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# Computer Can Choose Sowing Strategies for Containerized Seedlings 

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## INTRODUCTION

A crucial problem for nursery managers in growing containerized seedlings results from blank containers. If the overall germination and survival rate is low and a single seed is planted in each container, the number of blank containers will be high, and the cost of carrying these blanks may be unacceptable. Because of the high cost of growing containerized seedlings, efficient methods are needed to deal with the problem caused by blank containers.
The proportion of blank containers can be reduced by planting more than one seed in some or all containers. However, this remedial action requires the sowing of additional seeds and will result in multiple excess seedlings for some containers, which must be thinned and discarded or replanted.
Thus, the nursery manager needs a sowing strategy that deals effectively with the problem of blanks. But the strategy must not impose a severe penalty in terms of additional seed-related and thinning costs.

In this paper we present an interactive computer program (CONSOW) that enables the user to choose an optimal sowing strategy. The strategy is optimal in the sense that it minimizes a penalty function which reflects the cost of carrying blank containers and costs of remedial actions taken to combat the problem of blanks.

CONSOW, written in the programming language BASIC, was developed on a Hewlett-Packard 9830A microcomputer. The 9830A version is one of many versions of BASIC that exist today, and most are machine-dependent. Therefore, some modifications might be necessary to run CONSOW on other computers.

CONSOW was also written in ANS FORTRAN IV1977. This program is not interactive, as is the BASIC
version, but the input variables are entered on cards and the job is run in the batch mode. Both programs are punched on cards and copies can be acquired from the Southern Forest Experiment Station, Biometrics Branch, T-10210 U.S. Postal Services Building, 701 Loyola Avenue, New Orleans, Louisiana 70113.

## SOWING STRATEGIES

Choosing a sowing strategy amounts to specifying values $0 \leqslant X_{j} \leqslant 1, \Sigma_{j} X_{j}=1(j=1,2, \ldots, n)$ where $X_{j}=$ fraction of total number of containers in which $j$ seeds are planted. In practical container operations the number of seeds does not generally exceed three, so it is only necessary to choose values for $\mathrm{X}_{1}, \mathrm{X}_{2}$, and $\mathrm{X}_{3}$. In practice, containers may be grouped in trays or blocks of 70 to several hundred individuals, in which case $X_{j}$ refers to the fraction of trays or blocks in which each container receives j seeds.

In this paper, sowing is presumed to be accomplished by a widely used technique employing a vacuum-operated seeder (Carlson, 1979). This type of seeder uses a vacuum to hold single seeds over holes in a template. When the vacuum is released, the seeds drop into cavities beneath the holes. A sowing strategy is accomplished by making j passes over $(\mathrm{M})\left(\mathrm{X}_{\mathrm{j}}\right)$ trays, where $\mathrm{M}=$ total number of trays and $\mathrm{j}=1,2,3$.
The program provides two sowing options for producing a given number of seedlings: Option 1. Blank containers are not replanted, but excess seedlings are thinned. Option 2. Use a sowing strategy for which the predicted number of blanks does not exceed the predicted number of excess seedlings. Remove enough excess seedlings to replant blanks, insuring that each cell is occupied, and thin the remaining excess seedlings.

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## MATHEMATICAL BASIS OF THE PROGRAM

## Explanation of Symbols

The key quantities in a sowing strategy are total number of containers ( N ), total number of seeds ( S ), predicted number of plantable seedlings before thinning ( $\hat{\mathbf{Z}}$ ), predicted number of plantable seedlings after each occupied cell is thinned to one seedling (Y), predicted number of excess seedlings (E), and predicted number of blank containers ( $\hat{\mathrm{B}}$ ). These quantities are computed as follows in terms of germination and survival rates and sowing frequencies:
[1] $\quad \mathrm{S}=\mathrm{N}\left(\mathrm{X}_{1}+2 \mathrm{X}_{2}+3 \mathrm{X}_{3}\right), \overline{\mathrm{S}}=\mathrm{S} / \mathrm{N}$,
[2] $\quad \hat{\mathrm{Z}}=\mathrm{N}\left(\hat{\mathrm{P}}_{11} \mathrm{X}_{1}+\left(\hat{\mathrm{P}}_{12}+2 \hat{\mathrm{P}}_{29}\right) \mathrm{X}_{2}+\left(\hat{\mathrm{P}}_{13}+2 \hat{\mathrm{P}}_{23}\right.\right.$

$$
\left.\left.+3 \hat{\mathrm{P}}_{32}\right) \mathrm{X}_{3}\right), \mathrm{Z}=\hat{\mathrm{Z}} / \mathrm{N}
$$

