# Comparison of Methods to Determine Disk and Heartwood Areas 

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#### Abstract

The feasibility of using radius measurements on disks to determine cross-sectional areas of tree stems and the heartwood they contain was examined in sugar maple and red oak butt logs. Areas calculated from quadratic means of four stem radii and four heartwood radii were compared with areas measured with a planimeter. The lineal measurement method was less precise for disks taken from stumps than it was for disks taken from the tops of butt logs, probably because of bole irregularities. The radii measurements were more variable for heartwood than for complete disks in sugar maple but not in red oak. Areas computed from radii were the same as areas measured with a planimeter only for maple stump disks, red oak stump disks, and red oak heartwood.


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## Introduction

The subject of how to measure disk and heartwood size arose during research on the effect of silvicultural treatments on the quality of sugar maple and northern red oak. The amount and distribution of heartwood is an important determinant of wood value. Heartwood development in sugar maple is important because "white" maple is more valuable than maple with discolored heartwood. Unlike species that form normal heartwood, the wound-initiated discoloration called heartwood in sugar maple often is irregular in shape and off-center, particularly near the tree base. Disks from higher in the bole tend to have more regular, centered heartwood except when heartwood development is influenced by the proximity of branches or stem bifurcations. This situation is sometimes complicated by the presence of more than one pith in a cross section. Heartwood discoloration in maple may be absent or negligible, or it may occupy more than half of the crosssectional area. By contrast, red oak forms normal heartwood that is much more valuable than the sapwood. Red oak heartwood is symmetrical around the pith and the sapwood in mature trees usually is less than an inch wide.

The shape of an entire disk is affected by the size of the tree and the location of the disk. In small trees, cross section usually is circular with few irregularities. In large trees, flutes or buttresses can cause irregularly shaped cross sections with many deep indentations or protuberances; this condition also is pronounced at the tree base and decreases with height.

In this report we compare disk and heartwood areas determined by radius measurements and directly with a planimeter. The planimeter method can be used when disks are available, but it cannot be used in standing trees, and may be ineffective on stumps in the forest or on logs in a log yard. Radius measurements might be feasible in any situation, either from increment borings or by measurement of stumps and log ends using a tape measure or log scale. One also might use radius measurements on disks with large portions missing. The validity of using rays measured from a point inside an irregular shape was examined by Matérn (1956) and Gregoire and Valentine (1995). They showed that the quadratic mean of length measurements from a fixed point can be used to calculate an unbiased estimate of cross-sectional area, and that the closer the fixed point is to the geometric center of the shape, the more precisely area can be estimated.

## Methods

For this analysis, we used 60 Grade 1 sugar maple trees and 60 Grade 1 red oak trees (Hanks 1976) harvested from the Menominee Indian Reservation, Wisconsin. For each species there were 20 trees each in three groups:


Figure 1.-Method for determining radii using random angle.

14-18, 19-23, and 24+ inches in diameter at breast height (d.b.h.). The trees were felled leaving stumps of less than a foot and bucked to length based on the feller's assessment of optimum log length to maximize value; log lengths were recorded. Cross-sectional disks were cut from the stumps and from the top of each butt log. Disk size and the amount of heartwood were evaluated using a tape measure (linear measurement) and with a planimeter.

## Linear Method

For the linear measure, each disk was assigned an angle ranging from zero to 359 degrees in increments of 1 degree. The angles were generated using the random number generation tool in Microsoft Excel 2000. ${ }^{1} \mathrm{~A}$ circular protractor ( 8 -inch diameter) was placed with its center point at the pith of each disk, a mark was made at the random angle and at points $90^{\circ}, 180^{\circ}$, and $270^{\circ}$ from the first mark, and lines were drawn across the disks through the two diameters defined by these marks (Fig. 1). A tape measure was stretched across the entire cross section on each diameter, and length readings were taken at the bark-cambium, sapwood-heartwood, pith, heartwood-sapwood, and cambium-bark boundaries.

[^0]Where decay left a hollow in the center of a disk, the position of the pith was estimated from the direction of the wood rays. If a disk contained more than one pith, one was chosen randomly. If damage resulted in incomplete disks, the radii affected were estimated if the amount of missing wood was small, or not measured if the amount of missing wood was too great to allow an accurate estimate.

For each disk that was complete enough to provide four pith-to-bark measurements, inside-bark disk radii and heartwood radii were calculated by subtraction of appropriate pairs of measurements. Average disk and heartwood radii were computed as quadratic means (QMR) using the following equation:

$$
\mathrm{QMR}=\left(\left(\mathrm{r}_{1}^{2}+\mathrm{r}_{2}^{2}+\mathrm{r}_{3}^{2}+\mathrm{r}_{4}^{2}\right) / 4\right)^{1 / 2}
$$

where $r_{1^{\prime}}, r_{2^{\prime}} r_{3^{\prime}}$ and $r_{4}$ are the four radii.
For each disk, quadratic mean area (QMA) of each whole disk and its heartwood were calculated as $\pi(\mathrm{QMR}) .{ }^{2}$ The population of disks was subdivided into eight groups: maple and oak stumps, whole disks; maple and oak stumps, heartwood; maple and oak tops, whole disks; maple and oak tops, heartwood.

Mean QMR, the standard deviations of the quadratic mean radii, and coefficients of variability of the quadratic mean were computed for each group. Mean QMA for each group was computed as arithmetic mean.

## Planimeter Method

The heartwood and disk areas were measured to the nearest 0.1 square inch for each disk using a Lasico Model Series 1282 Planimeter/Digitizer. To check the calibration of the instrument, 20 circles and 18 irregular closed shapes in a range of sizes were generated and plotted using Autocad 14.01. These shapes were traced with the planimeter and their measured areas compared with areas calculated by the Autocad. Percentage errors between planimeter and Autocad areas were calculated by subtracting Autocad area from planimeter area, dividing the difference by Autocad area, and multiplying by 100 to obtain percent.

The planimeter was then used to determine areas (PA) of disks and heartwood for each disk for which measurements were possible. The disk area (inside the bark) was measured by following the contour of the cambium. Where small amounts of wood were missing, the contour was interpolated; disks with large amounts of missing wood were rejected. The heartwood area was the area inside the periphery marked by the dark more or less central area of the disk.

QMA was plotted as a function of PA for each group of disks, and the coefficient of determination ( $\mathrm{R}^{2}$ ) was
computed for each group. For each disk and its heartwood, percentage differences were calculated by subtracting PA from QMA, dividing by PA, and multiplying by 100 to obtain percent. PA was used as the base because as a direct measurement it was assumed to be more accurate than area calculated from radii. Mean percentage differences (MPD) and their standard deviations were computed for each group. Because the differences between areas computed using radii and those measured by planimeter were both positive and negative, we also computed mean absolute percent differences (MAPD).

Ninety-five percent confidence intervals were computed for differences between the Autocad and planimeter measurements, and for the MPD. The confidence intervals were inspected to determine if they bracketed zero. If they did, the two methods were judged not significantly different.

## Results

In the calibration tests, errors ranged from -0.4 to +2.0 percent. The planimeter method overestimated areas in 18 of the 20 circles and 17 of the 18 irregular shapes (Table 1). The precision for tracing irregular shapes was the same as for tracing circles, and the coefficient of determination between percent error and area was less than 0.01 , indicating no trend for circles or irregular shapes. However, inspection of 95-percent confidence intervals ( 0.62 to 1.01 percent for circles; 0.53 to 0.93 percent for shapes) verified that the areas measured by planimeter and Autocad differed for both circles and irregular shapes. The consistent overestimate of area for all but the smallest shape may have resulted from instrument error or from systematic error in tracing, e.g., tending toward the outside of the curve. Wenger (1984) reported that a planimeter can measure small areas within 1 percent of error, and larger areas within 0.1 percent. Mean percentage error in our calibration was 0.8 percent for circles and 0.7 percent for shapes (Table 1 ), so we believe that the degree of precision is acceptable for measuring areas on rough wood disks.

