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Ground-Water Resources Program

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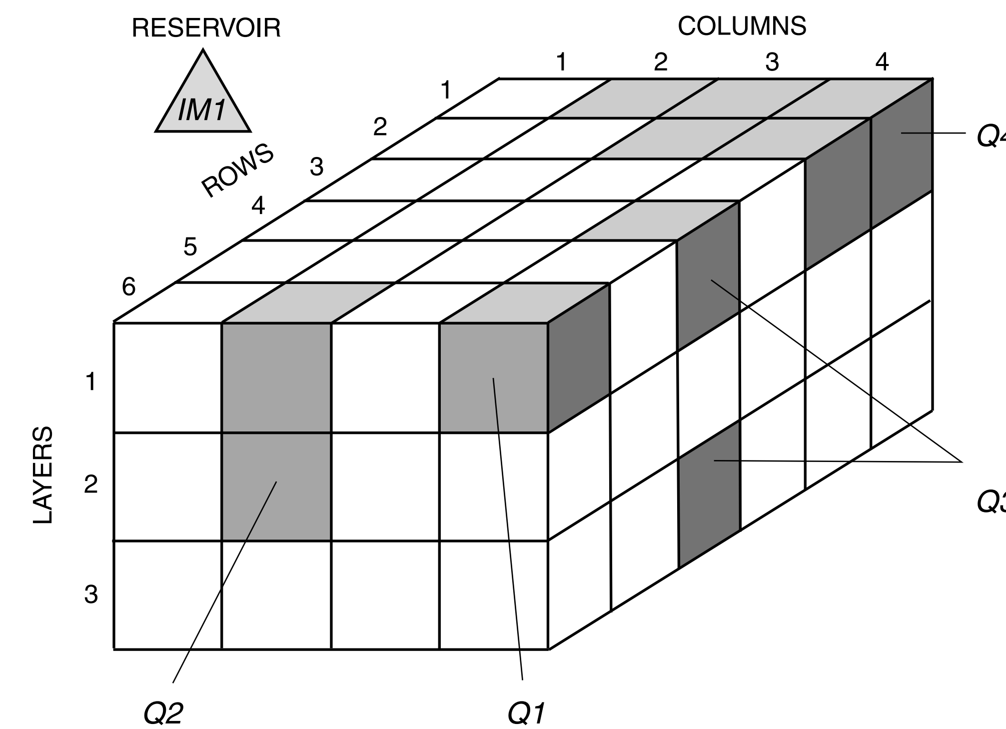

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U.S. Department of the Interior
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GWM—A Ground-Water Management Process for the U.S. Geological Survey Modular Ground-Water Model (MODFLOW-2000)

By David P. Ahlfeld (University of Massachusetts), Paul M. Barlow, and
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Preface

This report describes a Ground-Water Management Process (GWM) for the U.S. Geological Survey modular three-dimensional ground-water model, MODFLOW-2000. The performance of the program has been tested in a variety of applications. Future applications, however, might reveal errors that were not detected in the test simulations. Users are requested to notify the U.S. Geological Survey of any errors found in this report or the computer program by using the address on the inside of the back cover of the report. Updates might occassionally be made to both the report and to the computer program. Users can check for updates on the Internet at URL http://water.usgs.gov/software/ground_water.html/.

Contents

Figures

Conversion Factors and Abbreviations

Multiply	By	To obtain
cubic foot (ft^3)	0.02832	cubic meter
cubic foot per day (ft^3/d)	0.02832	cubic meter per day
foot (ft)	0.3048	meter
foot per day (ft/d)	0.3048	meter per day
square foot per day (ft^2/d)	0.09290	square meter per day

GWF	Ground-Water Flow
GWM	Ground-Water Management
LP	Linear programming
MPS	Mathematical Programming System
RMS	Response Matrix Solution
SLP	Sequential linear programming

GWM—A Ground-Water Management Process for the U.S. Geological Survey Modular Ground-Water Model (MODFLOW-2000)

By David P. Ahlfeld, Paul M. Barlow, *and* Ann E. Mulligan

Abstract

GWM is a Ground-Water Management Process for the U.S. Geological Survey modular three-dimensional ground-water model, MODFLOW-2000. GWM uses a response-matrix approach to solve several types of linear, nonlinear, and mixed-binary linear ground-water management formulations. Each management formulation consists of a set of decision variables, an objective function, and a set of constraints. Three types of decision variables are supported by GWM: flow-rate decision variables, which are withdrawal or injection rates at well sites; external decision variables, which are sources or sinks of water that are external to the flow model and do not directly affect the state variables of the simulated ground-water system (heads, streamflows, and so forth); and binary variables, which have values of 0 or 1 and are used to define the status of flow-rate or external decision variables. Flow-rate decision variables can represent wells that extend over one or more model cells and be active during one or more model stress periods; external variables also can be active during one or more stress periods. A single objective function is supported by GWM, which can be specified to either minimize or maximize the weighted sum of the three types of decision variables. Four types of constraints can be specified in a GWM formulation: upper and lower bounds on the flow-rate and external decision variables; linear summations of the three types of decision variables; hydraulic-head based constraints, including drawdowns, head differences, and head gradients; and streamflow and streamflow-depletion constraints.

The Response Matrix Solution (RMS) Package of GWM uses the Ground-Water Flow Process of MODFLOW to calculate the change in head at each constraint location that results from a perturbation of a flow-rate variable; these changes are used to calculate the response coefficients. For linear management formulations, the resulting matrix of response coefficients is then combined with other components of the linear management formulation to form a complete linear formulation; the formulation is then solved by use of the simplex algorithm, which is incorporated into the RMS Package. Nonlinear formulations arise for simulated conditions that include water-table (unconfined) aquifers or head-dependent boundary conditions (such as streams, drains, or evapotranspiration from the water table). Nonlinear formulations are solved by sequential linear programming; that is, repeated linearization of the nonlinear features of the management problem. In this approach, response coefficients are recalculated for each iteration of the solution process. Mixed-binary linear (or mildly nonlinear) formulations are solved by use of the branch and bound algorithm, which is also incorporated into the RMS Package.

Three sample problems are provided to demonstrate the use of GWM for typical ground-water flow management problems. These sample problems provide examples of how GWM input files are constructed to specify the decision variables, objective function, constraints, and solution process for a GWM run. The GWM Process runs with the MODFLOW-2000 Global and Ground-Water Flow Processes, but in its current form GWM cannot be used with the Observation, Sensitivity, Parameter-Estimation, or Ground-Water Transport Processes. The GWM Process is written with a modular structure so that new objective functions, constraint types, and solution algorithms can be added.

Introduction

Since the 1960s, numerical ground-water flow models have become increasingly important tools for the analysis of ground-water systems. More recently, ground-water flow models have been combined with optimization techniques to determine water-resource management strategies that best meet a particular set of management objectives and constraints. Optimization techniques are a set of mathematical programs that seek to find the optimal (or best) allocation of resources to competing uses. In the context of ground-water management, the resources are typically the ground- and surface-water resources of a basin and (or) the financial resources of the communities that depend on the water. The management objectives and constraints are stated (or formulated) mathematically in an optimization (management) model. Combined ground-water flow and optimization models have been applied to various ground-water management problems, including the control of water-level declines and land subsidence that could result from ground-water withdrawals, conjunctive management of ground-water and surface-water systems, capture and containment of contaminant plumes, and seawater intrusion. As applied in U.S. Geological Survey (USGS) studies, management agencies and other stakeholders provide information on water-resource objectives and constraints. The USGS then provides scientific data, analysis, and expertise in ground-water flow and optimization modeling to help decisionmakers understand how the characteristics of the hydrologic (ground-water and surface-water) system and the stated objectives and constraints interact to affect options for managing the resource.

A number of computer codes have been developed during the past two decades to facilitate linked flow and optimization modeling of ground-water flow systems (Lefkoff and Gorelick, 1987; Greenwald, 1998; Zheng and Wang, 2002; Ahlfeld and Riefler, 2003; Peralta, 2004). These codes differ in the numerical model used to represent the ground-water flow system, the types of ground-water management problems that can be solved, and the approaches used to solve the management problems.