[3] $\quad \hat{\mathrm{Y}}=\mathrm{N}\left(\hat{\mathrm{P}}_{11} \mathrm{X}_{1}+\left(\hat{\mathrm{P}}_{12}+\hat{\mathrm{P}}_{22}\right) \mathrm{X}_{2}+{ }^{\wedge}\left(\mathrm{P}_{13} \hat{+} \mathrm{P}_{23}\right.\right.$ $\left.\left.+\mathrm{P}_{33}\right) \mathrm{X}_{3}\right), \hat{\mathrm{Y}}=\hat{\mathrm{Y}} / \mathrm{N}$,
[4] $\quad \hat{\mathrm{E}}=\mathrm{N}\left(\hat{\mathrm{P}}_{22} \mathrm{X}_{2}+\left(\hat{\mathrm{P}}_{23}+2 \hat{\mathrm{P}}_{33}\right) \mathrm{X}_{3}\right), \hat{\overline{\mathrm{E}}}=\hat{\mathrm{E}} / \mathrm{N}$,
[5] $\quad \hat{\mathrm{B}}=\mathrm{N}\left(\hat{\mathrm{P}}_{01} \mathrm{X}_{1}+\hat{\mathrm{P}}_{02} \mathrm{X}_{2}+\mathrm{P}_{03} \mathrm{X}_{3}\right), \hat{\mathrm{B}}=\hat{\mathrm{B}} / \mathrm{N}$,
where
$\hat{\mathrm{P}}_{\mathrm{ij}}=$ estimated probability of producing i plantable seedlings given that j seeds were planted ( $\mathrm{i}=$ $0,1,2,3 ; \mathrm{j}=1,2,3 ; \mathrm{i} \leqslant \mathrm{j})$.
These probabilities may be estimated with the binomial formula $\hat{P}_{i j}=\frac{j!}{(j-i)!i!}(\hat{p})^{i}(1-\hat{\mathbf{p}})^{j-i} i=0,1,2, \ldots, j$, where $\hat{p}=$ estimated germination and survival rate. In practice, this estimate will often represent an overall average germination and survival rate for a composite of seedlots. Naturally, the reliability of the estimate is influenced by the amount of variation within and among seedlots, but will usually not be quantified because of the lack of estimates for these variances. For a discussion of the validity of using this model to compute the $\mathrm{P}_{\mathrm{ij}}$ 's see Pepper and Barnett (1981a).

## Cost Components of a Penalty Function

It is neither realistic nor necessary to identify all costs that might occur in the production of a containergrown seedling. Rather, a function is defined to reflect the penalty when blanks occur and remedial actions are taken.

As a point of departure, consider the extreme case of perfect seed and seedling performance with 100 percent germination and survival. In this case, a single seed is planted in N cells and N seedlings are produced; no thinning is necessary and no blanks occur. Thus, the seed-related cost for the first N seeds is not regarded as a penalty. But in reality, blanks do occur and the sowing strategy chosen aims to reduce blanks. The additional number of seed required is $\mathrm{S}-\mathrm{N}$ and the additional cost is $\left(\mathrm{C}_{\text {sd }}+\mathrm{C}_{\mathrm{sw}}\right)(\mathrm{S}-\mathrm{N})$ where
$\mathrm{C}_{\text {sd }}=$ cost per seed and
$\mathrm{C}_{\mathrm{sw}}=$ cost per seed for sowing.
The cost of carrying blanks and/or replanting blanks is $\left(\mathrm{C}_{\mathrm{b}}\right)$ (number of blanks carried) $+(\mathrm{C}$,) (number of blanks replanted) where
$\mathrm{C}_{\mathrm{b}}=$ cost per cell for carrying blanks and
$\mathrm{C}_{\mathrm{r}}=$ cost per cell for removing the required number of excess seedlings and replanting blanks.
The cost for thinning excess seedlings is $\left(\mathrm{C}_{\mathrm{t}}\right)$ (number of excess seedlings) where
$\mathrm{C}_{\mathrm{t}}=$ cost per seedling for thinning seedlings not used for replanting blanks.
It is assumed that the cost of sowing is the same for each seed. This assumption seems logical since the seeder must make a pass over the tray of cells for each seed planted per cell. The cost for thinning is assumed to be the same for each seedling. The cost of replanting a blank cell consists of the cost of removing an excess seedling from another cell and transplanting it to the blank cell. The mortality of transplanted seedlings is assumed to be negligible. The cost of carrying a blank cell is defined as the cost of the container plus the cost of the medium. Container costs vary according to the type and the number of times that they can be reused. For a discussion of the above assumptions see Pepper and Barnett (1981b).
The sum of the independent contributions of the components described above is the total penalty, including the cost of blanks and the cost of remedial actions to reduce blanks. CONSOW enables the user to choose a strategy ( $\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{3}$ ) to minimize the predicted total penalty.

The formula for calculating predicted penalty depends on the sowing option used. For Option 1 the predicted penalty is

$$
\begin{equation*}
\hat{\mathrm{P}}_{\mathrm{c}}=\left(\mathrm{C}_{\mathrm{sd}}+\mathrm{C}_{\mathrm{sw}}\right)(\mathrm{S}-\mathrm{N})+\mathrm{C}_{\mathrm{b}} \hat{\mathrm{~B}}+\mathrm{C}_{\mathrm{t}} \hat{\mathrm{E}} . \tag{6}
\end{equation*}
$$

For Option 2 the predicted penalty is
[7] $\quad \hat{\mathrm{P}}_{\mathrm{c}}=\left(\mathrm{C}_{\mathrm{sd}}+\mathrm{C}_{\mathrm{sw}}\right)(\mathrm{S}-\mathrm{N})+\mathrm{C}_{\mathrm{r}} \hat{\mathrm{B}}+\mathrm{C}_{\mathrm{t}}(\hat{\mathrm{E}}-\hat{\mathrm{B}})$.

