.)
      Y1=RL1*RM1/2.
      Q=2.*G/RL1
      V=(-2.*G1*T*T)+(6.*H1/(T**6.))
      H1=((U1 * W)-(W1*((3.4*U)-V)))*W
      H1=Q/(W**(1./3.4))*(H1+(Y1*T)-(.72*V*U))
      H2=(((RN1-T)*((RL*(RN1+T))+(RM*((ABS(RN1-T))* *8.)*(RN1+(9.*T)))))
```

```
    *-H)*P
    H3=(RN*(Z1+(12.*(T**11.))))/((Z1+(T**11.))**2.)
    H3=H3*(Q1+(((R1/2.)+(P/3.))*P))*P
    XH=H0+(RI1*(H1+H2-H3))
    TC=(T*T1)-273.16
    CALL SATLN(TC,PS)
    PS=PS/P1
    DV=.02414+(10.**(A2/(T-A3)))*(1.+((P-PS)*A4*(T-A5)))
    T=T*T1
    P=P*P1
    DV=DV*1.E-3
    T=T-273.16
    RETURN
    END
```

Subroutine SATLN(T,P)

```
    SUBROUTINE SATLN(T,P)
*   THIS SUBROUTINE CALCULATES THE SATURATION PRESSURE FOR A
*   GIVEN WATER TEMPERATURE. THE WATER TEMPERATURE IS IN C
AND
*   THE SATURATION PRESSURE IS RETURNED IN BARS.
    DATA RK,A,B,C,D,E,F/2.937E5,5.426651,-2005.1,1.3869E-4,1.1965E-11,
    *-.0044,-.0057148/
    T=T+273.16
    T2=T+.01
    Y=647.26-T
    X=(T2*T2)-RK
    T=T/647.3
    A1=A+(B/T2)+(C*X/T2*(10.**(D*X*X)-1.))+(E*(10.**(F*(Y* *1.25))))
    P=(1.01325*(10.**A1))+((T-.422)*(.577-T)*EXP(-12.+(T**4.))*
    *9.80665E-3)
    T=T*647.3
    T=T-273.16
    RETURN
    END
```

Subroutine ROMBRG(FUN,A,B,C,ERR,RES)

```
    SUBROUTINE ROMBRG(FUN,A,B,C,ERR,RES)
C    THIS SUBROUTINE COMPUTES INTEGRALS OF A USER SUPPLIED
C    FUNCTION USING ROMBERG’S METHOD. THIS SUBROUTINE IS FROM
C    “NUMERICAL METHODS FOR ENGINEERING APPLICATION”,J.H.
C    FERZIGER, JOHN WILEY AND SONS, 1981. THE ARGUMENT ARE:
C    FUN = THE FUNCTION TO BE INTEGRATED
C    A = LOWER LIMIT
C    B = UPPER LIMIT
C    C = ARRAY OF FUNCTION DEFINITION PARAMETERS IF REQUIRED
C    ERR = THE DESIRED ACCURACY
C    RES = THE RESULTING VALUE FOR THE INTEGRAL
```

```
      EXTERNAL FUN
C     Z IS THE ARRAY OF APPROXIMATIONS
      DIMENSION Z(10,10)
C     INITIALIZE AND COMPUTE THE FIRST APPROXIMATION.
      I=1
      DEL=B-A
      Z(1, 1)=.5*DEL*(FUN(A,C)+FUN(B,C))
C     THE MAIN LOOP. THE FIRST PART COMPUTES THE INTEGRAL USING A
C     2J+1 POINT TRAPEZOID RULE. THE METHODS MAKES MAXIMAL USE
C     OF THE VALUES ALREADY COMPUTED.
   10 J=2**(I-1)
      DEL=DEL/2.
      I=I+1
      Z(I,1)=.5*Z(I-l,l)
      DO 1 K=1,J
      X=A+(2.*K-1)*DEL
      Z(I,1)=Z(I,1)+DEL*FUN(X,C)
1     CONTINUE
C     NOW WE DO THE RICHARDSION EXTRAPOLATION
      DO 2 K=2,I
      Z(I,K)=(4.**(K-1)*Z(I,K-1)-Z(I-1,K-1))/(4.**(K-1)-1.)
2     CONTINUE
C     ERROR CONTROL
      DIFF=ABS(Z(I,I)-Z(I,I-1))
      PRINT 108, DIFF,Z(I,I)
  108 FORMAT(5X,2E15.5)
      IF(DIFF.LT.ERR) GO TO 20
C     THE MAXIMUM NUMBER OF ITERATIONS ALLOWED IS 10.
      IF(I.LT.10) GO TO 10
      PRINT 100
  100 FORMAT(‘ MORE THAN lO ITERATIONS REQUIRED, CHECK
PARAMETERS.’)
STOP
20    RES=Z(I,I)
      RETURN
      END
```

Function FUN(T,Z)