GWM is a new process for the USGS MODFLOW-2000 modular ground-water model (Harbaugh and others, 2000). The GWM Process solves several types of linear, nonlinear, and mixed-binary linear ground-water management problems. The response-matrix approach, which has been used widely in ground-water management modeling, is used to transform a ground-water management problem into an optimization formulation that can be solved by GWM. GWM uses the simplex and branch and bound optimization algorithms to solve the resulting formulations; these algorithms have been coded internally in GWM in the FORTRAN-90 computer language. In its current form, the GWM Process can only be used with the MODFLOW-2000 Ground-Water Flow (GWF) and Global Processes; it cannot be used with the Ground-Water Transport, Observation, Sensitivity, or Parameter-Estimation Processes. Currently (2005), MODFLOW-2000 (Harbaugh and others, 2000) is the most recent version of the MODFLOW code, which was originally developed in the

1980s (McDonald and Harbaugh, 1988; Harbaugh and McDonald, 1996). Unless otherwise noted, in the remainder of this report, the term MODFLOW will refer to the MODFLOW-2000 version of the code.

The origin of GWM is the MODOFC code developed by Ahlfeld and Riefler (2003). MODOFC is based on the MODFLOW-96 version of MODFLOW (Harbaugh and McDonald, 1996). Several modifications have been made in the transition from MODOFC to GWM. First, the types of decision variables and constraints that can be specified in a management-model formulation have been modified and expanded. For example, it is now possible to include decision variables that represent a source or sink of water that is external to the model domain. Second, the structure of the input and output files has been modified substantially. In the new structure, separate input files are used to define the decision variables, the objective function, the constraints, and the solution technique. The new structure is intended to facilitate the process of converting each management formulation into a format that can be solved by GWM; the revised structure also should make the process of developing new options and associated computer modules for GWM easier. Finally, many changes were made to the original MODOFC FORTRAN code; these changes included organizing the subroutines into modules and packages that are consistent with the overall MODFLOW-2000 structure.

This report describes the formulation of ground-water management problems that can be solved with GWM, the approaches that GWM uses for solving the management problems, and the input and output files associated with a GWM run. The report also includes three sample problems of the application of GWM to typical ground-water management problems. These sample problems provide examples of the input files needed by GWM and the output that is generated by GWM. This report, however, is not a guide to the application of optimization modeling for aquifer management. For detailed guides to the application of management models, refer to textbooks by Willis and Yeh (1987), Gorelick and others (1993), and Ahlfeld and Mulligan (2000), and to literature reviews by Gorelick (1983), Yeh (1992), Ahlfeld and Heidari (1994), and Wagner (1995).

The report begins with a brief overview of ground-water flow modeling using the MODFLOW Ground-Water Flow Process. The purpose of the overview is to introduce concepts that are important to the GWM Process. For more details on the theory and use of the Ground-Water Flow Process, the reader is referred to Harbaugh and others (2000).

Numerical Modeling of Ground-Water Flow with MODFLOW

The partial-differential equation of ground-water flow used in MODFLOW is (McDonald and Harbaugh, 1988)

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right)-W=S_s\frac{\partial h}{\partial t}\ , \tag{1}$$

where

h is the potentiometric head (L);

K_{xx}, K_{yy}, K_{zz} are values of hydraulic conductivity along the x, y, and z coordinate axes, which are assumed to be parallel to the major axes of hydraulic conductivity (L/T);

W is a volumetric flow rate per unit volume, and represents sources and/or sinks of water (T^{-1});

S_s is the specific storage of the porous material (L^{-1}); and

t is time (T).

Equation 1 describes three-dimensional ground-water flow of constant density under nonequilibrium conditions in a heterogeneous and anisotropic medium, provided the principal axes of hydraulic conductivity are aligned with the coordinate directions (McDonald and Harbaugh, 1988). The ground-water flow equation, when combined with a particular set of boundary and initial conditions, constitutes a mathematical representation of a ground-water flow system. A solution to equation 1 gives the distribution of head (the state variable) throughout the flow system as a function of space and time, $h(x, y, z, t)$.

There are three main types of boundary conditions that can be specified for a given location and time along the boundary of the model domain. These are (1) specified-head boundaries, (2) specified-flow boundaries, and (3) head-dependent flow boundaries. A fourth type of boundary condition is required if a free surface, or water table, is present. Because the water table is a free-surface boundary whose position is not fixed, the location of the boundary is not known. This variable boundary results in a nonlinear relation between the position of the water table and conditions along the boundary of, or stresses to, a ground-water system. The presence of free-surface or head-dependent boundary conditions has implications for the solution of the ground-water management problem, as discussed later in the report.

MODFLOW uses the finite-difference method to solve the partial-differential equation of ground-water flow. In this method, the continuous system described by equation 1 is converted into a finite set of simultaneous linear algebraic difference equations that define the state variable $h(x, y, z, t)$ at discrete node points and times. The basic concept of the finite-difference method is to approximate the derivatives in equation 1 using the difference in head at adjoining nodes during specified time intervals. To define the locations of the node points, the ground-water flow system is discretized into a mesh of blocks (called cells) that consists of a set of rows, columns, and layers. Each cell within the mesh is identified by its row, column, and layer position by the indexing scheme i, j, k (that is, row i, column j, and layer k). MODFLOW's block-centered formulation of the finite-difference equations places the node points at the center of the cells.

A finite-difference approximation also is defined for the time derivative of head on the right-hand side of equation 1, in which the continuous time derivative is replaced by a set of discretized time intervals called time steps. MODFLOW uses a backward-difference approximation to the time derivative because that approach is always numerically stable—that is, errors introduced at any time diminish progressively at succeeding times (McDonald and Harbaugh, 1988). MODFLOW defines stress periods as time intervals during which all of the external stresses to the simulated ground-water flow system are constant. Stress periods are divided into time steps to achieve adequate numerical accuracy. Stress periods defined for a particular MODFLOW simulation can be of variable lengths; ground-water management problems solved by GWM also can use stress periods that are of variable lengths.

Solution of the finite-difference equations gives an approximate value of head, which will be notated as $h_{i,j,k,t}$, at each of the discrete cells and for each of the discrete times. For steady-state problems, the subscript t can be ignored and the head is understood to be that at steady state. Most of the methods available in MODFLOW to solve the set of finite-difference equations are iterative methods, wherein the equations are solved repeatedly (iteratively) for each time step beginning with an initial estimate of the head distribution throughout the model domain and stopping when a head distribution is obtained that meets one or more convergence (or closure) criteria specified by the user. These convergence criteria most commonly include a requirement that the changes in calculated heads from one iteration to the next must be less than a specified value. Iterative procedures yield only approximate solutions to the finite-difference equations for each time step; the accuracy of these solutions depends on many factors, including the closure criteria specified by the user. Because of the importance of the accuracy of MODFLOW calculations to the solutions of ground-water management problems solved by the GWM Process, the issue of model accuracy will be discussed in more detail in the section “Solution of Ground-Water Management Problems with GWM.”

The source/sink term, W, in equation 1 is used to represent stresses imposed on a ground-water flow system. In MODFLOW, these stresses are flow rates applied to the simulated ground-water flow system at specified locations and stress periods, and, after multiplication by appropriate volumes, have dimensions of volume per unit time (L^3/T). One type of flow stress that can be represented by W is a withdrawal (discharge) or injection (recharge) rate at a simulated well site, Qw_n, where n represents both the location of the nth well site and the stress period (or periods) during which the well operates. As used in MODFLOW, Qw_n is positive ($Qw_n > 0.0$) for flow into the ground-water system at the well (injection) and negative ($Qw_n < 0.0$) for flow out of the ground-water system at the well (withdrawal).

Withdrawal and injection of water at simulated wells affect the head distribution of the simulated ground-water flow system. At a particular location and time in the modeled flow system, the relation between head and simulated stresses at the wells can be expressed mathematically as

$$h_{i,j,k,t} = h_{i,j,k,t}(\boldsymbol{Qw}) \quad , \tag{2}$$

where $\boldsymbol{Qw}$ represents the vector (set) of all withdrawal and injection rates at all well locations and all operative stress periods. Equation 2 simply states that there is a functional relation between the state variable $h_{i,j,k,t}$ calculated by MODFLOW and the simulated wells. Equations similar to equation 2 also could be written for other head-based state variables indirectly calculated by MODFLOW, such as head drawdowns and gradients.