## Constraints on Sowing Frequencies

The first step in formulating a container sowing problem specifies the required number of seedlings $\left(\mathrm{Y}_{\mathrm{R}}\right)$ to be produced and the number ( N ) of containers. In the case of Option 1, $\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{3}$ will be chosen to minimize the predicted penalty subject to the constraint that the predicted number of seedlings is not less than the required number. Mathematically, the problem is to minimize

$$
\hat{\mathrm{P}}_{\mathrm{c}}=(\mathrm{N})\left(\mathrm{C}_{1} \mathrm{X}_{1}+\mathrm{C}_{2} \mathrm{X}_{2}+\mathrm{C}_{3} \mathrm{X}_{3}\right) \text {, subject to }
$$

[8] $\mathrm{N}\left(\hat{\mathrm{P}}_{11} \mathrm{X}_{1}+\left(\hat{\mathrm{P}}_{12}+\hat{\mathrm{P}}_{22}\right) \mathrm{X}_{2}+\left(\hat{\mathrm{P}}_{13}+\hat{\mathrm{P}}_{23}+\hat{\mathrm{P}}_{33}\right) \mathrm{X}_{3}\right)$ $\Rightarrow \mathrm{Y}_{\mathrm{R}}$
$\mathrm{X}_{1}+\mathrm{x} 2+\mathrm{x} 3=1$
$\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{3} \geqslant 0$
where

$$
\begin{aligned}
& \mathrm{C}_{1}=\mathrm{C}_{\mathrm{b}} \mathrm{P}_{01} \\
& \mathrm{C}_{2}=\mathrm{C}_{\mathrm{sd}}+\mathrm{C}_{\mathrm{sw}}+\mathrm{C}_{\mathrm{b}} \hat{\mathrm{P}}_{02}+\mathrm{C}_{\mathrm{t}} \hat{\mathrm{P}}_{22}
\end{aligned}
$$

and

$$
\mathrm{C}_{3}=2\left(\mathrm{C}_{\mathrm{sd}}+\mathrm{C}_{\mathrm{sw}}\right)+\mathrm{C}_{\mathrm{b}} \hat{\mathrm{P}}_{03}+\left(\hat{\mathrm{P}}_{23}+2 \hat{\mathrm{P}}_{33}\right) \mathrm{C}_{\mathrm{t}}
$$

Dividing the first constraint by N yields the required number of seedlings per cell $\left(\overline{\mathrm{Y}}_{\mathrm{R}}\right)$ on the right hand side of the inequality. This quantity is to be distinguished from Y defined in [3] above.

With Option 2, the optimization problem is to minimize the predicted penalty subject to the constraint that the predicted number of blanks does not exceed the predicted number of excess seedlings. Mathematically, we minimize

$$
\hat{\mathrm{P}}_{\mathrm{c}}=(\mathrm{N})\left(\mathrm{K}_{1} \mathrm{X}_{1}+\mathrm{K}_{2} \mathrm{X}_{2}+\mathrm{K}_{3} \mathrm{X}_{3}\right)
$$

subject to

$$
\text { [9] } \begin{aligned}
& \mathrm{N}_{\hat{2}}\left\{-\hat{\mathrm{P}}_{01} \mathrm{X}_{1}+\left(\hat{\mathrm{P}}_{22}-\hat{\mathrm{P}}_{02}\right) \mathrm{X}_{2}+\left(\hat{\mathrm{P}}_{23}+2 \hat{\mathrm{P}}_{33}-\right.\right. \\
& \left.\left.\hat{\mathrm{P}}_{\mathrm{a}}\right) \mathrm{X}_{3}\right\} \geqslant 0 \\
& \mathrm{X}_{1}+\mathrm{X}_{2}+\mathrm{X}_{3}=1 \\
& \mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{3} \geqslant 0
\end{aligned}
$$

where

$$
\begin{aligned}
\mathrm{K}_{1}= & \left(\mathrm{C}_{\mathrm{r}}-\mathrm{C}_{\mathrm{t}}\right) \hat{\mathrm{P}}_{01} \\
\mathrm{~K}_{2}= & \mathrm{C}_{\text {sd }}+\mathrm{C}_{\mathrm{sw}}+\left(\mathrm{C}, \quad \mathrm{C}_{\mathrm{t}}\right) \hat{\mathrm{P}}_{02}+\mathrm{C}_{\mathrm{t}} \hat{\mathrm{P}}_{22} \\
\mathrm{~K}_{3}= & 2\left(\mathrm{C}_{\mathrm{sdx}}+\mathrm{C}_{\mathrm{sw}}\right)+\left(\mathrm{C},-\mathrm{C}_{\mathrm{t}}\right) \hat{\mathrm{P}}_{03}+\mathrm{C}_{\mathrm{t}}\left(\hat{\mathrm{P}}_{23}\right. \\
& \left.+2 \mathrm{P}_{33}\right) .
\end{aligned}
$$

Since the penalty function and constraints are linear in the X's and the constraints define a convex set of admissable solutions, linear programming can be used to determine the optimal solution. Solutions at corners or vertices of the set of admissable solutions are termed extreme points in linear programming problems and are known to be the only candidates for optimality. Since the number of extreme point solutions is small, the optimal one can be found by inspecting the value of the penalty function at each of these.