Results of the radius and planimeter measurements, areas calculated from radii, and percentage differences between pairs of values are listed in Tables 2 through 5, respectively, for sugar maple stump disks, sugar maple top disks, red oak stump disks, and red oak top disks. Measurement of area with a planimeter was not possible in some instances because a disk was missing large pieces (missing data indicated by dashes). Figure 2 shows the percentage of disks in each absolute percentage difference category for each measurement group. The QMA method was much more accurate for top disks than for stump disks. In top disks, errors exceeded 20 percent only for maple heartwood, whereas in stump disks, some errors exceeded 70 percent in every measurement group.


Figure 2.-Percentage of measurements, for each measurement group, with indicated differences between quadratic mean area (QMA) and planimeter area (PA). Labels on abscissa denote midpoints of 10percent difference categories. Bars indicate percentage of measurements in each category; lines indicate cumulative percentages.


Figure 3.-Disk (A) and heartwood (B) quadratic mean areas (ordinates) versus planimeter areas (abscissas) in sugar maple stumps.
underestimates in 21. The difference was substantial in many cases: it exceeded 25 percent in 7 of the 43 disks and 80 percent in one disk (Table 2). These large differences likely were due to the irregular shape of the stump disks. Further, for all 43 disks taken together, quadratic mean seemed to overestimate disk area, as is evident in the difference of +6.0 percent in the MPD (Table 6), though this difference was not statistically significant. Overall, QMA differed from PA by 13.9 percent (MAPD, Table 6). Figure 3A shows the relationship between area determined by radial measurements and area measured by planimeter; the coefficient of determination was 0.81 (Table 6).

There also were large differences between the two measurement methods with respect to heartwood area. Of the 51 disks whose heartwood was measurable with a planimeter, the quadratic means method overestimated heartwood area in 29 and underestimated it in 21 (one disk had no heartwood). Differences between the two methods exceeded 25 percent in 21 disks and exceeded 70 percent in one disk (Table 2). Overall, QMA differed from PA by 26.2 percent (MAPD, Table 6). This difference was an overestimate of QMA as indicated by the statistically significant difference of +10.3 percent in the MPD (Table 6). Figure 3B shows the relationship between stump disk heartwood areas measured by the


Figure 4.-Stump disk of sugar maple, showing irregularity in disk shape, and extreme irregularity in heartwood shape.
two methods. The relationship between area determined by radii and area determined by planimeter was stronger for heartwood than for whole disks; the coefficient of determination was 0.92 (Table 6).

Comparing the MAPDs between areas determined from QMR and planimeter (Table 6) shows that the lineal measurement method gave closer estimates of disk areas than heartwood areas. That the MAPD for heartwood was nearly twice that for entire disks may be due to the often extreme irregularity in heartwood shape. It is possible that heartwood shape is affected by both disk shape and the wound compartmentalization process (see Figure 4). This result is supported by comparison of standard deviations, which shows that percentage differences are more variable for heartwood (SD of MPD $=31.6$ percent) than for whole disks (SD of MPD $=20.8$ percent) (Table 6).

## Sugar Maple Top Disks

The top disks were located 8.6 to 18.1 feet above the stump disks (Table 3). Comparison of disk areas computed from radii with areas measured directly with the planimeter showed a strong tendency toward overestimation by the radius method, and the confidence interval showed a significant difference between methods (Table 6). Of the 57 top disks that were complete enough for planimeter measurements of disk area, the QMR method overestimated area in 54. However, in none of the 57 disks did the difference between the two areas exceed 25 percent; the greatest
difference was 18.5 percent (Table 3). For all 57 disks taken together, QMA tended to overestimate (difference of +8.4 percent in MPD, Table 6 ). Overall, QMA differed from PA by 8.8 percent (MAPD, Table 6). Figure 5A shows the strong relationship ( $\mathrm{r}^{2}=0.99$, Table 6) between area determined by radial measurements and area measured by planimeter in maple top disks.

Differences between the two area measures were larger for heartwood than whole disks. Of the 58 disks whose heartwood was measurable with the planimeter, differences exceeded 25 percent in 5 (Table 3). The quadratic means method overestimated heartwood area in 51 disks and underestimated it in 7. The tendency toward overestimation is seen in the statistically significant difference of +11.0 percent in the MPD (Table 6 ). Overall, QMA differed from PA by 12.0 percent (MAPD, Table 6). Figure 5B shows the strong relationship ( $\mathrm{r}^{2}=0.99$, Table 6) between heartwood areas measured by the two methods.

Comparing MAPD shows that the lineal measurement method gave closer estimates for disk than for heartwood areas. The MAPD for heartwood (12.0 percent) was one-third greater than the MAPD for entire disks ( 8.8 percent; Table 6). Although this difference was smaller than for stump disks, it supports the contention that maple heartwood shape is strongly affected by the wound compartmentalization process; a comparison of standard deviations also shows that differences are higher for heartwood (SD of MPD $=9.8$ percent) than for whole disks (SD of MPD $=4.2$ percent) (Table 6).

## Red Oak Stump Disks

The oak stump disks had much larger heartwood radii than the maple stump disks even though the disks were about the same size. Comparisons of QMR show that disk radii were nearly equal (14.5 inches for oak, 15.0 inches for maple) but that the heartwood radii were much larger in oak (13.3 inches for oak, 3.4 inches for maple). The coefficients of variation also were similar for the disks ( 0.25 inch for oak, 0.24 inch for maple) but differed greatly for heartwood (0.27 inch for oak, 0.71 inch for maple) (Table 6).

All 60 oak stump disks were complete enough to measure disk and heartwood areas with the planimeter. QMR gave overestimates of disk area in 24 disks and underestimates in 36; the maximum difference was 76 percent (Table 4). Differences exceeded 25 percent in 10 disks (Table 4). As with maple, these large differences likely were due to the irregular shape of the stump disks. QMR did not bias the disk area, as seen in the low MPD (-0.1 percent; Table 6) and the confidence interval showed no difference between the two methods. Overall, QMA differed from PA by 13.4 percent (MAPD, Table 6). Figure 6A shows the relationship between areas measured by the two methods; as in maple the coefficient of determination was 0.81 (Table 6).


Figure 5.-Disk (A) and heartwood (B) quadratic mean areas (ordinates) versus planimeter areas (abscissas) in sugar maple top disks.

QMR overestimated heartwood area in 24 oak disks and underestimated it in 36; the differences between the two methods exceeded 25 percent in 10 disks (Table 4).
Overall, QMA differed from PA by 13.3 percent (MAPD, Table 6), though the confidence interval showed no statistically significant difference. Figure 6B shows the relationship between stump disk heartwood areas measured by the two methods. The relationship between heartwood area determined by radii and by planimeter is nearly the same as for whole disk areas (coefficients of determination are 0.82 and 0.81 , respectively; Table 6).

Comparing MAPDs also shows that the lineal measurement method gave similar estimates for whole
disk and heartwood areas; MAPD was 13.0 percent for heartwood and 13.4 percent for disks (Table 6). Unlike heartwood in maple, oak heartwood is concentric with disk perimeter, so its variability is affected by disk shape only. As a result, the standard deviation of the percent differences in heartwood areas is nearly the same as that of whole disks (Table 6).

## Red Oak Top Disks

The top disks were 8.5 to 16.7 feet above the stumps (Table 5). As with the stump disks, the oak top disks had much larger heartwood radii than the maple top disks even though the disks were about the same size.


Figure 6.-Disk (A) and heartwood (B) quadratic mean areas (ordinates) versus planimeter areas (abscissas) in red oak stump disks.