Ground-water flow systems commonly interact with streams and other surface-water features that are in hydraulic connection with the underlying ground-water system. For the case of streams, this interaction occurs as seepage of water across the ground-water/streambed interface. The rate of seepage to or from a particular stream reach at a particular time is dependent upon the conductance of the streambed and the head gradient between the ground-water flow system and the stream; when ground-water head is above the stream bottom, the seepage rate is described by

$$Qsb_{r,t} = Csb_r(Hs_{r,t} - h_{i,j,k,t}) \quad , \tag{3}$$

where

$Qsb_{r,t}$ is the flow rate (seepage rate) between the ground-water flow system at cell i,j,k and stream reach r at time t (L^3/T);
Csb_r is the streambed conductance in stream reach r (L^2/T);
$Hs_{r,t}$ is the head in stream reach r at time t (L); and
$h_{i,j,k,t}$ is the ground-water head in cell i,j,k at time t (L).

Prudic (1989) developed a Streamflow-Routing (STR1) Package for MODFLOW that simulates hydraulic interactions between the simulated ground-water system and adjoining streams, and also keeps track of the amount of water within each simulated stream reach, Qsf_r, where r represents both the location of the rth stream reach and the stress period of flow in the reach. Each stream reach corresponds to an individual cell in the finite-difference grid used to simulate ground-water flow. The flow in each stream reach consists of inflows from adjacent upstream reaches, diversions within the reach, and the gain or loss of streambed seepage.

The functional relation between ground-water head in a model cell and the seepage rate between the aquifer and stream reach is shown schematically in figure 1. An important characteristic of the relation is that stream seepage is a nonlinear function of head. If head in the aquifer is above the bottom of the streambed (SBOT in fig. 1), then there is a linear relation between head in the model cell and seepage to or from the stream. If head in the aquifer falls below the bottom of the streambed, however, then the ground-water head is set to the elevation of the bottom of the streambed, and the seepage rate from the stream to the model cell depends only on stream stage (as long as there is flow in the stream reach to meet the seepage rate).

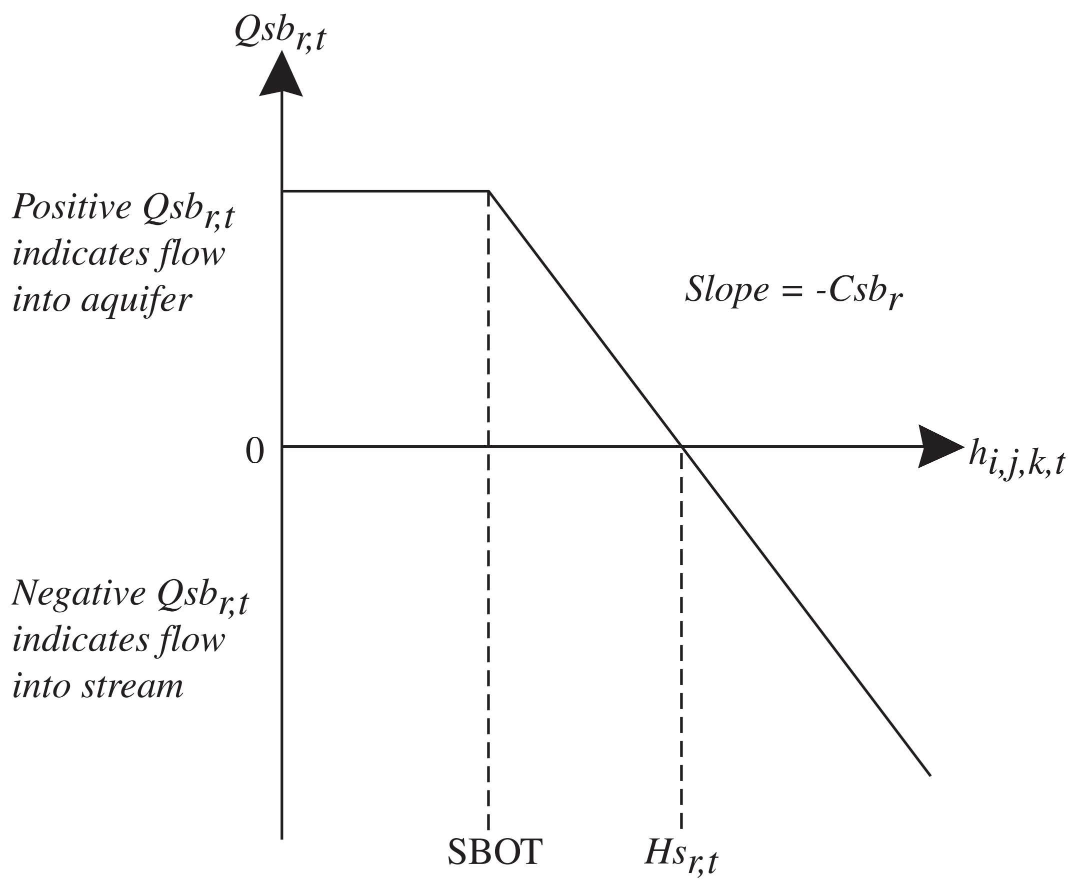

Figure 1. Seepage rate, $Qsb_{r,t}$, through a streambed as a function of ground-water head, $h_{i,j,k,t}$, in a model cell, where $SBOT$ is the streambed bottom, Csb_r is streambed conductance, and $Hs_{r,t}$ is head in the stream. Figure modified from Prudic (1989).

Because streamflow calculated by the STR1 Package is a function of seepage between each simulated stream reach and the modeled ground-water flow system, and seepage, in turn, is a function of head in the ground-water system (eq. 3), calculated streamflows are also a function of the head in the aquifer. Therefore, streamflows calculated by the STR1 Package can be considered to be state variables of the flow model, and, as with heads, a functional relation between streamflows and simulated wells can be defined:

$$Qsf_r = Qsf_r\,(\boldsymbol{Qw})\,. \tag{4}$$

Prudic and others (2004) developed a second Streamflow-Routing (SFR1) Package that can be used instead of the original STR1 Package. Because the SFR1 Package had not been released during the development of GWM, the functional relations between streamflow and simulated wells are calculated in GWM by use of the STR1 Package.

A calibrated MODFLOW ground-water flow model provides the basis by which GWM develops specific functional relations between the simulated wells and calculated heads and streamflows described in general form by equations 2 and 4. These relations are referred to as response coefficients (or response functions), and are described in more detail in the solution-procedures section of the report.

Formulation of Ground-Water Management Problems with GWM

This section describes the components available in the GWM Process for formulating ground-water management problems. A ground-water management formulation consists of three components: decision variables, an objective function, and a set of constraints. Together, these three components define a mathematical model of the management decision-making (or design) process (Ahlfeld and Mulligan, 2000; Hillier and Lieberman, 2001).

The decision variables of the management problem are the quantifiable controls (or decisions) that are to be determined by the model, such as the withdrawal rates at a set of managed wells. The values determined by GWM for these control variables define the solution of the problem. The objective function of the problem, which is stated in terms of one or more of the decision variables, is a measure of the performance of the design process; the objective function is used to identify the best solution among many possible solutions. This function may be maximized or minimized, depending upon the GWM application. The third component of the management problem is a set of constraints that impose restrictions on the values that can be taken by the decision variables. The solution of a well-defined ground-water management formulation consists of values for the decision variables that optimize the objective function while satisfying all constraints on decision-variable values (Ahlfeld and Mulligan, 2000).

Decision Variables

GWM supports three types of decision variables: flow-rate decision variables, external decision variables, and binary variables.

Flow-Rate Decision Variables

The primary type of decision variable is a withdrawal (discharge) or injection (recharge) flow rate at a managed well site, Qw_n. All flow-rate decision variables are treated as positive values in GWM, whether they represent withdrawal or injection. For flow-rate variables that represent withdrawal, GWM internally converts to the correct sign for consistency with the sign convention of the MODFLOW GWF Process. Each flow-rate decision variable can extend over one or more model cells and can be active during one or more stress periods; the stress periods for which a decision variable are defined do not need to be of equal length. In the simplest case, Qw_n is the withdrawal or injection rate at a single model cell during a single stress period. In more complicated problems, Qw_n might consist of a set of vertical cells that simulate withdrawal from a well screened over several model layers, or a set of cells within a single model layer that represent the aggregated withdrawal from several wells in a subarea of the model domain. Although each decision variable can be defined to extend over multiple cells and multiple stress periods, the flow rate determined for the decision variable is a single value that is constant for each stress period during which the decision variable is active, and is apportioned to each cell within the areal extent of the decision variable on a percentage basis specified by the user.

Each flow-rate decision variable represents either withdrawal or injection at the well site; a single decision variable cannot be used for both withdrawal and injection. However, the user can specify two flow-rate decision variables for a well site that are defined by the same set of model cells and stress periods, with one of the decision variables defined as a withdrawal well and the other defined as an injection well. Moreover, for transient models, a single well site may have more than one flow-rate decision variable associated with it, wherein each decision variable for the site has the same location but is active during different stress periods. This is a common situation for wells at which withdrawal rates vary with time.