In linear programming problems of greater complexity with large numbers of extreme points, iterative procedures, usually computer-oriented, lead to optimal solutions without examining each extreme point in the set of admissable points.

## USING THE PROGRAM

## User-Supplied Information

CONSOW prompts the user for values of 13 variables that deal with costs, alternatives for program changes, sowing options, and rates of survival. The first variable needed in the program is the option variable. A value of 1 (transplanting-no) or 2 (transplant-ing-yes) is entered for Option 1 or 2 respectively.

The next required variable is the germination and survival rate ( p ), which is entered as a decimal fraction between 0.00 and 1.00 .
In Option 1, four different costs are entered. They are the cost per thousand seeds ( $\mathrm{C}_{\text {sd }}$ ), cost per thousand seeds for sowing ( $\mathrm{C}_{\mathrm{sw}}$ ), cost per thousand seedlings for thinning seedlings not used for replanting blanks $\left(\mathrm{C}_{\mathrm{t}}\right)$, and the cost per thousand cells for carrying blank cells ( $\mathrm{C}_{\mathrm{b}}$ ).

In Option 2, three of the costs are identical: Cost per thousand seeds, cost per thousand seeds for sowing, and cost per thousand seedlings for thinning seedlings not used for replanting blanks. The only difference is that the cost of carrying blank cells is replaced by the cost per thousand seedlings for removing the required number of excess seedlings and replanting blank cells $\left(\mathrm{C}_{\mathrm{r}}\right)$. In Option 1, blank cells are not replanted.

The required number of seedlings per container $\left(\overline{\mathrm{Y}}_{\mathrm{R}}\right)$, found by dividing the required number of seedlings by the total number of containers available, must be entered. This number is to be distinguished from the predicted number of seedlings ( $\overline{\mathrm{Y}}$ ) per cell.
The remaining variables deal with altering the user-supplied variables. There are a series of 5 questions to be answered yes or no. The first question deals with whether or not changes will be made. If not, the program will end. Any combination of the remaining four independent alternatives is possible. The alternatives are: (1) begin a new page, (2) change the germination and survival rate, (3) change the cost coefficients and (4) change the required number of seedlings per container in Option 1.

## Operations Performed by the Program

CONSOW is divided into two sections, a section for Option 1 and a section for Option 2. The operations in each section are similar; only the sets of cost coefficients are different, and the predicted number of seedlings is calculated after thinning for Option 1 and before thinning for Option 2. The operations can be identified by comment statements in the listing of the program in Appendix 1.
The program performs the following operations:
(1) Prompts the user for a choice of options and prints the appropriate headings.
(2) Given the germination and survival rate, the binomial probabilities, needed in the constraints and prediction equations, are calculated and summed.
(3) Prompts the user for the cost of seeds, the cost of sowing, and the cost of thinning excess seedlings not used for replanting blanks.
(4) For Option 1, prompts user for required number of seedlings per container to be produced and for the cost per thousand cells of carrying blank cells. For Option 2, the user supplies the cost per thousand seedlings for removing the required number of excess seed-
lings and replanting blank cells. The required number of seedlings per container is set to one in Option 2 because all of the blank cells are replanted with the thinned seedlings.
(5) Determines all extreme point solutions, i.e. sets of values for $X_{1}, X_{2}$, and $X_{3}$ satisfying specified constraints.
(6) Determines the expected number of excess seedlings per cell, the predicted number of plantable seedlings per cell after thinning for Option 1 and before thinning for Option 2, and the average number of seeds per cell.
(7) Calculates cost coefficients for the penalty function, the penalty per thousand cells in dollars, and the penalty per thousand seedlings in dollars. The final tables are printed for the option selected.
(8) Alternatives are given to end the program or to analyse a new problem with new linear programming model parameters.

## An Example

Suppose Option 1 is used in an operation requiring 300,000 seedlings. Assume that $p=0.7$ and cost values are $\mathrm{C}_{\mathrm{sd}}=\$ 1.70$ per 1,000 seeds, $\mathrm{C}_{\mathrm{sw}}=\$ 4.04$ per 1,000 seeds, $\mathrm{C}_{\mathrm{t}}=\$ 8.37$ per 1,000 seedlings' and $\mathrm{C}_{\mathrm{b}}$ $=\$ 13.00$ per 1,000 cells'. The decision is made to use $\mathrm{N}=375,000$ containers for a mean yield of 300,000 / $375,000=0.8$ seedlings per container.