Mean QMR was nearly equal in whole disks (9.1 inches for oak, 8.9 inches for maple) but heartwood was much smaller in maple ( 8.7 inches for oak, 4.1 inches for maple; Table 6). Coefficients of variation also were similar for the whole disks ( 0.22 inches for oak, 0.23 inches for maple) but differed greatly for heartwood ( 0.23 inches for oak, 0.45 inches for maple) (Table 6).

Fifty-nine oak top disks were complete enough for measurements of disk and heartwood areas with a planimeter. Area by QMR was overestimated in 51 disks and underestimated in 8; the largest difference was 18 percent (Table 5). Unlike the stump disks, top disks had greater QMA than PA as shown by the MPD of +4.6
percent (Table 6); the confidence interval showed that the difference was significant. Overall, QMA differed from PA by 5.2 percent (MAPD; Table 6). Figure 7A shows a strong ( $\mathrm{r}^{2}=0.99$; Table 6) relationship between area determined by radial measurements and area measured by planimeter in oak top disks.

There were slightly larger differences between the two measures for heartwood than for whole disk area (MAPD 5.9 percent for heartwood, 5.2 percent for disks; Table 6), though the largest difference was 18 percent in both cases (Table 5). The quadratic means method overestimated heartwood area in 52 disks and underestimated it in 7. The tendency toward overestimation (MPD $=+5.5$ percent) also is seen in


Figure 7.—Disk (A) and heartwood (B) quadratic mean areas (ordinates) versus planimeter areas (abscissas) in red oak top disks.

Table 6; the confidence interval showed that the difference between the two methods was significant (Table 6). Figure 7B shows this strong ( $\mathrm{r}^{2}=0.99$; Table 6) relationship.

## Discussion and Conclusions

Although the QMR can be used to obtain an unbiased estimate of area, it overestimated area in seven of eight measurement groups. These overestimates were statistically significant in five groups: maple stump heartwood and top disks and heartwood in both maple and oak. The probable cause of these overestimates is
related to the measurement methodology. Linear measurements were taken on disks immediately after they arrived at the Northeastern Station's Forestry Sciences Laboratory at Princeton, WV. Upon completion of these measurements, areas were measured with a planimeter. The loss of moisture during this interval probably resulted in a reduction in disk and heartwood size. Because many of the stump disks were much thicker (sometimes exceeding 8 inches) than the top disks (mostly less than 2 inches), they would have dried more slowly and had less shrinkage than the top disks. This accounts for the statistically significant differences between QMA and PA for top disks but not for stump disks.

Coefficients of determination between QMA and PA were 0.81 to 0.92 for stumps and 0.99 for tops. The lower values for stumps can be attributed to the much greater irregularity in shape of both the disks and the heartwood they contain.

Except for maple heartwood, at least half of the QMAs were within 10 percent of the PAs, as seen in Figure 2. Overall, QMA estimated PA to within 10 percent for 59 percent of the measurements, to within 20 percent for 83 percent of the measurements, and to within 30 percent for 92 percent of the measurements. The largest discrepancies between QMA and PA were for heartwood areas in maple stump disks; the smallest were for disk and heartwood areas in oak top disks. Stump disks were more irregular in shape than top disks, and maple heartwood was more irregular in shape than oak heartwood. The disks or heartwood areas with the largest errors were easily identified by their highly irregular shapes.

Modifying the QMA procedure might increase its level of precision in disks or heartwood areas that are highly irregular in shape. One possibility is to increase the number of radii measured to achieve a desired level of precision if the sample contains irregularly shaped disks. An example might be to measure an additional four radii - giving a total of eight equally spaced measurements - if the ratio of the longest to shortest radius in any disk exceeds a chosen value. The chosen value would depend on the characteristics of the area being measured and the degree of accuracy and level of precision required.

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Table 1.-Data on circles and irregular shapes generated during calibration of the planimeter (linear and area measurements in inches and square inches)

| Item | Autocad |  | Planimeter | Percent Error |
| :---: | :---: | :---: | :---: | :---: |
|  | Radius | Area |  |  |
| Circle 1 | 1.784 | 10 | 10.2 | 2.00 |
| Circle 2 | 2.523 | 20 | 20.1 | 0.50 |
| Circle 3 | 3.090 | 30 | 30.0 | 0.00 |
| Circle 4 | 3.568 | 40 | 40.0 | 0.00 |
| Circle 5 | 3.989 | 50 | 50.4 | 0.80 |
| Circle 6 | 5.642 | 100 | 100.8 | 0.80 |
| Circle 7 | 6.910 | 150 | 151.7 | 1.13 |
| Circle 8 | 7.979 | 200 | 201.6 | 0.80 |
| Circle 9 | 8.921 | 250 | 253.0 | 1.20 |
| Circle 10 | 9.772 | 300 | 302.5 | 0.83 |
| Circle 11 | 10.555 | 350 | 352.8 | 0.80 |
| Circle 12 | 11.284 | 400 | 403.7 | 0.92 |
| Circle 13 | 11.968 | 450 | 454.5 | 1.00 |
| Circle 14 | 12.616 | 500 | 505.4 | 1.08 |
| Circle 15 | 13.231 | 550 | 553.4 | 0.62 |
| Circle 16 | 13.820 | 600 | 604.8 | 0.80 |
| Circle 17 | 14.384 | 650 | 655.4 | 0.83 |
| Circle 18 | 14.927 | 700 | 705.1 | 0.73 |
| Circle 19 | 15.451 | 750 | 755.7 | 0.76 |
| Circle 20 | 15.958 | 800 | 805.3 | 0.66 |
| Mean | 9.419 | 345.000 | 347.820 | 0.814 |
| Shape 1 | - | 10.04 | 10.0 | -0.40 |
| Shape 2 | - | 20.23 | 20.4 | 0.84 |
| Shape 3 | - | 30.12 | 30.6 | 1.59 |
| Shape 4 | - | 40.00 | 40.3 | 0.75 |
| Shape 5 | - | 49.89 | 50.2 | 0.62 |
| Shape 6 | - | 100.03 | 101.3 | 1.27 |
| Shape 7 | - | 150.25 | 151.5 | 0.83 |
| Shape 8 | - | 200.20 | 202.4 | 1.10 |
| Shape 9 | - | 250.14 | 251.0 | 0.34 |
| Shape 10 | - | 300.38 | 302.4 | 0.67 |
| Shape 11 | - | 350.36 | 353.4 | 0.87 |
| Shape 12 | - | 400.22 | 403.0 | 0.69 |
| Shape 13 | - | 450.34 | 453.0 | 0.59 |
| Shape 14 | - | 499.93 | 502.8 | 0.57 |
| Shape 15 | - | 549.97 | 553.3 | 0.61 |
| Shape 16 | - | 599.78 | 604.2 | 0.74 |
| Shape 17 | - | 650.09 | 654.8 | 0.72 |
| Shape 18 | - | 700.36 | 705.2 | 0.69 |
| Mean |  | 297.352 | 299.433 | 0.728 |