```
      FUNCTION FUN(T,Z)
C     THIS FUNCTION CONTAINS THE INTEGRAND OF THE I2 PARAMETER.
      A=0.135
      B=0.161
      C=-0.0555
      PVFE=9.077
      PVFH=9.077
      EPS=5.E-5
      PL=1000.
      FRD=100.
```

```
    CE=7.E-5
    CH=3.4E-5
    AEDA=0.9
    TSD=120.
    TRD=60.
    CALL  WTRTBL(TRD,PRD,RHORD,RHD,RMUD)
    CALL  WTRTBL(TSD,PSD,RHOSD,SHD,SMUD)
    RHOD=(RHOSD+RHORD)/2.
    TS=120.
    TA=20.
    A12=A*PVFE*PL*(EPS**B)*(1.273240**(2+C))
    QF=(0.425*COS(6.283*T/8760.))+0.575
    CALL CLMTD(TS,TA,QF,FRF,TR,TRG,FRFG)
    FT=FRF
C   CALCULATE QUANITIES WHICH MAY BE A FUNCTIONS OF TIME.
    CALL WTRTBL(TS,PS,RHOS,SH,SMU)
    CALL WTRTBL(TR,PR,RHOR,RH,RMU)
    A7=(((1./(SMU**C))/(RHOS*RHOS))+((1./(RMU**C))/(RHOR*RHOR)))/2.
    RHOA=(RHOS+RHOR)/2.
    FUN=A7*((((CE*RHOA)/(AEDA*RHOD))*(FT* *(2+C)))-((PVFH/PVFE)
    &*CH*(FT**(3+C))))
    FUN=FUN*A12*(FRD**(3+C))
    PRINT 102, T,FT,FUN
102 FORMAT(5X,3E 15.4)
RETURN
END
```

Program I2

```
    PROGRAM I2
    EXTERNAL FUN
    CALL ROMBRG(FUN,0.,4380.,0.,1.E-7,PI2)
    PRINT 101,PI2+2.
101 FORMAT(l0X,F15.5)
    STOP
    END
```

Program I2-C-GMT

```
PROGRAM I2-C-GMT

    SUBROUTINE ROMBRG(FUN,A,B,C,ERR,RES)
C   THIS SUBROUTINE COMPUTES INTEGRALS OF A USER SUPPLIED
FUNCTION
C   USING ROMBERG'S METHOD. THIS SUBROUTINE IS FROM
"NUMERICAL
C   METHODS FOR ENGINEERING APPLICATION",J.H. FERZIGER, JOHN
WILEY
C   AND SONS, 1981. THE ARGUMENTS ARE:
C   FUN = THE FUNCTION TO BE INTEGRATED
C   A = LOWER LIMIT
```

```
C    B = UPPER LIMIT
C    C = ARRAY OF FUNCTION DEFINITION PARAMETERS IF REQUIRED
C    ERR = THE DESIRED ACCURACY
C    RES = THE RESULTING VALUE FOR THE INTEGRAL

     EXTERNAL FUN
C    Z IS THE ARRAY OF APPROXIMATIONS
     DIMENSION Z(10,10)
C    INITIALIZE AND COMPUTE THE FIRST APPROXIMATION.
     I=1
     DEL=B-A
     Z(1,1)=.5*DEL*(FUN(A,C)+FUN(B,C))
C    THE MAIN LOOP. THE FIRST PART COMPUTES THE INTEGRAL USING A
C    2J+1 POINT TRAPEZOID RULE. THE METHODS MAKES MAXIMAL USE
OF THE
C    VALUES ALREADY COMPUTED.
  10 J=2**(I-1)
     DEL=DEL/2.
     I=I+1
     Z(I,1)=.5*Z(I-l,l)
     DO 1 K=1,J
     X=A+(2.*K-1)*DEL
     Z(I,l)=Z(I,l)+DEL*FUN(X,C)
1    CONTINUE
C    NOW WE DO THE RICHARDSION EXTRAPOLATION
     DO 2 K=2,I
     Z(I,K)=(4.**(K-1)*Z(I,K-l)-Z(I-l,K-1))/(4.**(K-1)-1.)
2    CONTINUE
C    ERROR CONTROL
     DIFF=ABS(Z(I,I)-Z(I,I-1))
     PRINT 108, DIFF,Z(I,I)
  108 FORMAT(5X,2E15.5)
     IF(DIFF.LT.ERR) GO TO 20
C    THE MAXIMUM NUMBER OF ITERATIONS ALLOWED IS 10.
     IF(I.LT.10) GO TO 10
     PRINT 100
  100 FORMAT(‘ MORE THAN 10 ITERATIONS REQUIRED, CHECK
PARAMETERS.’)
     STOP
  20 RES=Z(I,I)
     RETURN
     END
```

Function FUN(T,C)