Appendix 2 provides a computer printout containing an exhaustive list of four extreme point solutions for this problem. For each extreme point solution penalty per thousand containers is computed

$$
\hat{\mathrm{P}}_{\mathrm{c}}=\mathrm{C}_{1} \mathrm{X}_{1}+\mathrm{C}_{2} \mathrm{X}_{2}+\mathrm{C}_{3} \mathrm{X}_{3}
$$

where $C_{j}$ 's are calculated as shown in equation [8]. Penalty per thousand seedlings produced is $\overline{\mathrm{P}}_{\mathrm{s}}=\overline{\mathrm{P}}_{\mathrm{c}} / \hat{\overline{\mathrm{Y}}}$. By computing penalty values at each extreme point we

[^1]find that $\left(\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{3}\right)=(0.52,0.48,0.00)$ produces the smallest value. The theory of linear programming assures us that this choice of X's produces the absolute minimum penalty for the given set of constraints.

Suppose Option 2 is used in an operation requiring a yield of 375,000 seedlings with 375,000 containers. This obviously implies $\overline{\mathrm{Y}}_{\mathrm{R}}=1$. Assume that cost values are the same as above except that $\mathrm{C}_{\mathrm{b}}$ is replaced by $\mathrm{C}_{\mathrm{r}} /=\$ 16.79$ per thousand cells ${ }^{2}$. Appendix 2 shows an exhaustive list of four extreme point solutions for $p$ $=0.7$ and $\overline{\mathrm{Y}}_{\mathrm{R}}=1$. The optimal solution is $\left(\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{3}\right)$ $=(0.57,0.43,0.00)$. For Option 2, the total number of plantable seedlings per cell ( $\hat{\mathbf{Z}}$ ) includes excess seedlings which are used to replant blanks.

The example above shows how an optimal sowing strategy is found for a specified germination and survival rate and a given set of costs. The program can also be used to study the effects of changing these parameters. For example, a user might wish to see how optimality changes for a given set of costs as the germination and survival rate is improved, or how optimality and penalty change as costs change.

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## APPENDIX l-Listing of Program CONSOW

30 REM
40 REM 50 REM
60 REM
70 REM
80 REM 90 REM
100 REM
110 REM
120 REM
130 REM
140 REM
150 REM
160 REM WITH EXCESS SEEDLINGS. EACH SOWING STRATEGY IS REQUIRED
170 REM TO PRODUCE A SUFFICIENT NUMBER OF EXCESS SEEDLINGS TO
180 REM REPLANT ALL BLANK CELLS. ANY EXCESS SEEDLINGS NOT USED
190 REM FOR REPLANTING BLANK CELLS ARE THINNED AND DISCARDED.
200 REM THE PROGRAM WILL PROMPT THE USER FOR INFORMATION
210 REM REGARDING THE COST COMPONENTS, THE ESTIMATED GERMINATION
220 REM AND SURVIVAL RATES, AND THE REQUIRED NUMBER OF SEEDLINGS
230 REM PER CONTAINER. THE REQUIRED NUMBER OF SEEDLINGS PER
240 REM CONTAINER IS DETERMINED BY DIVIDING THE REQUIRED NUMBER
250 REM OF SEEDLINGS BY THE TOTAL NUMBER OF AVAILABLE CONTAINERS.
260 REM THE FOLLOWING VARIABLE NAMES APPEAR IN THE COLUMN
270 REM HEADINGS IN TABLES PRODUCED BY THE PROGRAM:
280 REM
290 REM
300 REM
310 REM
320 REM
330 REM
340 REM
350 REM
360 REM
370 REM
380 REM
390 REM
400 REM
410 REM
420 REM
430 REM
440 REM
450 REM
460 REM
470 REM
480 REM
490 REM
500 REM
510 REM
520 REM
530 REM
540 REM
550 REM

P - ESTIMATED GERMINATION AND SURVIVAL RATE
YBAR - REQUIRED NUMBER OF SEEDLINGS PER CELL
Xl - FRACTION OF TOTAL NUMBER OF CELLS RECEIVING 1 SEED
x2 - FRACTION OF TOTAL NUMBER OF CELLS RECEIVING 2 SEEDS
x3 - FRACTION OF TOTAL NUMBER OF CELLS RECEIVING 3 SEEDS

SBAR - AVERAGE NUMBER OF SEEDS PER CELL
YHAT - PREDICTED NUMBER OF PLANTABLE SEEDLINGS AFTER EXCESS SEEDLINGS HAVE BEEN THINNED AND REPLANTED OR DISCARDED