Table 2.-Data on 60 sugar maple stump disks by tree d.b.h. (linear and area measurements in inches and square inches)

|  | Tree d.b.h. | Quadratic mean area |  | Disk area |  |  | Heartwood area |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Disk | Heartwood | Quadratic mean | Planimeter | Percent difference | Quadratic mean | Planimeter | Percent difference |
|  | 13.6 | 13.4 | 1.2 | 564.9 | 312.5 | 80.8 | 4.5 | 2.7 | 66.1 |
|  | 14.2 | 9.1 | 0.5 | 261.6 | 243.8 | 7.3 | 0.6 | 0.7 | -7.5 |
|  | 14.3 | 9.8 | 1.2 | 301.3 | 321.8 | -6.4 | 4.3 | 5.7 | -25.0 |
|  | 14.6 | 8.6 | 0.9 | 231.3 | 226.1 | 2.3 | 2.6 | 2.9 | -10.9 |
|  | 15.0 | 11.0 | 1.6 | 379.2 | 305.3 | 24.2 | 8.0 | 7.2 | 10.7 |
|  | 15.0 | 12.6 | 1.0 | 499.5 | 484.2 | 3.2 | 2.9 | 3.6 | -19.5 |
|  | 15.1 | 12.4 | 0.9 | 479.8 | 422.9 | 13.5 | 2.8 | 2.9 | -4.4 |
|  | 15.5 | 10.5 | 2.1 | 346.2 | 349.3 | -0.9 | 13.7 | 17.2 | -20.5 |
|  | 15.6 | 10.7 | 1.4 | 361.7 | 376.9 | -4.0 | 6.5 | 3.8 | 72.3 |
|  | 15.9 | 9.5 | 1.1 | 283.0 | 305.2 | -7.3 | 3.5 | 2.7 | 28.6 |
|  | 16.0 | 14.1 | 1.6 | 624.2 | 584.1 | 6.9 | 7.8 | 9.1 | -14.5 |
|  | 16.1 | 10.8 | 2.2 | 369.1 | 308.4 | 19.7 | 14.9 | 9.2 | 62.2 |
|  | 16.1 | 12.0 | 2.0 | 455.8 | 535.3 | -14.8 | 12.5 | 9.6 | 30.3 |
|  | 16.2 | 11.1 | 3.0 | 390.2 | 389.2 | 0.2 | 28.9 | 21.4 | 34.9 |
|  | 16.3 | - | - | - | - | - | - | - | - |
|  | 16.4 | 10.0 | 1.6 | 317.2 | 322.5 | -1.6 | 8.4 | 11.8 | -29.1 |
|  | 16.5 | 10.8 | 0.5 | 367.6 | 374.7 | -1.9 | 0.8 | 0.5 | 62.2 |
|  | 17.8 | 13.8 | 0.6 | 597.8 | 566.2 | 5.6 | 1.2 | 1.2 | 1.2 |
|  | 17.9 | 11.2 | 4.2 | 396.5 | 440.6 | -10.0 | 55.0 | 41.0 | 34.1 |
|  | 18.0 | 14.0 | 1.7 | - | - | - | 8.6 | 23.3 | -63.0 |
|  | 18.0 | 14.4 | 2.9 | 651.3 | 665.5 | -2.1 | 27.0 | 30.8 | -12.2 |
|  | 18.1 | 11.1 | 5.5 | 384.5 | 466.2 | -17.5 | 94.3 | 62.5 | 50.9 |
|  | 18.3 | 18.4 | 2.5 | 1064.4 | 618.8 | 72.0 | 20.0 | 23.5 | -14.8 |
|  | 18.7 | - | - | - | - | - | - | - | - |
|  | 18.9 | - | - | - | - | - | - | - | - |
|  | 19.8 | 17.4 | 0.0 | 953.7 | 757.0 | 26.0 | 0.0 | 0.0 | 0.0 |
|  | 19.9 | 17.1 | 1.8 | - | - | - | 10.2 | 9.2 | 10.8 |
|  | 20.2 | 14.0 | 2.1 | - | - | - | 13.4 | 8.3 | 61.4 |
|  | 20.4 | 15.2 | 2.1 | 724.8 | 626.1 | 15.8 | 13.5 | 10.7 | 26.6 |
|  | 21.1 | 15.2 | 5.1 | 724.8 | 684.5 | 5.9 | 81.7 | 65.2 | 25.2 |
|  | 21.8 | 13.7 | 1.4 | 586.6 | 578.3 | 1.4 | 6.2 | 7.6 | -18.7 |
|  | 22.0 | 14.2 | 4.0 | 630.4 | 733.4 | -14.0 | 50.0 | 47.2 | 5.9 |
|  | 22.0 | 14.3 | 4.0 | 643.0 | 709.9 | -9.4 | 50.8 | 36.3 | 40.0 |
|  | 22.2 | 16.1 | 1.4 | 814.5 | 829.5 | -1.8 | 6.3 | 17.6 | -64.0 |
|  | 22.3 | 17.2 | 4.3 | - | - | - | 59.2 | 61.1 | -3.1 |
|  | 22.8 | 16.5 | 2.6 | 856.0 | 735.9 | 16.3 | 22.0 | 17.0 | 29.4 |
|  | 22.9 | 18.6 | 1.8 | 1090.3 | 860.7 | 26.7 | 10.2 | 13.3 | -23.6 |
|  | 22.9 | - | - | - | - | - | - | - | - |
|  | 23.2 | 18.9 | 5.6 | 1121.5 | 857.2 | 30.8 | 98.9 | 64.6 | 53.0 |
|  | 23.7 | 16.6 | 4.1 | 864.4 | 906.9 | -4.7 | 54.0 | 44.5 | 21.4 |
|  | 24.1 | 14.7 | 3.9 | 680.7 | 713.7 | -4.6 | 46.7 | 44.3 | 5.3 |
|  | 24.3 | 14.3 | - | - | - | - | - | - | - |
|  | 24.5 | 14.3 | 5.3 | 644.7 | 772.7 | -16.6 | 87.6 | 74.7 | 17.3 |
|  | 24.7 | 13.8 | 6.2 | 601.5 | 621.7 | -3.3 | 120.4 | 100.2 | 20.2 |
|  | 24.9 | 13.5 | 3.9 | 574.8 | 566.2 | 1.5 | 46.7 | 38.7 | 20.7 |
|  | 25.3 | 21.5 | - | - | - | - | - | - | - |
|  | 25.5 | 19.0 | 4.4 | 1136.8 | 870.2 | 30.6 | 59.6 | 58.4 | 2.1 |
|  | 27.1 | - | - | - | - | - | - | - | - |
|  | 27.2 | 18.3 | 6.2 | 1050.5 | 824.8 | 27.4 | 122.1 | 97.7 | 25.0 |
|  | 28.1 | 23.2 | 7.0 | , | - | - | 153.0 | 99.2 | 54.2 |
|  | 28.2 | 19.6 | 5.8 | 1207.6 | 1256.1 | -3.9 | 105.2 | 134.3 | -21.7 |
|  | 28.3 | 18.2 | 4.6 | 1043.5 | 1189.0 | -12.2 | 65.5 | 89.5 | -26.8 |
|  | 29.0 | 20.0 | 6.4 | 1260.9 | 1139.4 | 10.7 | 128.5 | 92.1 | 39.5 |
|  | 29.1 | 14.6 | 8.4 | - | - | - | 221.7 | 221.8 | -0.1 |
|  | 29.3 | 17.6 | 3.8 | - | - | - | 45.5 | 38.2 | 19.1 |
|  | 29.4 | 18.1 | 5.4 | 1031.1 | 1310.4 | -21.3 | 91.1 | 99.5 | -8.5 |
|  | 30.2 | 17.6 | 7.0 | 969.6 | 1092.1 | -11.2 | 155.4 | 171.1 | -9.2 |
|  | 30.4 | 22.3 | 6.9 | - | - | - | 149.8 | 165.0 | -9.2 |
|  | 30.4 | 23.2 | 11.8 | - | - | - | - | - | - |
|  | 33.3 | - | - | - | - | - | - | - | - |
| Means | 21.48 | 14.97 | 3.41 | 663.70 | 633.84 | 6.03 | 47.13 | 43.54 | 10.28 |