It is important to recognize the difference between flows simulated by the Well (WEL) or Multi-Node Well (MNW) Packages of the GWF Process and those defined as decision variables for a GWM problem. For the purpose of a GWM problem, flows simulated by the WEL (Harbaugh and others, 2000) or MNW (Halford and Hanson, 2002) Packages have user-defined withdrawal or injection rates, and are referred to as unmanaged wells. These unmanaged withdrawal and injection rates are considered to be background stresses, in the sense that they will contribute to the total stress of the ground-water flow system in the absence of managed withdrawals and injections. In contrast, for a flow-rate decision variable defined in a GWM problem, the withdrawal or injection rate is unknown at the start of the GWM problem, and is determined as part of the solution process. Flow rates defined as decision variables are referred to as managed flows, because they are part of the management solution. At the start of a GWM problem, the managed flows are candidates for possible selection into the final set of active (that is, nonzero) flow-rate decision variables that compose the solution of the problem. It is possible that some of the candidate decision variables may not be selected as part of the solution to the problem, in which case the decision variables will be inactive (calculated withdrawal or injection flow rates of zero).

GWM allows for the simultaneous use of both managed and unmanaged wells at model cells. For example, the user might specify an unmanaged withdrawal rate (that is, a background stress) of 1.0 ft^3/s at a particular cell with the WEL Package; the user also could define a managed withdrawal at the same cell by use of a flow-rate decision variable in GWM. The total withdrawal rate at the cell at the end of the GWM run would then equal the sum of the unmanaged withdrawal rate (1.0 ft^3/s) and the managed withdrawal rate determined by GWM for the decision variable.

Although flow-rate decision variables have been described with reference to a well site (and are simulated in the GWM Process in a manner that is identical to a well simulated by the MODFLOW WEL Package), these variables could represent other types of managed stresses to a ground-water system, such as the recharge rate to an artificial recharge basin or the discharge rate to an irrigation drain. Several examples of how flow-rate decision variables can be defined in a GWM problem are provided in Appendix 1 of this report. Those examples also can be used as a guide during preparation of the input files for GWM.

External Decision Variables

The second type of decision variable that can be specified in GWM is a source or sink of water that does not have a direct effect on the state variables of the ground-water flow system. All external variables are treated as positive values in GWM, whether they represent a source or sink of water. These decision variables are external to the ground-water flow system, and therefore, are called external variables, Ex_m, where m represents the mth external variable. In contrast to the flow-rate decision variables, external variables are not part of the MODFLOW GWF Process, and response coefficients between the variables and MODFLOW state variables are not determined by GWM. Sources of external water to the management model are referred to as imports, whereas sinks of water are referred to as exports. An example of a source of water to the management model is an interbasin transfer from a surface-water reservoir that is external from the ground-water basin; additional examples of sources of water are treated wastewater available for artificial recharge or desalinated water from a desalination plant. An example of an export of water is a fraction of the ground water withdrawn from the simulated ground-water basin that is exported out of the basin and, therefore, is unavailable to meet within-basin water-supply demands.

Binary Variables

The third type of decision variable supported by GWM is a binary variable, which is defined to indicate the status of associated sets of flow-rate and external decision variables. One or more flow-rate or external decision variables, or combinations of flow-rate and external decision variables, can be associated with a single binary variable (see example 3 in Appendix 1). Binary variables are notated as I_l (where l represents the lth binary variable), and have values of 0 or 1 (written $I_l = 0, 1$). If I_l equals 1, at least one of the flow-rate or external decision variables associated with the binary variable is active (that is, the site has been constructed or is operational); if I_l equals 0, all of the associated flow-rate and external variables are inactive. The binary variables are not directly associated with specific stress periods of the model; rather, the stress periods are implied by those assigned to the associated flow-rate and external decision variables.

Binary variables often are used to model the construction costs of a water-supply facility (a well site or external facility), but can have other uses in a management model. For example, there may be a requirement that the total number of active well sites constructed by a water-supply agency must be less than or equal to the total number of developable sites owned by the agency; alternatively, the water agency might require that the withdrawal rate at each well site either be zero (that is, the site is not constructed) or, if the site is constructed, greater than some minimum, nonzero withdrawal rate.

Binary variables can impose a significant computational burden on the GWM solution process, and they should be used with caution when nonlinear responses are present in the ground-water model.

Objective Function

GWM supports a single objective function, which is to minimize or maximize the weighted sum of the three types of decision variables:

$$\sum_{n=1}^{N} \beta_n Qw_n T_{Qw_n} + \sum_{m=1}^{M} \gamma_m Ex_m T_{Ex_m} + \sum_{l=1}^{L} \kappa_l I_l \ , \tag{5}$$

where

β_n is the cost or benefit per unit volume of water withdrawn or injected at well site n;

γ_m is the cost or benefit per unit volume of water imported or exported at external site m;

κ_l is the unit cost or benefit associated with the binary variable I_l;

T_{Qw_n} is the total duration of flow at well site n;

T_{Ex_m} is the total duration of flow at external site m; and

N, M, L are the total number of flow-rate, external, and binary decision variables, respectively.

T_{Qw_n} and T_{Ex_m} are calculated by GWM by summing the duration of all stress periods during which the *n*th or *m*th decision variable is active. (Again, note that GWM does not require that stress periods specified in a MODFLOW simulation be of equal length.) The coefficients β_n, γ_m, and κ_l are called the objective-function coefficients.

Equation 5 can be considered in terms of economic costs or benefits of the water withdrawn, injected, imported, or exported in the management model, so that each of the three terms of equation 5 has monetary units (such as dollars). The first term in equation 5 is the cost (or benefit) of withdrawing or injecting water at the flow-rate decision variables, the second term is the cost (or benefit) of importing or exporting water at the external variables, and the third term is the cost (or benefit) of making the flow-rate or external variables associated with each binary variable active. Note that the coefficients in the first two terms in the equation (β_n and γ_m) imply that the cost (or benefit) of water is linearly proportional to the volume of water withdrawn, injected, imported, or exported.

Although equation 5 can be considered in monetary terms, it is often difficult to assign economic costs and benefits to all components of a water-resource management problem. In cases where economic costs and benefits are unknown or are not required in the management formulation, the user can specify the β_n and γ_m coefficients as relative costs or benefits among the different flow-rate or external variables. For example, the objective might be to maximize the water withdrawn at n well sites, in which case the β_n coefficients could be set to dimensionless values of 1.0. Equation 5 then would become

$$\text{Maximize } \sum_{n=1}^{N} 1.0 Qw_n T_{Qw_n} \ , \tag{6}$$

and the resulting objective value is in terms of total volume of water withdrawn. Alternatively, the user might want to weight the withdrawal from one well site as being twice as beneficial as the withdrawal from a second site, in which case the objective function would simplify to

$$\text{Maximize } 2.0 Qw_1 T_{Qw_1} + 1.0 Qw_2 T_{Qw_2} \, . \tag{7}$$

Regardless of the meaning assigned to the coefficients β_n, γ_m, and κ_l in equation 5, the user must ensure that the units of these coefficients are consistent. When GWM solves the management problem, it is the relative magnitude of each term in equation 5 that will determine the solution obtained.

Constraints

GWM supports four general types of management-model constraints that are described in the four subsections that follow. These constraints can be divided broadly into two types: those for which response coefficients need not be generated (constraints on the decision variables themselves and linear-summation constraints), and those for which response coefficients between the decision variables and ground-water flow system state variables must be generated (the hydraulic-head and streamflow constraints). Constraints that do not require generation of response coefficients are described first.

Decision-Variable Constraints

Lower and upper bounds are commonly placed on the flow-rate and external decision variables of a ground-water management model to express bounds on the yield of a well, injection rate of a well, and minimum and maximum capacities of an external source or sink of water. These lower and upper bounds are written mathematically as

$$Qw_n^l \le Qw_n \le Qw_n^u \tag{8}$$

and

$$Ex_m^l \le Ex_m \le Ex_m^u, \tag{9}$$

where Qw_n^l and Qw_n^u are the lower and upper bounds on flow-rate decision variable n, respectively, and Ex_m^l and Ex_m^u are the lower and upper bounds on external variable m, respectively. GWM requires that Qw_n^l, Qw_n^u, Ex_m^l, and Ex_m^u all be greater than or equal to zero, and that $Qw_n^l \le Qw_n^u$ and $Ex_m^l \le Ex_m^u$.