ZHAT - TOTAL NUMBER OF PLANTABLE SEEDLINGS BEFORE THINNING

EHAT - PREDICTED NUMBER OF EXCESS SEEDLINGS
PENALTY - ECONOMIC INDEX OF EFFICIENCY

INPUT Y
590 PRINT
600 PRINT
610 REM *** PRINT HEADINGS ***
620 IF $\mathrm{Y}=2$ THEN 650
630 PRINT TAB8" OPTION ONE SOWING STRATEGIES"
640 GOT0 660
650 PRINT TAB8" OPTION TWO SOWING STRATEGIES"
660 PRINT
670 PRINT
680 IF Y-2 THEN 740
$690 \operatorname{WRITE}(15,710) "$ ";
700 WRITE $(15,720)$
710 FORMAT 16X,"EXTREME POINT SOLUTIONS",10X,"YHAT",3X,"EHAT"
720 FORMAT 5X,"PENALTY"
730 GOT0 780
740 WRITE $(15,760) "$ ";
750 WRITE $(15,770)$
760 FORMAT 16X,"EXTREME POINT SOLUTIONS",10X,"ZHAT", 3X,"EHAT"
770 FORMAT 5X, "PENALTY"
780 WRITE $(15,790) "$ "
790 FORMAT 49X,"PER", 4X,"PER", 4X,"PER", 5X,"PER"
$800 \operatorname{WRITE}(15,820) "$ n
810 WRITE $(15,830) " \mathrm{~m}$;
820 FORMAT 8X,"P",3X,"YBAR", 4X,"X1",5X,"X2",5X,"X3",5X,"SBAR"
830 FORMAT 4X,"CELL",2X,"CELL",3X,"CELL",4X,"M SDLGS"
840 PRINT
850 IF $\mathbf{B} \$={ }^{\mathbf{n}} \mathrm{N}$ " THEN 1010
860 REM *** FIND THE GERMINATION AND SURVIVAL RATE ***
870 DISP "ENTER ESTIMATED GERMINATION AND"
880 WAIT 3000
890 DISP "SURVIVAL RATE":
900 INPUT P
910 REM *** CALCULATE BINOMIAL PROBABILITIES ***
920 Pl=1-P
930 P2=P
940 P 3 $=(1-P)^{\wedge} 2$
950 P4=2*P*(1-P)
960 P5=(1-P) ^3
970 P6=3*P* (1-P) ^2
980 P7=P*P
990 P8=3*P*P*(1-P)
1000 P9=P ^3
1010 IF C\$="N" THEN 1110
1020 REM *** DETERMINE THE COST COEFFICIENTS ***
1030 DISP "ENTER COST PER 1000 SEEDS":
1040 INPUT Sl
1050 DISP "ENTER COST / M SEEDS FOR SOWING":
1060 INPUT S2
1070 DISP "ENTER COST / M SEEDLINGS FOR"
1080 WAIT 3000
1090 DISP "THINNING SEEDLINGS NOT USED";
1100 INPUT T
1110 REM *** BEGINNING OF OPTION 1 SOWING STRATEGY ***
1120 IF $\mathrm{Y}=\mathbf{2}$ THEN 2250
1130 IF $\mathbf{A} \$=" Y$ " THEN 1150
1140 IF D $\$=$ "N" THEN 1250

```
1150 REM *** DETERMINE THE NUMBER OF SEEDLINGS PER CELL ***
1160 DISP "ENTER REQUIRED NUMBER"
1170 WAIT 3000
1180 DISP "OF SEEDLINGS PER CONTAINER";
1190 INPUT A4
1200 REM *** CALCULATE PROPORTION CONSTRAINT COEFFICIENTS ***
1210 Al=P2
1220 A2 = P4 +P7
1230 A3 = P6 +P8+P9
1240 REM *** CALCULATE THE SEED PROPORTIONS ***
1250 Dl=A3-A2
1260 D2=A3-A1
1270 D3=A2-A1
1280 X2=(A3/D1)-(A4/D1)
1290 X3= (-A2/D1)+(A4/D1)
1300 Yl = (A3/D2)-(A4/D2)
1310 Y3=(-A1/D 2)+(A4/D 2)
1320 Z1=(A2/D3)-(A4/D3)
1330 Z2=(-Al/D3) + (A4/D3)
1340 IF A$="Y" THEN 1360
1350 IF C$="N" THEN 1400
1360 DISP "ENTER COST PER 1000 CELLS"
1370 WAIT 3000
1380 DISP "FOR CARRYING BLANKS";
1390 INPUT B
1400 x2=x2+0.000001
1410 x3=x3+0.000001
1420 Yl=Y1+0.000001
1430 Y 3 =Y 3+0.000001
1440 21=21+0.000001
1450 Z2=Z2+0.000001
1460 REM *** CALCULATE PENALTY COST COEFFICIENTS ***
1470 Cl=B*P1
1480 C2=Sl+S2+B*P3+T*P7
1490 C 3=2* (S1+S2) +B*P 5+T*P8+2*T*P9
1500 REM *** CALCULATE THE EXPECTED NUMBER OF EXCESS
1510 REM SEEDLINGS PER CELL
1520 El=P7
1530 B1=P3
1540 E2=P8+2*P9
1550 B2=P5
1560 E3=P7**2+P8*X3+2*P9*X3
1570 B3=P3*X2+P5*X3
1580 E4=P8*Y3+2*P9*Y 3
1590 B4=P1*Y1+P5*Y3
1600 E5=P7*Z2
1610 B5=P1*Z1+P3*Z2
1620 REM *** CALCULATE THE PENALTY PER THOUSAND CELLS
1630 REM
1640 Wl=C2
1650 W2=C 3
1660 W3=C2*X2+C3*X3
1670 W4=C1*Y1+C3*Y3
1680 W5=Cl* Z1+C2*Z2
1690 REM *** CALCULATE THE PREDICTED NUMBER OF PLANTABLE
1700 REM SEEDLINGS PER CELL AFTER THINNING ***
1710 Tl=A2
```