Table 3.-Data on 60 sugar maple top disks by tree d.b.h. (linear and area measurements in inches and square inches unless otherwise indicated)

|  | Distance above stump | Tree d.b.h. | Quadratic mean radius |  | Disk area |  |  | Heartwood area |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (feet) |  | Disk | Heartwood | Quadratic mean | Planimeter | Percent difference | Quadratic mean | Planimeter | Percent difference |
|  | 12.7 | 13.6 | 5.8 | 2.3 | 107.2 | 96.6 | 10.9 | 16.4 | 13.7 | 20.0 |
|  | 10.7 | 14.2 | 5.9 | 1.8 | 108.7 | 102.6 | 5.9 | 10.7 | 8.8 | 21.0 |
|  | 11.3 | 14.3 | 6.0 | 2.3 | 114.8 | 107.1 | 7.2 | 17.3 | 18.7 | -7.3 |
|  | 8.6 | 14.6 | 6.4 | 2.1 | 130.2 | 119.0 | 9.4 | 13.3 | 10.9 | 21.6 |
|  | 9.6 | 15.0 | 7.1 | 1.5 | 158.8 | 139.6 | 13.8 | 7.4 | 6.5 | 14.2 |
|  | 11.7 | 15.0 | 6.2 | 2.4 | 121.2 | 109.9 | 10.3 | 18.3 | 16.9 | 8.2 |
|  | 9.6 | 15.1 | 6.6 | 1.9 | 136.7 | 122.5 | 11.6 | 11.7 | 11.9 | -1.3 |
|  | 12.6 | 15.5 | 6.3 | 2.7 | 123.2 | 113.5 | 8.5 | 23.1 | 20.9 | 10.4 |
|  | 12.6 | 15.6 | 6.5 | 1.8 | 131.7 | 119.7 | 10.0 | 10.0 | 9.4 | 6.4 |
|  | 9.7 | 15.9 | 6.6 | 2.0 | 138.3 | 133.9 | 3.3 | 12.2 | 13.5 | -9.6 |
|  | 10.7 | 16.0 | 7.1 | 2.5 | 158.1 | 146.4 | 8.0 | 20.0 | 17.2 | 16.4 |
|  | 10.7 | 16.1 | 7.1 | 2.5 | 158.3 | 143.3 | 10.4 | 20.1 | 17.2 | 16.6 |
|  | 10.7 | 16.1 | 6.8 | 2.3 | 147.3 | 130.7 | 12.7 | 16.2 | 14.0 | 15.9 |
|  | 12.5 | 16.2 | 6.9 | 4.7 | 147.4 | 132.4 | 11.4 | 69.2 | 52.0 | 33.1 |
|  | 9.5 | 16.3 | 6.9 | 2.9 | 147.7 | 137.4 | 7.5 | 26.4 | 26.5 | -0.5 |
|  | 12.8 | 16.4 | 7.4 | 2.0 | 171.2 | 156.4 | 9.5 | 12.8 | 13.8 | -7.3 |
|  | 8.8 | 16.5 | 6.6 | 1.9 | 138.0 | 147.9 | -6.7 | 11.9 | 11.0 | 8.1 |
|  | 12.7 | 17.8 | 7.4 | 1.9 | 172.0 | 157.8 | 9.0 | 11.4 | 8.3 | 37.2 |
|  | 9.5 | 17.9 | 7.7 | 3.7 | 183.9 | 168.1 | 9.4 | 42.1 | 41.9 | 0.4 |
|  | 9.5 | 18.0 | 8.0 | 2.7 | 200.9 | 180.7 | 11.2 | 23.4 | 22.4 | 4.4 |
|  | 13.6 | 18.0 | 7.5 | 4.2 | 178.6 | 163.3 | 9.4 | 56.3 | 51.1 | 10.1 |
|  | 9.7 | 18.1 | 7.6 | 5.5 | 183.3 | 163.9 | 11.8 | 95.0 | 73.8 | 28.8 |
|  | 11.7 | 18.3 | 7.6 | 3.4 | 179.2 | 163.5 | 9.6 | 36.3 | 32.1 | 13.1 |
|  | 10.8 | 18.7 | 8.0 | 2.4 | 202.5 | 181.8 | 11.4 | 18.6 | 16.2 | 14.5 |
|  | 14.5 | 18.9 | 8.2 | 3.7 | 213.6 | 196.2 | 8.9 | 43.2 | 31.5 | 37.1 |
|  | 16.8 | 19.8 | 7.8 | 2.4 | 192.8 | 188.1 | 2.5 | 18.6 | 16.6 | 11.9 |
|  | 9.6 | 19.9 | 8.7 | 3.0 | 236.9 | 222.0 | 6.7 | 27.4 | 23.2 | 18.3 |
|  | 16.8 | 20.2 | 8.9 | 3.5 | 249.6 | 235.4 | 6.0 | 38.1 | 35.9 | 6.1 |
|  | 16.7 | 20.4 | 8.0 | 3.4 | 200.5 | 207.1 | -3.2 | 36.8 | 36.3 | 1.3 |
|  | 10.6 | 21.1 | 7.8 | 3.9 | 192.9 | 187.1 | 3.1 | 49.0 | 49.8 | -1.7 |
|  | 16.6 | 21.8 | 9.1 | 3.5 | 260.8 | 236.5 | 10.3 | 38.0 | 33.5 | 13.4 |
|  | 9.5 | 22.0 | 9.5 | 4.8 | 284.6 | 267.0 | 6.6 | 71.5 | 69.4 | 3.0 |
|  | 11.6 | 22.0 | 9.4 | 4.6 | 278.8 | 257.2 | 8.4 | 66.2 | 59.6 | 11.1 |
|  | 11.5 | 22.2 | 9.7 | 3.3 | 296.5 | 264.7 | 12.0 | 34.0 | 30.1 | 12.8 |
|  | 10.7 | 22.3 | 9.6 | 4.5 | 288.9 | 269.0 | 7.4 | 64.4 | 59.8 | 7.6 |
|  | 15.7 | 22.8 | 9.3 | 3.4 | 270.0 | 238.8 | 13.1 | 37.2 | 32.6 | 14.2 |
|  | 9.6 | 22.9 | 9.9 | 4.4 | 307.8 | 278.4 | 10.6 | 60.3 | 49.0 | 23.1 |
|  | 10.7 | 22.9 | 9.8 | 3.2 | 301.3 | 273.1 | 10.3 | 31.4 | 31.8 | -1.3 |
|  | 11.5 | 23.2 | 10.1 | 4.5 | 318.2 | 277.1 | 14.8 | 64.4 | 58.4 | 10.2 |
|  | 16.3 | 23.7 | - | - | - | - | - | - | - | - |
|  | 14.5 | 24.1 | 9.6 | 5.1 | 287.8 | 269.0 | 7.0 | 82.2 | 72.7 | 13.0 |
|  | 12.4 | 24.3 | - | - | - | - | - | - | - | - |
|  | 13.7 | 24.5 | 10.3 | 5.4 | 331.8 | 307.6 | 7.9 | 92.5 | 85.9 | 7.7 |
|  | 9.9 | 24.7 | 9.7 | 5.0 | 294.2 | 268.0 | 9.8 | 79.7 | 70.7 | 12.8 |
|  | 11.8 | 24.9 | 9.3 | 5.3 | 274.2 | 257.5 | 6.5 | 88.9 | 87.1 | 2.1 |
|  | 9.7 | 25.3 | 10.7 | 4.9 | 361.8 | 333.3 | 8.5 | 76.5 | 72.8 | 5.1 |
|  | 14.8 | 25.5 | 9.8 | 4.0 | 303.6 | 282.8 | 7.4 | 50.8 | 47.1 | 7.9 |
|  | 12.5 | 27.1 | 11.5 | 7.7 | - | - | - | 186.0 | 168.2 | 10.6 |
|  | 15.6 | 27.2 | 11.4 | 5.8 | 409.3 | 379.5 | 7.8 | 105.7 | 103.7 | 2.0 |
|  | 13.5 | 28.1 | 12.0 | 7.3 | 455.8 | 418.3 | 9.0 | 168.6 | 146.7 | 14.9 |
|  | 10.7 | 28.2 | 12.1 | 5.7 | 457.2 | 385.9 | 18.5 | 102.5 | 92.5 | 10.8 |
|  | 16.8 | 28.3 | 10.8 | 5.3 | 368.2 | 333.8 | 10.3 | 86.9 | 77.2 | 12.6 |
|  | 16.5 | 29.0 | 11.8 | 6.1 | 438.3 | 416.1 | 5.3 | 118.4 | 111.0 | 6.7 |
|  | 18.1 | 29.1 | 11.9 | 7.4 | 443.5 | 452.9 | -2.1 | 170.4 | 144.3 | 18.1 |
|  | 12.6 | 29.3 | 11.7 | 5.3 | 431.7 | 386.9 | 11.6 | 87.8 | 70.2 | 25.1 |
|  | 13.5 | 29.4 | 12.6 | 6.4 | 498.6 | 451.2 | 10.5 | 127.0 | 111.8 | 13.6 |
|  | 13.6 | 30.2 | 12.4 | 7.4 | 481.7 | 430.8 | 11.8 | 173.3 | 161.7 | 7.2 |
|  | 12.6 | 30.4 | 12.4 | 6.9 | 484.6 | 459.3 | 5.5 | 151.7 | 139.2 | 9.0 |
|  | 17.6 | 30.4 | 12.2 | 8.3 | 470.0 | 425.7 | 10.4 | 216.1 | 200.2 | 7.9 |
|  | 10.5 | 33.3 | 12.5 | 8.5 | 493.5 | 485.3 | 1.7 | 228.1 | 207.7 | 9.8 |
| Means | 12.37 | 21.48 | 8.91 | 4.10 | 259.98 | 239.99 | 8.43 | 63.34 | 57.19 | 11.01 |