If a flow-rate or external decision variable has not been associated with a binary variable in GWM, the lower bound for each variable (Qw_n^l or Ex_m^l) must be defined as 0. Equations 8 and 9 then become

$$0 \le Qw_n \le Qw_n^u \tag{10}$$

and

$$0 \le Ex_m \le Ex_m^u. \tag{11}$$

The user can specify nonzero lower bounds for decision variables that are not associated with a binary variable by use of linear-summation constraints described in the next section of the report.

If a flow-rate or external decision variable has been associated with a binary variable, then enhanced lower-bound constraints can be placed on the variable. Specifically, each variable can be required to be inactive, with a volumetric flow rate of zero, or active, with a nonzero lower bound (Qw_n^l or Ex_m^l). This either/or type of constraint can be written with binary variables as

$$Qw_n \geq Qw_n^l I_l \tag{12}$$

and

$$Ex_m \geq Ex_m^l I_l , \tag{13}$$

where I_l is the binary variable associated with decision variables Qw_n and (or) Ex_m. Note that if $I_l = 0$, then constraints 12 and 13 imply that the variable lower bound is 0, whereas if $I_l = 1$, then the lower bound is the specified nonzero bound value. Constraints 12 and 13 are applied separately to each decision variable associated with a given binary variable. Additional constraints are automatically added by GWM to force the decision variable to be 0 if the binary variable is 0 and to allow the decision variable to take any value in the range described by equations 8 and 9 if the binary variable is 1.

Linear-Summation Constraints

Constraints are available in GWM for linear summation of the three types of decision variables. These constraints have the general form

$$\sum_{p=1}^{P} a_p GV_p \leq b_p , \tag{14}$$

$$\sum_{p=1}^{P} a_p GV_p \geq b_p , \tag{15}$$

and

$$\sum_{p=1}^{P} a_p GV_p = b_p , \tag{16}$$

where a_p and b_p are specified coefficients, GV_p is any of the three types of decision variables (Qw_n, Ex_m and I_l), and P is the total number of terms in the summation. Because all flow-rate and external decision variables are treated in GWM as positive values whether they are withdrawals, injections, imports, or exports, the user must ensure that the sign of each coefficient in equations 14–16 is defined to achieve the desired constraint form. The user also must ensure that the units used throughout a GWM formulation, including the coefficients in equations 14–16, are consistent.

When specifying a linear-summation constraint in GWM, it is not necessary to specify the stress period (or periods) for which the constraint is active, because the time period(s) for which each constraint applies is implicitly defined by the stress period(s) for which the flow-rate or external decision variables have been defined. The generalized nature of these constraints requires that the user ensure that the constraint definition makes sense. For example, if a withdrawal variable has been defined for a well site as Qw_1 in stress period 1 and a second withdrawal variable has been defined for the well site as Qw_2 for stress period 2, there may be a requirement that withdrawal at the site not decrease by more than 25 percent from the first to the second stress period, that is, $Qw_2 \geq 0.75Qw_1$. This constraint is specified in GWM as

$$-0.75Qw_1 + 1.0Qw_2 \geq 0\,, \tag{17}$$

with the stress periods for which the constraint applies implied by the stress periods associated with the two decision variables.

Summation constraints described by equations 14–16 can be used in a variety of ways. A few examples of the use of the summation constraints are described in the next paragraphs.

Water-supply demands: Water-management models often include constraints that describe the demands on the water-supply system. These constraints can be written in a general way as

$$\sum_{n=1}^{N} a_{Qw_n} Qw_n + \sum_{m=1}^{M} a_{Ex_m} Ex_m \geq D_t\,, \tag{18}$$

where a_{Qw_n} and a_{Ex_m} are the coefficients (weights) associated with withdrawals Qw_n and sources Ex_m, respectively, N and M are the total number of withdrawal and source locations, respectively, and D_t is the water-supply demand during stress period t.

As an example of the use of equation 18, an irrigation district might require that the total ground water withdrawn from its three wells, in addition to the amount that is available from an out-of-basin source (an import), must meet the district's total irrigation-season water-supply demands. If the three wells from which the water is withdrawn during the irrigation season are designated Q_1, Q_2, and Q_3, all sources of water are assigned equal weights (that is, all coefficients equal 1.0), the amount available from the out-of-basin source is designated as IM_1, and the total demand during the irrigation season is designated D, then the constraint can be written as

$$Q_1 + Q_2 + Q_3 + IM_1 \geq D\,.$$

Net stress constraints: An extension of equation 18 is to require that the difference between the total withdrawal and total injection rates from a set of candidate withdrawal and injection wells be constrained between upper and lower limits. To impose such a constraint, equations 14 and 15 can be written as

$$\sum_{nw=1}^{NW} Qw_{nw} - \sum_{ni=1}^{NI} Qw_{ni} \leq Qw^u \tag{19}$$

and

$$\sum_{nw=1}^{NW} Qw_{nw} - \sum_{ni=1}^{NI} Qw_{ni} \geq Qw^l\,, \tag{20}$$

where Qw_{nw} and Qw_{ni} are the candidate withdrawal and injection wells, NW and NI are the total numbers of candidate withdrawal and injection wells, and Qw^u and Qw^l are the upper and lower bounds on the net stress during the stress period (or periods), respectively.

Stress ratio constraints: Instead of constraining the total net stress from a set of candidate withdrawal and injection wells, as in the previous example, there may be a desire to define the ratio between total withdrawals from and total injections to a set of candidate wells. These constraints can be written as

$$\frac{\sum_{nw=1}^{NW} Qw_{nw}}{\sum_{ni=1}^{NI} Qw_{ni}} \geq a \tag{21}$$

and

$$\frac{\sum_{nw=1}^{NW} Qw_{nw}}{\sum_{ni=1}^{NI} Qw_{ni}} \leq \frac{1}{b}, \tag{22}$$

where a and b are specified stress ratios and other terms are defined as they are for equations 19 and 20. Equations 21 and 22 are rearranged for use in GWM, resulting in

$$\sum_{nw=1}^{NW} Qw_{nw} - a \sum_{ni=1}^{NI} Qw_{ni} \geq 0.0 \tag{23}$$

and

$$\sum_{ni=1}^{NI} Qw_{ni} - b \sum_{nw=1}^{NW} Qw_{nw} \geq 0.0\,. \tag{24}$$

Equations 23 and 24 can be used, for example, in the design of a capture and containment system of a ground-water contamination plume, in which a fraction of the water withdrawn to contain the plume is chemically treated and then reinjected into the ground-water system.

Constraints on the total number of active well sites: Upper and lower bounds on the total number of active (that is, nonzero) well sites can be specified by use of binary variables and summation constraints. These constraints are written in the general form

$$\sum_{l=1}^{L} I_l \leq N^U \tag{25}$$

and

$$\sum_{l=1}^{L} I_l \geq N^L, \tag{26}$$

where N^U and N^L are the upper and lower bounds on the number of active well sites and L is the total number of candidate well sites. For example, a water district might have access to a total of eight possible sites at which wells could be constructed, but funding is available for a maximum of five wells only. In addition, a minimum of three wells is desired by the water district to guard against the risk of one or two of the wells being taken out of service because of damage to the wells or contamination of the ground water. These requirements could be specified as

$$I_1 + I_2 + I_3 + I_4 + I_5 + I_6 + I_7 + I_8 \leq 5 \tag{27}$$

and

$$I_1 + I_2 + I_3 + I_4 + I_5 + I_6 + I_7 + I_8 \geq 3\,. \tag{28}$$

Hydraulic-Head Constraints

GWM supports four types of hydraulic-head constraints. The first type of constraint is an absolute lower and (or) upper bound placed on a head at a specific location and stress period:

$$h_{i,j,k,t} \geq h^{l}_{i,j,k,t} \tag{29}$$

and

$$h_{i,j,k,t} \leq h^{u}_{i,j,k,t}, \tag{30}$$

where $h^{l}_{i,j,k,t}$ and $h^{u}_{i,j,k,t}$ are the specified lower and upper bounds on head at location i, j, k (fig. 2*A*) <u>at the end</u> of stress period t (that is, at the end of the last time step in the stress period). These constraints can be used to control excessive lowering (eq. 29) or mounding (eq. 30) of the potentiometric surface.