1730 T3 $=$ A 2 * $\times 2+A 3^{*} \times 3$
$1740 \mathrm{~T} 3=\mathrm{T} 3+0.000001$
1750 T4 $=$ Al*Y1 + A $3 * Y 3$
$1760 \mathrm{~T} 4=\mathrm{T} 4+0.000001$
1770 T5=A1*Z1+A2*Z2
1780 T5=T5+0.000001
1790 REM *** CALCULATE NUMBER OF SEEDS PER CELL ***
1800 Q1 $=2$
$1810 \quad$ Q2 $=3$
1820 Q3 $=2$ * $\mathbf{X} 2+3$ * $\mathbf{X 3}$
1830 Q4 $=\mathbf{Y} 1+3 * \mathrm{Y} 3$
1840 Q $5=\mathrm{Zl}+2$ * 22
1850 REM *** CALCULATE PENALTY PER THOUSAND SEEDLINGS
1860 REM IN DOLLARS ***
1870 Vl=Wl/Tl
1880 V2=W2/T2
1890 V3=W3/T3
$1900 \quad \mathrm{~V} 4=\mathrm{W} 4 / \mathrm{T} 4$
1910 V5=W5/T5
1920 REM *** PRINT THE RESULTS ***
1930 PRINT
1940 IF T1<A 4 THEN 2000
1950 WRITE $(15,1970) \mathrm{P}, \mathrm{A} 4 ;$
1960 WRITE (15,1980) Q1,T1,El,Wl,V1;
1970 FORMAT 6X,F4.1,1X,F5.2,3X,"0.00", 3X,"1.00", 3X,"0.00"
1980 FORMAT 3X,F5.2,3X,F5.2,2X,F5.2,2X,F6.2,2X,F6.2
1990 PRINT
2000 IF T2<A4 THEN 2060
2010 WRITE (15,2030)P,A4;
2020 WRITE $(15,2040)$ Q2,T2,E2,W2,V2;
2030 FORMAT 6X,F4.1,1X,F5.2,3X," 0.00",3X," 0.00", 3X,"1.00"
2040 FORMAT 3X,F5.2,3X,F5.2,2X,F5.2,2X,F6.2,2X,F6.2
2050 PRINT
2060 IF T3<A4 OR X2<0.0001 OR X3<0.0001 THEN 2120
2070 WRITE $(15,2090) P, A 4, X 2, X 3 ;$
2080 WRITE $(15,2100)$ Q3,T3,E3,W3,V3;
2090 FORMAT 6X,F4.1,1X,F5.2,3X,"O.00",2X,F5.2,2X,F5. 2
2100 FORMAT 3X,F5.2,3X,F5.2,2X,F5.2,2X,F6.2,2X,F6.2
2110 PRINT
2120 IF T4<A4 OR Yl<0.0001 OR Y $3<0.0001$ THEN 2180
2130 WRITE $(15,2150) P, A 4, Y 1, Y 3$;
2140 WRITE $(15,2160)$ Q4,T4,E4,W4,V4;
2150 FORMAT 6X,F4.1,1X,F5.2,2X,F5.2,3X,"0.00", 2X,F5. 2
2160 FORMAT 3X,F5.2,3X,F5.2,2X,F5.2,2X,F6.2,2X,F6.2
2170 PRINT
2180 IF T5<A4 OR Z1<0.0001 OR Z2<0.0001 THEN 2230
2190 WRITE $(15,2210) P, A 4,21,22$;
2200 WRITE $(15,2220)$ Q5,T5,E5,W5,V5;
2210 FORMAT 6X,F4.1,1X,F5.2,2X,F5.2,2X,F5.2,3X,"0.00"
2220 FORMAT 3X,F5.2,3X,F5.2,2X,F5.2,2X,F6.2,2X,F6.2
2230 PRINT
2240 GOTO 3360
2250 REM *** OPTION 2 SOWING STRATEGY ***
2260 Al=P2
2270 A $2=\mathrm{P} 4+2$ * P 7
2280 A $3=P 6+2 * P 8+3 * P 9$

```
2290 A4=1
2300 Bl=-Pl
210 B2=P7-P 3
2320 B3=P8+(2*P9)-P5
2330 IF A$="Y" THEN 2350
2340 IF C$="N" THEN 2440
2350 DISP "ENTER COST PER 1000 SEEDLINGS"
2360 WAIT 3000
2370 DISP "FOR REMOVING THE REQUIRED NUMBER"
2380 WAIT 3000
2390 DISP "OF EXCESS SEEDLINGS AND"
2400 WAIT 3000
2410 DISP "REPLANTING BLANKS";
2420 INPUT R
2430 REM *** CALCULATE PENALTY COST COEFFICIENTS ***
2440 Cl=(R-T)*Pl
2450 C2=S1+S2+(R-T)*P 3+T*P7
2460 C 3 = 2* (S 1+S2) +(R-T)*P5+T*P8+2*T* P9
2470 Dl=B3-B2
2480 D2=B3-B1
2490 D3=B2-B1
2500 rem *** CALCUlATe sEed PrOpOrTIONS ***
2510 X2=B3/D1
2520 X2=X2+0.000001
2530 X3=-B2/D1
2540 X3=X3+0.000001
2550 Yl=B3/D2
2560 Yl=Y1+0.000001
2570 Y3=-B1/D2
2580 Y 3=Y3+0.000001
2590 2l=B2/D3
2600 Zl=21+0.000001
2610 Z2=-Bl/D3
2620 Z2=Z2+0.000001
2630 REM *** CALCULATE EXCESS SEEDLINGS ***
2640 El=P7
2650 Ol=P3
2660 E2=P8+2*P9
2670 02=P5
2680 E3=P7*X2+P8*X3+2*P 9*X3
2690 03=P3*X2+P5*X3
2700 E4=P8*Y 3+2*P9*Y 3
2710 O4=Pl*Yl+P5*Y3
2720 E5=P7*Z2
2730 05=P1*Z1+P3*Z2
2 7 4 0 ~ R E M ~ * * * ~ C A L C U L A T E ~ T H E ~ P E N A L T Y ~ P E R ~ T H O U S A N D ~ C E L L S ~
2 7 5 0 ~ R E M ~ I N ~ D O L L A R S ~
2760 Wl=C2
2770 W2=C 3
2780 W W = C2**2+C3**3
2790 W4=Cl*Yl+C3*Y3
2800 W5=Cl* Zl+C2*Z2
2810 REM *** CALCULATE NUMBER OF SEEDS PER CELL ***
2820 Q1=2
2830 Q2=3
2840 Q3=2**2+3**3
2850 Q4=Y1+3*Y3
```

```
3430 DISP "TO BEGIN NEW PAGE, TYPE Y";
3440 INPUT A$
3450 WAIT 3000
3 4 6 0 ~ D I S P ~ " T O ~ C H A N G E ~ V A L U E ~ O F ~ G E R M I N A T I O N " ~
3470 WAIT 3000
3480 DISP "AND SURVIVAL RATE, TYPE Y";
3490 INPUT B$
3500 WAIT 3000
3510 DISP " TO CHANGE THE COST COMPONENTS,"
3520 WAIT 3000
3530 DISP "TYPE Y";
3540 INPUT C$
3550 WAIT 3000
3 5 6 0 ~ D I S P ~ " T O ~ C H A N G E ~ V A L U E ~ O F ~ R E Q U I R E D " ~
3570 WAIT 3000
3580 DISP "NUMBER OF SEEDLINGS PER CELL"
3590 WAIT 3000
3600 DISP "IN OPTION 1, TYPE Y";
3610 INPUT D$
3620 WAIT 2000
3 6 3 0 ~ G O T O ~ 5 6 0 ~
3640 END
3620 WAIT 2000
3 6 3 0 ~ G O T O ~ 5 6 0 ~
3640 END
```