Table 4.-Data on 60 red oak stump disks by tree d.b.h. (linear and area measurements in inches and square inches)

|  | Tree d.b.h. | $\underline{\text { Quadratic mean radius }}$ |  | Disk area |  |  | Heartwood |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Disk | Heartwood | Quadratic mean | Planimeter | Percent difference | Quadratic mean | Planimeter | Percent difference |
|  | 13.7 | 9.0 | 8.1 | 256.4 | 313.7 | -18.3 | 207.5 | 240.0 | -13.5 |
|  | 15.4 | 10.4 | 8.8 | 339.1 | 340.4 | -0.4 | 241.7 | 250.6 | -3.6 |
|  | 15.5 | 12.3 | 11.5 | 473.7 | 417.0 | 13.6 | 417.2 | 356.4 | 17.1 |
|  | 15.6 | 12.8 | 11.3 | 512.8 | 481.8 | 6.4 | 399.2 | 409.3 | -2.5 |
|  | 15.8 | 10.0 | 8.9 | 317.2 | 321.5 | -1.3 | 249.1 | 258.9 | -3.8 |
|  | 15.8 | 10.9 | 9.9 | 375.2 | 374.3 | 0.2 | 309.8 | 317.0 | -2.3 |
|  | 15.9 | 10.2 | 9.5 | 328.3 | 360.2 | -8.9 | 281.3 | 308.2 | -8.7 |
|  | 16.0 | 11.3 | 9.7 | 403.8 | 363.0 | 11.2 | 292.9 | 272.2 | 7.6 |
|  | 16.1 | 11.0 | 9.7 | 379.9 | 399.4 | -4.9 | 296.0 | 306.6 | -3.5 |
|  | 16.2 | 9.6 | 8.7 | 289.9 | 443.3 | -34.6 | 236.8 | 361.1 | -34.4 |
|  | 16.5 | 11.1 | 10.2 | 388.1 | 403.4 | -3.8 | 325.8 | 328.0 | -0.7 |
|  | 16.8 | 10.9 | 10.0 | 376.1 | 411.9 | -8.7 | 313.8 | 341.5 | -8.1 |
|  | 16.8 | 13.4 | 11.5 | 564.5 | 457.6 | 23.4 | 418.8 | 360.3 | 16.2 |
|  | 17.0 | 10.5 | 9.8 | 343.5 | 482.1 | -28.8 | 300.2 | 409.6 | -26.7 |
|  | 17.2 | 10.4 | 9.5 | 338.6 | 482.3 | -29.8 | 284.6 | 398.2 | -28.5 |
|  | 17.3 | 11.3 | 10.4 | 400.5 | 395.2 | 1.3 | 338.8 | 337.2 | 0.5 |
|  | 17.3 | 12.2 | 10.2 | 466.6 | 392.9 | 18.8 | 323.7 | 295.0 | 9.7 |
|  | 17.3 | 13.6 | 12.1 | 581.4 | 578.2 | 0.5 | 463.5 | 452.1 | 2.5 |
|  | 17.7 | 10.9 | 9.9 | 370.5 | 374.6 | -1.1 | 309.2 | 302.1 | 2.3 |
|  | 18.0 | 12.6 | 11.1 | 500.5 | 422.7 | 18.4 | 385.5 | 333.5 | 15.6 |
|  | 18.8 | 13.2 | 11.9 | 544.2 | 563.4 | -3.4 | 445.2 | 460.4 | -3.3 |
|  | 19.2 | 13.0 | 11.9 | 531.8 | 481.4 | 10.5 | 448.1 | 399.6 | 12.1 |
|  | 19.6 | 13.1 | 12.2 | 540.6 | 672.9 | -19.7 | 468.9 | 576.3 | -18.6 |
|  | 20.0 | 11.6 | 10.8 | 422.7 | 440.1 | -4.0 | 363.7 | 377.5 | -3.7 |
|  | 20.0 | 15.0 | 14.0 | 709.4 | 657.6 | 7.9 | 612.7 | 502.9 | 21.8 |
|  | 20.1 | 11.9 | 11.1 | 447.6 | 655.6 | -31.7 | 386.5 | 557.4 | -30.7 |
|  | 20.1 | 13.8 | 12.4 | 597.3 | 574.2 | 4.0 | 483.2 | 468.6 | 3.1 |
|  | 20.2 | 13.8 | 12.6 | 595.3 | 602.4 | -1.2 | 496.3 | 506.7 | -2.0 |
|  | 20.4 | 12.6 | 11.2 | 497.4 | 625.3 | -20.5 | 395.3 | 496.5 | -20.4 |
|  | 20.5 | 13.1 | 12.2 | 543.1 | 639.5 | -15.1 | 465.7 | 539.2 | -13.6 |
|  | 20.6 | 12.9 | 11.0 | 525.6 | 666.7 | -21.2 | 379.1 | 488.5 | -22.4 |
|  | 21.1 | 15.1 | 13.6 | 714.8 | 649.0 | 10.1 | 582.0 | 539.5 | 7.9 |
|  | 21.7 | 15.1 | 14.1 | 714.9 | 668.6 | 6.9 | 622.2 | 589.3 | 5.6 |
|  | 21.7 | 17.4 | 16.3 | 950.4 | 897.7 | 5.9 | 833.6 | 773.9 | 7.7 |
|  | 22.0 | 15.3 | 14.6 | 738.6 | 743.3 | -0.6 | 668.1 | 651.2 | 2.6 |
|  | 22.5 | 14.6 | 13.7 | 671.7 | 725.6 | -7.4 | 587.7 | 628.3 | -6.5 |
|  | 22.5 | 15.1 | 14.2 | 715.8 | 749.9 | -4.5 | 634.6 | 658.0 | -3.6 |
|  | 22.6 | 18.2 | 16.5 | 1038.2 | 773.8 | 34.2 | 859.0 | 648.0 | 32.6 |
|  | 23.0 | 16.7 | 15.7 | 878.0 | 664.3 | 32.2 | 778.5 | 587.2 | 32.6 |
|  | 23.4 | 15.6 | 14.2 | 767.9 | 795.4 | -3.5 | 632.0 | 652.8 | -3.2 |
|  | 23.8 | 15.4 | 14.6 | 747.1 | 773.2 | -3.4 | 666.4 | 675.8 | -1.4 |
|  | 24.0 | 16.2 | 15.3 | 827.1 | 892.6 | -7.3 | 738.3 | 770.6 | -4.2 |
|  | 24.3 | 21.3 | 19.9 | 1430.1 | 813.8 | 75.7 | 1244.1 | 710.3 | 75.1 |
|  | 24.5 | 15.3 | 14.0 | 730.9 | 819.5 | -10.8 | 611.9 | 671.0 | -8.8 |
|  | 24.6 | 16.2 | 15.0 | 821.5 | 857.9 | -4.2 | 709.8 | 739.8 | -4.1 |
|  | 24.7 | 17.7 | 16.4 | 981.3 | 909.7 | 7.9 | 847.6 | 777.8 | 9.0 |
|  | 25.0 | 13.9 | 13.3 | 610.8 | 632.9 | -3.5 | 559.8 | 581.0 | -3.6 |
|  | 25.3 | 18.3 | 16.8 | 1048.0 | 873.8 | 19.9 | 881.9 | 732.6 | 20.4 |
|  | 25.5 | 18.7 | 17.3 | 1098.5 | 1170.7 | -6.2 | 945.3 | 999.0 | -5.4 |
|  | 26.0 | 13.5 | 12.6 | 576.3 | 889.5 | -35.2 | 500.2 | 749.9 | -33.3 |
|  | 26.5 | 16.1 | 14.0 | 809.6 | 893.8 | -9.4 | 616.3 | 722.9 | -14.7 |
|  | 27.0 | 17.4 | 16.4 | 949.4 | 1054.2 | -9.9 | 845.5 | 914.8 | -7.6 |
|  | 27.1 | 16.6 | 15.9 | 867.6 | 918.3 | -5.5 | 792.1 | 804.8 | -1.6 |
|  | 28.0 | 19.9 | 18.4 | 1241.2 | 1184.3 | 4.8 | 1068.1 | 1066.7 | 0.1 |
|  | 30.5 | 12.6 | 12.3 | 500.7 | 442.3 | 13.2 | 471.7 | 413.9 | 14.0 |
|  | 31.4 | 18.7 | 17.5 | 1098.2 | 1331.2 | -17.5 | 961.9 | 1208.1 | -20.4 |
|  | 31.4 | 23.4 | 22.0 | 1717.9 | 1341.8 | 28.0 | 1519.7 | 1184.4 | 28.3 |
|  | 31.6 | 20.6 | 19.6 | 1329.8 | 1516.1 | -12.3 | 1208.5 | 1334.3 | -9.4 |
|  | 33.3 | 26.2 | 25.4 | 2155.9 | 1508.0 | 43.0 | 2026.8 | 1349.5 | 50.2 |
|  | 33.5 | 20.6 | 19.1 | 1326.8 | 1441.7 | -8.0 | 1140.4 | 1262.6 | -9.7 |
| Means | 21.60 | 14.50 | 13.34 | 700.35 | 695.46 | -0.14 | 598.24 | 590.16 | 0.11 |