The second type of head constraint is drawdown of head at a specific location and stress period. Drawdowns are defined by $dd_{i,j,k,t}$ and are equal to the difference between an initial head at location i, j, k at the end of stress period t, $(h_{i,j,k,t})^0$, and the head calculated at the location at the end of stress period t after implementation of the optimal management strategy, $h_{i,j,k,t}$:

$$dd_{i,j,k,t} = (h_{i,j,k,t})^0 - h_{i,j,k,t} \; . \tag{31}$$

The drawdown constraints are written as

$$dd_{i,j,k,t} \geq dd^{\,l}_{i,j,k,t} \tag{32}$$

and

$$dd_{i,j,k,t} \leq dd^{\,u}_{i,j,k,t} \; , \tag{33}$$

where $dd^{\,l}_{i,j,k,t}$ and $dd^{\,u}_{i,j,k,t}$ are specified lower and upper bounds on drawdown at location i, j, k at the end of stress period t (fig. 2*B*). The use of drawdown constraints in a GWM formulation requires that a reference withdrawal or injection rate be specified for each of the candidate well sites. These reference rates are used to calculate the initial head, $(h_{i,j,k,t})^0$, at each drawdown constraint location.

The third type of head constraint is a lower bound on the difference in head between two model locations $(i, j, k)_1$ and $(i, j, k)_2$ at the end of stress period t:

$$(h_{i,j,k,t})_1 - (h_{i,j,k,t})_2 \geq (hdif_{i,j,k,t})_{1,2} \, , \tag{34}$$

where $(h_{i,j,k,t})_1$ and $(h_{i,j,k,t})_2$ are heads calculated for locations $(i, j, k)_1$ and $(i, j, k)_2$ at the end of stress period t and $(hdif_{i,j,k,t})_{1,2}$ is a specified lower bound on the difference in head between locations 1 and 2 at the end of stress period t (fig. 2*C*). The two specified locations may be vertically or horizontally separated and need not be adjacent cells. GWM requires that the head calculated at the second location be lower than the head at the first location by an amount that is at least $(hdif_{i,j,k,t})_{1,2}$.

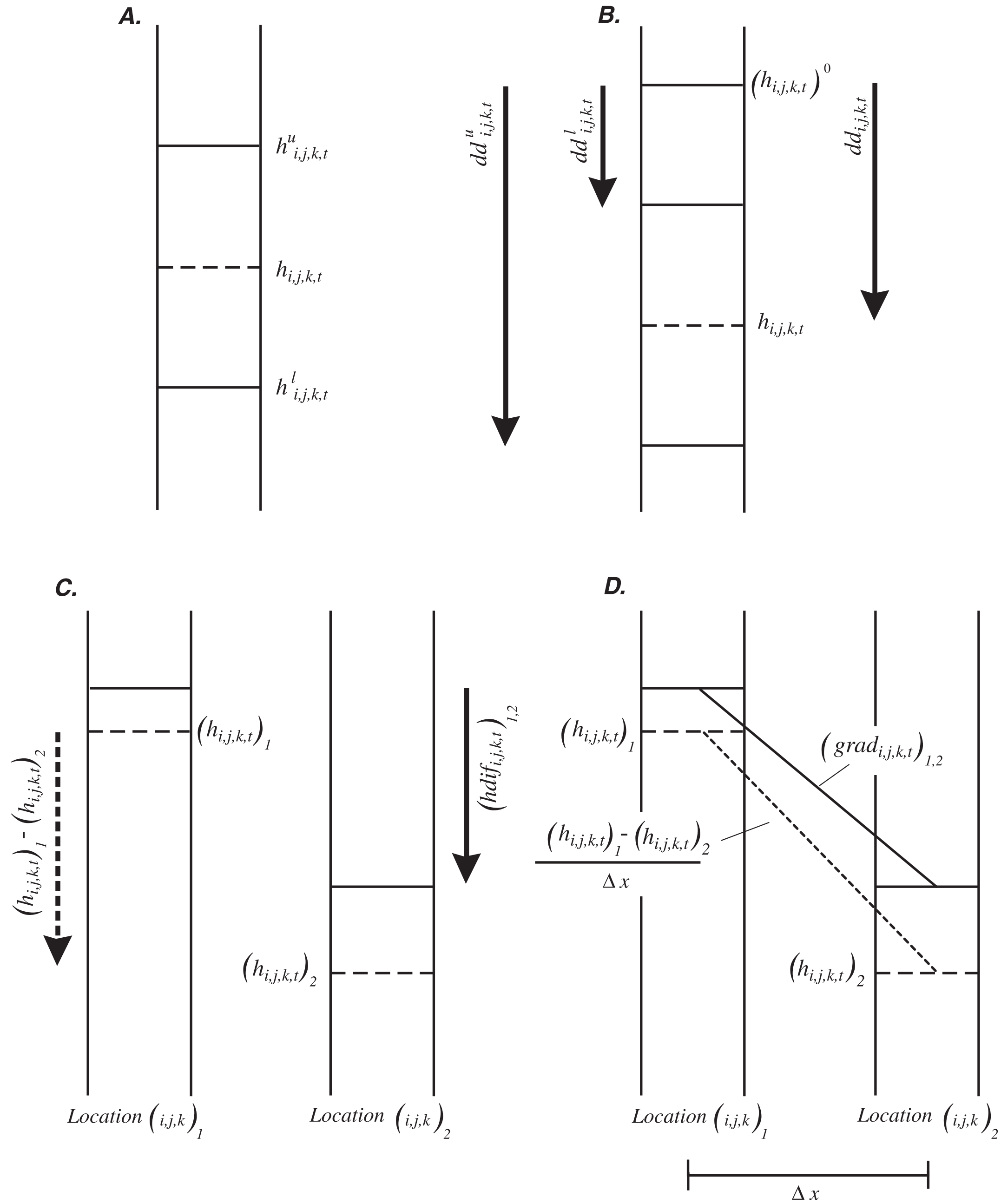

Figure 2. Types of hydraulic-head constraints supported by GWM: *A,* Lower ($h^l_{i,j,k,t}$) or upper ($h^u_{i,j,k,t}$) bound on head ($h_{i,j,k,t}$) at location i, j, k at the end of stress period t; *B,* Lower ($dd^l_{i,j,k,t}$) or upper ($dd^u_{i,j,k,t}$) bound on drawdown ($dd_{i,j,k,t}$) at location i, j, k at the end of stress period t; $(h_{i,j,k,t})^0$ is the initial head at i, j, k; *C,* Specified, $(hdif_{i,j,k,t})_{1,2}$, and model-calculated, ($(h_{i,j,k,t})_1 - (h_{i,j,k,t})_2$), difference in head between two model locations $(i, j, k)_1$ and $(i, j, k)_2$ at the end of stress period t; and *D*, Specified $(grad_{i,j,k,t})_{1,2}$, and model-calculated $[(h_{i,j,k,t})_1 - (h_{i,j,k,t})_2]/\Delta x$, gradient in head between two model locations $(i, j, k)_1$ and $(i, j, k)_2$ at the end of stress period t; Δx is the specified distance between the two locations.

The fourth type of head constraint is a lower bound on the gradient in head between two model locations $(i, j, k)_1$ and $(i, j, k)_2$ at the end of stress period t. The head gradient between two model locations ($(grad_{i,j,k,t})_{1,2}$) can be written as

$$\frac{(h_{i,j,k,t})_1 - (h_{i,j,k,t})_2}{\Delta x}, \tag{35}$$

where $(h_{i,j,k,t})_1$ and $(h_{i,j,k,t})_2$ are heads calculated for locations $(i, j, k)_1$ and $(i, j, k)_2$, and Δx is the distance between the two locations (fig. 2*D*). The constraint that is implemented in GWM is

$$\frac{(h_{i,j,k,t})_1 - (h_{i,j,k,t})_2}{\Delta x} \geq (grad_{i,j,k,t})_{1,2}. \tag{36}$$

The user must specify two model locations (with head at the second location lower than head at the first location), a distance between the two model locations (which could be the distance between the midpoint of each of the two model cells), and a specified lower bound on the gradient ($(grad_{i,j,k,t})_{1,2}$). The two model locations need not be adjacent.

Streamflow Constraints

Two types of streamflow constraints can be specified in a GWM management model, both of which are based on streamflows calculated by the Streamflow-Routing (STR1) Package developed for MODFLOW by Prudic (1989). The first type of constraint is an absolute lower and upper bound placed on streamflow Qsf_r at a specific stream location and stress period:

$$Qsf_r \geq Qsf_r^l \tag{37}$$

and

$$Qsf_r \leq Qsf_r^u, \tag{38}$$

where Qsf_r^l and Qsf_r^u are the specified lower and upper bounds on streamflow at stream location r at the end of stress period t.