```
     FUNCTION FUN(T,C)
C    THIS FUNCTION CONTAINS THE INTEGRAND OF THE I2 PARAMETER.
C    It has been modified to include the effect of the consumers model
C    using the GMTD model. See EQ. 3.25 for explanation of symbols
C    used in flow rate equation.
```

```
      TSD=120.
      TRD=55.
      TA=20.
      TS=120.
      TGMTD=59.1608
      RN1=1.3
      A13=1.0
      PL=1000.
      DMF=4.
      TF=(0.425*COS(6.283 *T/8760.))+0.575
      TOP=(TSD-TRD)*TF
      BOTTOM=TS-TA-((1/(TS-TA))*(TGMTD*TGMTD)*((TF/A13)**(2/RN1)))
      A=TOP/BOTTOM
C  "A" is the normalized mass flow rate, m/md.
      FUN=(2.043E-8*DMF*((DMF*A)**1.9432))-(8.924E-9*((A*DMF)**2.9432))
      FUN=FUN*PL/1000
      PRINT 102, T,A,FUN
  102 FORMAT(5X,3E15.4)
      RETURN
      END
```

Program I2

```
      PROGRAM I2
      EXTERNAL FUN
      CALL ROMBRG(FUN,0.,4380.,0.,1.E-9,PI2)
      PRINT 101,PI2*2.
  101 FORMAT(l0X,E15.5)
      STOP
      END
```

Program I1EQ3-26

```
PROGRAM I1EQ3-26

      FUNCTION FUN1(T,Z)
C    THIS FUNCTION CONTAINS THE INTEGRAND OF THE I1 PARAMETER.
C    It uses equation 3.26 which was derived using the GMTD appx.
C    Modified from "IlFT.for" on 1/12/94.
      RN1=1.3
      A14=0.575
      A15=0.425
      A13=1.0
      FUN1=(((A15*COS(6.283*T/8760.))+A14)/A13)**(2/RN1)
      PRINT 102, T,FUN1
102 FORMAT(5X,2E15.4)
      RETURN
      END
```

Program I1

```
      PROGRAM I1
      EXTERNAL FUN1
      CALL ROMBRG(FUN1,0.,4380.,0.,1.E-7,PI1)
      PVFH=9.077
      PL=1000.
      CH=3.4E-5
      TS=120.
      TM=6.4
      TA=20.
      TCINS=0.030
      AT=8760
      TGMTD=59.1608
      A16=PVFH*PL*12.56637*TCINS*CH
      RI1=A16*(((PI1*2.)*(TGMTD *2/(2*(TS-TA))))+((((TS+TA)/2)-TM)*AT))
      PRINT 101,RI1
  101 FORMAT(10X,E15.7)
      STOP
      END
```

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First, a method for determining the optimal size for a single pipe segment in a district heating system is developed. The method is general enough to allow for any set of economic or physical parameter values. In addition, any form of load management, i.e., temperature or flow modulation, or both, can be accommodated by the integral form of the coefficients in the cost equation. An example is presented that shows a 17% savings in life cycle costs over a design based on a common rule of thumb. Next the heat consumer and his effects on the piping system are studied. A new model is developed for the consumer's heat exchanger that uses the geometric mean temperature difference as an approximation for the logarithmic mean temperature difference. The new consumer model is integrated into the previous single pipe model and, for a sample case, its effect is determined. For systems having multiple pipes and consumers, the constraints are first developed and then the general solution strategy. The method makes use of the solution to the unconstrained problem as a starting point for the constrained solution. Monotonicity analysis is then used to prove activity of some of the constraints, and thus simplify the problem. Finally, the branch-and-bound technique is shown to be suitable for finding a design with discrete values for all the pipe diameters. A simple example is provided. In addition, a method is also demonstrated for further refinement of the pipe network to eliminate excessive throttling losses in the consumer's control valves. The method developed here should be feasible for designing the piping networks for district heating systems of moderate size, and its major advantage is its flexibility.

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