Table 5.-Data on 60 red oak top disks by tree d.b.h. (linear and area measurements in inches and square inches unless otherwise indicated)

|  | Dist. above <br> stump <br> (feet) | Tree d.b.h. | Quadratic mean radius |  | Disk area |  |  | Heartwood area |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Disk | Heartwood | Quadratic mean | Planimeter | Percent difference | Quadratic mean | Planimeter | Percent difference |
|  | 12.7 | 13.7 | 5.7 | 5.4 | 103 | 108 | -4.4 | 92 | 96.5 | -4.2 |
|  | 12.7 | 15.4 | 6.8 | 6.4 | 147 | 134 | 9.9 | 127 | 114.3 | 11.0 |
|  | 11.6 | 15.5 | 7.3 | 7.0 | 169 | 143 | 17.9 | 154 | 130.0 | 18.3 |
|  | 10.6 | 15.6 | 6.8 | 6.5 | 145 | 145 | 0.6 | 134 | 130.4 | 2.7 |
|  | 14.8 | 15.8 | 6.4 | 6.0 | 130 | 135 | -3.8 | 114 | 116.9 | -2.4 |
|  | 15.8 | 15.8 | 6.4 | 6.1 | 129 | 128 | 1.2 | 116 | 115.5 | 0.7 |
|  | 8.6 | 15.9 | 7.1 | 6.9 | 160 | 158 | 1.0 | 148 | 145.6 | 1.4 |
|  | 10.6 | 16.0 | 6.8 | 6.4 | 147 | 144 | 2.1 | 129 | 126.4 | 2.4 |
|  | 12.7 | 16.1 | 7.0 | 6.6 | 154 | 150 | 2.4 | 139 | 133.2 | 4.2 |
|  | 10.7 | 16.2 | 7.0 | 6.6 | 156 | 142 | 9.9 | 135 | 125.9 | 7.5 |
|  | 10.6 | 16.5 | 7.1 | 6.7 | 159 | 157 | 1.2 | 142 | 137.7 | 2.9 |
|  | 9.7 | 16.8 | 7.3 | 7.0 | 169 | 170 | -0.2 | 155 | 152.3 | 1.9 |
|  | 12.7 | 16.8 | 7.0 | 6.7 | 155 | 147 | 5.4 | 142 | 133.1 | 6.3 |
|  | 15.6 | 17.0 | 7.1 | 6.7 | 157 | 151 | 3.6 | 140 | 135.7 | 3.5 |
|  | 10.7 | 17.2 | 7.5 | 7.1 | 176 | 162 | 8.6 | 160 | 145.8 | 9.7 |
|  | 9.5 | 17.3 | 7.6 | 7.1 | 180 | 172 | 5.0 | 160 | 150.9 | 6.2 |
|  | 10.6 | 17.3 | 7.1 | 6.7 | 157 | 162 | -3.1 | 140 | 144.4 | -2.8 |
|  | 14.6 | 17.3 | 7.0 | 6.5 | 153 | 152 | 0.7 | 134 | 132.8 | 1.0 |
|  | 10.6 | 17.7 | 7.7 | 7.2 | 185 | 183 | 1.2 | 164 | 160.5 | 2.4 |
|  | 12.5 | 18.0 | 7.4 | 7.0 | 172 | 167 | 2.7 | 152 | 145.8 | 4.5 |
|  | 14.6 | 18.8 | 7.8 | 7.3 | 191 | 182 | 4.9 | 168 | 163.9 | 2.4 |
|  | 10.2 | 19.2 | 8.4 | 8.0 | 221 | 211 | 4.9 | 202 | 191.5 | 5.2 |
|  | 13.7 | 19.6 | 7.9 | 7.6 | 196 | 194 | 1.5 | 182 | 174.2 | 4.5 |
|  | 14.6 | 20.0 | 8.7 | 8.3 | 237 | 215 | 10.4 | 218 | 196.7 | 11.1 |
|  | 16.6 | 20.0 | 8.3 | 7.9 | 215 | 200 | 7.5 | 194 | 180.2 | 7.9 |
|  | 9.7 | 20.1 | 8.6 | 8.3 | 234 | 226 | 3.4 | 214 | 204.6 | 4.6 |
|  | 10.6 | 20.1 | 8.6 | 8.1 | 233 | 235 | -0.7 | 207 | 208.0 | -0.4 |
|  | 8.6 | 20.2 | 8.8 | 8.4 | 244 | 222 | 9.8 | 222 | 198.5 | 11.9 |
|  | 8.7 | 20.4 | 9.2 | 8.9 | 268 | 261 | 2.6 | 246 | 230.4 | 6.9 |
|  | 13.6 | 20.5 | 8.5 | 8.1 | 228 | 214 | 6.7 | 208 | 193.0 | 8.0 |
|  | 14.7 | 20.6 | 8.4 | 7.8 | 220 | 219 | 0.3 | 192 | 191.9 | -0.1 |
|  | 15.8 | 21.1 | 9.1 | 8.7 | 258 | 246 | 4.6 | 237 | 221.0 | 7.1 |
|  | 10.6 | 21.7 | 9.0 | 8.6 | 256 | 243 | 5.2 | 231 | 217.4 | 6.3 |
|  | 14.7 | 21.7 | 8.9 | 8.5 | 248 | 246 | 1.2 | 226 | 219.7 | 2.9 |
|  | 10.7 | 22.0 | 9.5 | 9.0 | 282 | 263 | 7.2 | 257 | 239.0 | 7.5 |
|  | 10.6 | 22.5 | 9.6 | 9.3 | 292 | 265 | 10.3 | 269 | 241.0 | 11.7 |
|  | 13.3 | 22.5 | 9.4 | 8.9 | 275 | 256 | 7.6 | 251 | 230.7 | 8.8 |
|  | 14.7 | 22.6 | 9.2 | 8.7 | 267 | 246 | 8.6 | 238 | 219.9 | 8.1 |
|  | 9.5 | 23.0 | 9.3 | 8.8 | 273 | 261 | 4.4 | 245 | 234.9 | 4.4 |
|  | 13.7 | 23.4 | 10.1 | 9.7 | 320 | 309 | 3.7 | 294 | 281.5 | 4.4 |
|  | 13.6 | 23.8 | 9.5 | 9.2 | 282 | 276 | 2.1 | 265 | 254.8 | 3.8 |
|  | 11.4 | 24.0 | 10.6 | 10.2 | 352 | 340 | 3.5 | 324 | 314.5 | 3.0 |
|  | 9.3 | 24.3 | 10.8 | 10.4 | 367 | 355 | 3.5 | 340 | 325.8 | 4.4 |
|  | 10.8 | 24.5 | 10.4 | 9.8 | 337 | 313 | 7.6 | 303 | 279.1 | 8.6 |
|  | 13.6 | 24.6 | 10.3 | 9.7 | 332 | 314 | 5.5 | 298 | 280.7 | 6.1 |
|  | 8.5 | 24.7 | 10.7 | 10.2 | 357 | 330 | 8.3 | 329 | 299.2 | 9.8 |
|  | 10.5 | 25.0 | 11.0 | 10.6 | 377 | 385 | -2.0 | 356 | 366.0 | -2.7 |
|  | 10.6 | 25.3 | 11.0 | 10.4 | 379 | 339 | 11.8 | 338 | 301.8 | 12.0 |
|  | 10.6 | 25.5 | 10.8 | 10.3 | 366 | 336 | 8.9 | 334 | 306.1 | 9.0 |
|  | 10.6 | 26.0 | 11.2 | 10.7 | 391 | 370 | 5.6 | 360 | 334.9 | 7.5 |
|  | 14.8 | 26.5 | 11.0 | 10.3 | 381 | 336 | 13.4 | 336 | 294.6 | 14.1 |
|  | 14.7 | 27.0 | 11.2 | 10.8 | 394 | 352 | 11.9 | 367 | 329.6 | 11.3 |
|  | 12.6 | 27.1 | 11.3 | 10.8 | 398 | 401 | -0.6 | 369 | 365.0 | 1.2 |
|  | 8.6 | 28.0 | 11.9 | 11.6 | 442 | 398 | 11.2 | 420 | 374.8 | 12.1 |
|  | 10.4 | 30.5 | 11.2 | 10.8 | 396 | 396 | 0.1 | 369 | 362.0 | 1.9 |
|  | 9.6 | 31.4 | 13.8 | 13.3 | 596 | 560 | 6.4 | 556 | 521.4 | 6.7 |
|  | 9.6 | 31.4 | 13.4 | 13.0 | 563 | 572 | -1.6 | 530 | 534.6 | -0.8 |
|  | 9.6 | 31.6 | 13.7 | 13.2 | 593 | 544 | 8.9 | 546 | 506.5 | 7.7 |
|  | 16.7 | 33.3 | 14.0 | 13.7 | 618 | 599 | 3.2 | 592 | 556.6 | 6.3 |
|  | 10.3 | 33.5 | - | - | - | - | - | - | - | - |
| Mean | 11.89 | 21.60 | 9.07 | 8.66 | 271 | 258 | 4.63 | 248 | 234 | 5.47 |