The second type of streamflow constraint is a streamflow depletion at a specific stream location and stress period. Streamflow depletion is defined by Qsd_r, and is equal to the difference between an initial streamflow at stream location r at the end of stress period t, $(Qsf_r)^0$, and the streamflow calculated at the location at the end of stress period t after implementation of the optimal management strategy, Qsf_r:

$$Qsd_r = (Qsf_r)^0 - Qsf_r. \tag{39}$$

The streamflow-depletion constraints are written as

$$Qsd_r \geq Qsd_r^l \tag{40}$$

and

$$Qsd_r \leq Qsd_r^u, \tag{41}$$

where Qsd_r^l and Qsd_r^u are specified lower and upper bounds on streamflow depletion at location r at the end of stress period t. The use of streamflow-depletion constraints in a GWM formulation requires that a reference withdrawal or injection rate be specified for each of the candidate well sites. These reference rates are used to calculate the initial streamflow, $(Qsf_r)^0$, at each streamflow-depletion constraint location.

Complete Statement of Ground-Water Management Formulation Solved by GWM

Combining all objective and constraint functions described above, the complete statement of the ground-water management problem that can be formulated and solved by use of the GWM Process is

Maximize or minimize

$$\sum_{n=1}^{N} \beta_n Qw_n T_{Qw_n} + \sum_{m=1}^{M} \gamma_m Ex_m T_{Ex_m} + \sum_{l=1}^{L} \kappa_l I_l , \tag{42}$$

subject to

$$0 \le Qw_n \le Qw_n^u , \tag{43}$$

$$0 \le Ex_m \le Ex_m^u , \tag{44}$$

$$Qw_n \ge Qw_n^l I_l , \tag{45}$$

$$Ex_m \ge Ex_m^l I_l , \tag{46}$$

$$\sum_{p=1}^{P} a_p GV_p \le b_p , \tag{47}$$

$$\sum_{p=1}^{P} a_p GV_p \ge b_p , \tag{48}$$

$$\sum_{p=1}^{P} a_p GV_p = b_p , \tag{49}$$

$$h_{i,j,k,t} \ge h^l_{i,j,k,t} , \tag{50}$$

$$h_{i,j,k,t} \le h^u_{i,j,k,t} , \tag{51}$$

$$dd_{i,j,k,t} \ge dd^l_{i,j,k,t} , \tag{52}$$

$$dd_{i,j,k,t} \le dd^u_{i,j,k,t} , \tag{53}$$

$$(h_{i,j,k,t})_1 - (h_{i,j,k,t})_2 \ge (hdif_{i,j,k,t})_{1,2} , \tag{54}$$

$$\frac{(h_{i,j,k,t})_1 - (h_{i,j,k,t})_2}{\Delta x} \ge (grad_{i,j,k,t})_{1,2} , \tag{55}$$

$$Qsf_r \ge Qsf^l_r , \tag{56}$$

$$Qsf_r \le Qsf^u_r , \tag{57}$$

$$Qsd_r \ge Qsd^l_r , \tag{58}$$

and

$$Qsd_r \le Qsd^u_r . \tag{59}$$

If binary variables are not used, then each binary variable I_l in equations 42, 45, and 46 is set to 0. If binary variables are present, then GWM automatically adds additional constraints, not shown here, that ensure that binary variables take only a value of 0 or 1, and that flow-rate and external variables can only be nonzero when their associated binary variable has a value of 1.

Solution of Ground-Water Management Problems with GWM

The complete ground-water management formulation defined by equations 42–59 includes linear-programming, nonlinear-programming, and binary-programming aspects. A linear program is an optimization formulation in which the objective function and all constraints are linear functions of the decision variables. A nonlinear program is one in which the objective function and (or) one or more of the constraints are nonlinear functions of the decision variables. A binary program is one that includes binary decision variables, which in this case are the I_l variables. An optimization formulation that combines a linear program with binary variables is called a mixed-binary linear program. A specific management formulation that is developed from the complete formulation will include one or more of these program types. GWM provides capabilities to solve linear, nonlinear, and mixed-binary linear formulations. Mixed-binary nonlinear formulations also can be solved if the nonlinearities of the system are mild. The presence of binary variables in a nonlinear formulation, however, can make the solution process difficult, and may lead to suboptimal solutions or spurious results.

This section describes the three approaches used by the RMS (Response Matrix Solution) Package of GWM to solve linear, nonlinear, and mixed-binary linear (and mildly nonlinear) formulations. Users of GWM are encouraged to read the section on solution of linear formulations before proceeding to the other solution techniques. Additional detailed information on the theoretical background of the solution techniques can be found in Gorelick and others (1993) and Ahlfeld and Mulligan (2000).

Linear Formulations

If the ground-water management problem to be solved is linear, then a highly efficient, reliable, and widely used method for solving linear optimization problems called the simplex method can be used to solve the management problem. Use of the algorithm requires that the ground-water management formulation be stated in the form of a linear program. With respect to the complete ground-water management formulation described by equations 42–59, this means, first, that the state variables of the ground-water system (heads and streamflows) must respond linearly to changes in the stress rates imposed at the flow-rate decision variables (that is, at each Qw_n), and, second, that no binary variables be present in the formulation. For the first condition to be met, the ground-water flow system must be simulated with constant transmissivity in every layer (LAYCON equal 0 or 2), all boundary conditions must be linear, and GWF Process packages that contain other nonlinearities must not be present. Head-dependent boundary conditions such as those specified along simulated streams and illustrated by the graphical relation in figure 1 can induce nonlinear responses. Therefore, although there may be instances where seepage rates between a ground-water system and adjoining streams are linear (or mildly nonlinear) functions of ground-water heads and imposed flow rates, the constraints on streamflow and streamflow depletion (eqs. 56–59) are dropped from the management formulation for strictly linear programs; these constraints are described below for nonlinear formulations.

Eliminating the binary variables from the formulation results in constraint equations 45 and 46 being dropped and a modified objective function

Maximize or minimize

$$\sum_{n=1}^{N} \beta_n Qw_n T_{Qw_n} + \sum_{m=1}^{M} \gamma_m Ex_m T_{Ex_m}, \tag{60}$$

subject to constraint equations 43, 44, and 47–55.

The final step in transforming the ground-water management problem into a form that can be solved by use of the simplex method is to establish functional relations between the stresses imposed at the managed wells (the flow-rate decision variables) and the resulting changes in heads at the constraint locations. These functional relations, which were described in a general form by equation 2, are necessary to rewrite the head constraints (eqs. 50–55) in terms of the flow-rate decision variables. This transformation requires use of a first-order Taylor series expansion to define head at each constraint location and stress period as a function of the new withdrawal and injection stresses:

$$h_{i,j,k,t}(\boldsymbol{Qw}) = h^0_{i,j,k,t}(\boldsymbol{Qw}^0) + \sum_{n=1}^{N} \frac{\partial h_{i,j,k,t}}{\partial Qw_n}(\boldsymbol{Qw}^0)(Qw_n - Qw_n^0), \tag{61}$$

where

$h_{i,j,k,t}(\boldsymbol{Qw})$ is head at constraint location i, j, k and stress period t for a new vector (that is, a new set) of withdrawal and injection flow rates $\boldsymbol{Qw}$ having individual elements Qw_n;

$h^0_{i,j,k,t}(\boldsymbol{Qw}^0)$ is head at constraint location i, j, k and stress period t for an original vector (that is, a base-condition set) of withdrawal and injection flow rates $\boldsymbol{Qw}^0$ having individual elements Qw_n^0;

$\frac{\partial h_{i,j,k,t}}{\partial Qw_n}(\boldsymbol{Qw}^0)$ is the change in head at location i, j, k and stress period t for a change in withdrawal or injection flow rate for the *n*th flow-rate decision variable, evaluated at the original vector of flow rates $\boldsymbol{Qw}^0$; and

N is the total number of flow-rate decision variables.

Equation 61 states that the head at each constraint location is equal to the head at the constraint location for a base condition of withdrawal and injection rates $\boldsymbol{Qw}^0$ plus the sum of the head changes that result from the changes in withdrawal or injection rate at each of the n well sites. The linear summation defined by the right-hand side of the equation reflects the assumed linearity of the ground-water flow system and consequent linear response of heads to changes in withdrawal or injection rates. The partial derivatives in equation 61, ($\partial h_{i,j,k,t} / \partial Qw_n$), which are called the response coefficients, provide information on the response of ground-water heads to stresses at each withdrawal or injection site. For linear systems, each partial derivative is a constant whose value does not change with changes in the distribution of withdrawal and injection rates throughout the ground-water flow system.