## APPENDIX 2—Output of Sample Problems

## OPTION ONE SOWING STRATEGIES

|  |  | EXTREME | POINT | SOLUTIONS |  | YHAT <br> PER | EHAT <br> PER | PENALTY PENALTY <br> PER |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P | YBAR | XI | x 2 | x 3 | SBAR | CELL | CELL | M CELLS | M SDLGS |
| 0.7 | 0.80 | 0.00 | 1.00 | 0.00 | 2.00 | 0.91 | 0.49 | 11.01 | 12.10 |
| 0.7 | 0.80 | 0.00 | 0.00 | 1.00 | 3.00 | 0.97 | 1.13 | 21.26 | 21.85 |
| 0.7 | 0.80 | 0.63 | 0.00 | 0.37 | 1.73 | 0.80 | 0.41 | 10.26 | 12.83 |
| 0.7 | 0.80 | 0.52 | 0.48 | 0.00 | 1.48 | 0.80 | 0.23 | 7.29 | 9.11 |

OPTION TWO SOWING STRATEGIES

EXTREME POINT SOLUTIONS

|  |  |  |  |  |  | PER | PER | PER | PER |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P | YBAR | Xl | x2 | x 3 | SBAR | CELL | CELL | M CELLS | M SDLGS |
| 0.7 | 1.00 | 0.00 | 1.00 | 0.00 | 2.00 | 1.40 | 0.49 | 10.60 | 7.57 |
| 0.7 | 1.00 | 0.00 | 0.00 | 1.00 | 3.00 | 2.10 | 1.13 | 21.14 | 10.07 |
| 0.7 | 1.00 | 0.79 | 0.00 | 0.21 | 1.43 | 1.00 | 0.24 | 6.51 | 6.51 |
| 0.7 | 1.00 | 0.57 | 0.43 | 0.00 | 1.43 | 1.00 | 0.21 | 5.99 | 5.99 |

PEPPER, WILLIAM D., and W. DOUGLAS HODGE.
1982. Computer can choose sowing strategies for containerized seedlings. U.S. Dep. Agric. For. Serv. Gen. Tech. Rep. SO-39, 12 p. South. For. Exp. Stn., New Orleans, La.
This paper presents an interactive computer program to help select an optimal sowing strategy to deal with the problem of blank containers in growing containerized seedlings.


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[^1]:    'These estimates were provided by the Stuart Containerized Seedling Project, Kisatchie National Forest, Pollock, LA. This project is administered by the Range, Timber and Wildlife Division, National Forest System, Region 8, Atlanta, GA.
    ${ }^{2}$ These estimates were provided by Research Work Unit SO-1102, Southern Forest Experiment Station, Alexandria, LA.