Table 6.-Summary data for disks measured in this study (linear and area measurements in inches and square inches)

| Statistic ${ }^{\text {a }}$ | Sugar maple |  |  |  | Red oak |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Stump |  | Top |  | Stump |  | Top |  |
|  | Disk | Heartwood | Disk | Heartwood | Disk | Heartwood | Disk | Heartwood |
| RADII |  |  |  |  |  |  |  |  |
| $\mathrm{N}_{\mathrm{R}}$ | 54 | 52 | 58 | 58 | 60 | 60 | 59 | 59 |
| Mean QMR | 15.0 | 3.4 | 8.9 | 4.1 | 14.5 | 13.3 | 9.1 | 8.7 |
| CV of QMR | 0.24 | 0.71 | 0.23 | 0.45 | 0.25 | 0.27 | 0.22 | 0.23 |
| AREAS |  |  |  |  |  |  |  |  |
| $\mathrm{N}_{\mathrm{A}}$ | 43 | 51 | 57 | 58 | 60 | 60 | 59 | 59 |
| Mean QMA | 664 | 47 | 260 | 63 | 700 | 598 | 271 | 248 |
| Mean PA | 634 | 44 | 240 | 57 | 695 | 590 | 258 | 234 |
| MPD | +6.0 | +10.3 | +8.4 | +11.0 | -0.1 | +0.1 | +4.6 | +5.5 |
| SD of MPD | 20.8 | 31.6 | 4.2 | 9.8 | 19.1 | 19.0 | 4.6 | 4.5 |
| LCL | -0.4 | 1.4 | 7.3 | 8.4 | -5.1 | -4.8 | 3.4 | 4.3 |
| UCL | 12.4 | 19.2 | 9.6 | 13.6 | 4.8 | 5.0 | 5.8 | 6.7 |
| MAPD | 13.9 | 26.2 | 8.8 | 12.0 | 13.4 | 13.0 | 5.2 | 5.9 |
| $\mathrm{R}^{2}$ | 0.81 | 0.92 | 0.99 | 0.99 | 0.81 | 0.82 | 0.99 | 0.99 |

${ }^{a} N_{R}=$ number of disks used in radius analysis; $N A=$ number of disks used in area analysis; Mean $Q M R=$ mean quadratic mean radius; CV of $\mathrm{QMR}=$ coefficient of variation of QMR; Mean $\mathrm{QMA}=$ mean quadratic mean area; Mean PA=mean area by planimeter; MPD=Mean percentage difference between QMA and PA; SD of MPD=standard deviation of MPD; LCL=lower 95-percent confidence limit for MPD;
UCL=upper 95-percent confidence limit for MPD; MAPD=Mean absolute percentage difference; $\mathrm{R} 2=$ coefficient of determination between QMA and PA.

Wiemann, Michael C.; Brown, John P.; Bennett, Neal D. 2002. Comparison of methods to determine disk and heartwood areas. Res. Pap. NE-720. Newtown Square, PA: U.S. Department of Agriculture, Forest Service, Northeastern Research Station. 15 p .

The feasibility of using radius measurements on disks to determine cross-sectional areas of tree stems and the heartwood they contain was examined in sugar maple and red oak butt logs. Areas calculated from quadratic means of four stem radii and four heartwood radii were compared with areas measured with a planimeter. The lineal measurement method was less precise for disks taken from stumps than it was for disks taken from the tops of butt logs, probably because of bole irregularities. The radii measurements were more variable for heartwood than for complete disks in sugar maple but not in red oak. Areas computed from radii were the same as areas measured with a planimeter for only maple stump disks and red oak stump disks and heartwood.

Keywords: planimeter, red oak, sugar maple


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