Equation 61 is substituted for each head term in the head constraints described by equations 50 and 51; similar substitutions are made for other head constraints described by equations 52–55. For a more detailed description of the mathematical steps involved in these substitutions, the reader is directed to Ahlfeld and Mulligan (2000, p. 64–66).

Calculation of Response Coefficients

The partial derivatives that define the response coefficients are not calculated directly; instead, they are approximated by a first-order, finite-difference perturbation method. The derivative of head with respect to each flow-rate decision variable is approximated by the forward-difference equation

$$\frac{\partial h_{i,j,k,t}}{\partial Qw_n} \approx \frac{\Delta h_{i,j,k,t}}{\Delta Qw_n} = \frac{h_{i,j,k,t}(\boldsymbol{Qw}_{\Delta n}) - h^0_{i,j,k,t}(\boldsymbol{Qw}^0)}{Qw_{\Delta n}}, \tag{62}$$

where $Qw_{\Delta n}$ is the perturbation value for the *n*th flow-rate decision variable and $h_{i,j,k,t}(\boldsymbol{Qw}_{\Delta n})$ is the head at constraint location i, j, k and stress period t computed by using a vector of withdrawal and injection stress rates $\boldsymbol{Qw}_{\Delta n}$ that differs from the original vector of stress rates $\boldsymbol{Qw}^0$ only in the *n*th element, which is changed by an amount $Qw_{\Delta n}$.

To calculate each response coefficient defined by equation 62, the GWF Process of MODFLOW is run a total of N+1 times. In the first run, which is called the base-condition run, the head is calculated at each constraint location and stress period for the set of base-condition withdrawal and injection rates [that is, each $h^0_{i,j,k,t}(\boldsymbol{Qw}^0)$]. In each of the remaining N runs, the head for each constraint location is calculated on the basis of the change (perturbation) in the withdrawal or injection rate for the *n*th flow-rate decision variable. For each of these runs, the withdrawal or injection rate at each of the remaining N-1 well sites is kept at the base-condition value. The N runs and consequent computations of the response coefficients result in a matrix of response coefficients that is used to solve the optimization problem.

The perturbation values $Qw_{\Delta n}$ are calculated from the equation

$$Qw_{\Delta n} = \delta^0 Qw_n^u, \tag{63}$$

where δ^0 is a user-specified perturbation variable and Qw_n^u is the specified upper bound on withdrawal or injection rate at well site n. The perturbation variable δ^0 can be a positive or negative value: a positive value of δ^0 implies an increase in flow rate for $Qw_{\Delta n}$ (referred to as a forward-difference calculation), whereas a negative value of δ^0 implies a decrease in flow rate for $Qw_{\Delta n}$ (referred to as a backward-difference calculation).

As described above, calculation of the response coefficients requires conducting one flow-process run to obtain base values of the state variables and one additional flow-process run for each perturbed flow-rate decision variable. Because these runs are done automatically by GWM, it is important that the GWF Process input files produce runs that are stable for a range of values of the flow-rate decision variables. If the flow-process simulation is subject to convergence failure, excessive dewatering, or other instabilities, GWM may not be able to successfully compute response coefficients.

GWM has mechanisms for adjusting the perturbation value to attempt to overcome instabilities in the GWF Process. For each flow-process run, GWM checks for successful completion of the GWF Process. If failure occurs on a perturbation run, then GWM changes the perturbation value by a user-specified factor and re-executes the GWF Process. For a single flow-rate decision variable, re-adjustment of the perturbation value can be repeated until a successful run is achieved or a user-specified maximum is reached. See “Description of Selected Conventions, Options, and Variables in GWM” for more details on this feature.

Accuracy and Precision of the Response Coefficients

The response coefficients generated by the MODFLOW GWF Process are a critical link between the physics of the ground-water flow system and the results of a ground-water management model represented in GWM. As a consequence, the accuracy and precision of the response coefficients play an important role in the solution of a ground-water management problem. The accuracy of the response coefficients—that is, their ability to reflect the actual response of the aquifer—depends on at least two factors.

First, the accuracy of the heads calculated by MODFLOW for a particular ground-water flow system is a reflection of the accuracy with which the geologic framework, hydraulic properties, boundary conditions, and other flow-system characteristics are known and represented in the model. Clearly, the accuracy of the response coefficients increases as the level of understanding of the flow system and representation of the flow system with a numerical model improve.

The second issue that affects response-coefficient accuracy is approximation of the partial-differential equation of ground-water flow by a set of finite-difference equations. As described previously in this report (“Numerical Modeling of Ground-Water Flow with MODFLOW”), solution of the finite-difference equations that are used by MODFLOW to describe the head at each cell within the model domain gives only approximate values of the “true” head distribution in the aquifer as described by the partial-differential equation of ground-water flow. The error that is introduced by this approximation is proportional to the size of the grid cells that compose the discretized model domain and the size of the time steps used for transient conditions. In general, the approximation error increases as the grid becomes coarser and (or) the time steps become longer. Therefore, approximation error can be decreased by decreasing the size of the model grid cells and (or) time steps, although such actions typically increase computer run time.

The precision of the response coefficients is an indication of their ability to reflect the actual response of the calculated system state to changes in stress. A measure of response-coefficient precision is the number of significant digits in the value of the response coefficient. Maintaining adequate precision in computed response coefficients is essential for successful solution of the optimization problem by the RMS Package. Even if the flow-process run is not particularly accurate, it is important that the precision of each response coefficient be maintained. Precision of the response coefficients is affected in part by the size of the stress-rate perturbation values (that is, the size of each $Qw_{\Delta n}$) used in equation 62. Although the response coefficients for the linear systems considered here are constants whose values are independent of the size of the perturbation values, the precision of the response coefficients, and therefore the precision of the management solution, depends on the number of significant digits carried for each real number in the computation of each response coefficient. The relation between perturbation size and response-coefficient precision is related to roundoff error when the difference in heads is taken in the numerator of equation 62. If the two computed heads are very close in value, then significant precision can be lost. The issue is even more important when considering the use of perturbation for calculating the response coefficients for head-difference or gradient constraints (eqs. 54 and 55).

Most of the methods available to solve the flow-process finite-difference equations are iterative. Values of head are iteratively generated until the maximum calculated change in head at any model cell is less than a specified convergence criterion between iterations. The precision of the resulting heads can be estimated to be of the same magnitude as the convergence criterion. As a result, the precision of the response coefficients depends upon the convergence criterion used by the flow process.

There are two approaches for improving the precision of the response coefficients calculated for linear systems. The first approach is to lower the head convergence criterion in the GWF Process iterative solver, although this is likely to increase the computer time required to solve each GWF Process run. The second approach is to use relatively large perturbation values to ensure large head differences in the numerator of equation 62 (Reifler and Ahlfeld, 1996). Perturbation values equal to 500 percent of the expected solution stress rate are recommended as an initial guess; the perturbation values can then be varied in a series of GWM runs to evaluate how the management solutions are affected, if at all, by the size of the stress-rate perturbations.

Simplex Algorithm

Generation of the response-coefficient matrix by the RMS Package completes the transformation of the ground-water management problem into a form that can be solved using linear programming techniques. The revised linear formulation of the ground-water management problem consists of the objective function defined by equation 60, subject to constraints defined by equations 43, 44, 47–49, and revised constraints 50–55, in which heads have been rewritten in terms of the flow-rate decision variables (eq. 61). The revised linear formulation can be expressed in an equivalent vector form as

$$\text{Minimize } Z = \boldsymbol{c}^t\boldsymbol{x} \tag{64}$$

$$\text{subject to } \boldsymbol{Ax} = \boldsymbol{b} \tag{65}$$

$$\boldsymbol{0} \le \boldsymbol{x} \le \boldsymbol{u}, \tag{66}$$

where Z is the value of the objective function; $\boldsymbol{c}^t$ is a transposed column vector of objective-function coefficients associated with the decision variables; $\boldsymbol{x}$ is a column vector of decision variables with upper bounds $\boldsymbol{u}$; $\boldsymbol{A}$ is a matrix of coefficients that includes the response matrix for head; and $\boldsymbol{b}$ is a column vector of right-hand-side coefficients associated with the constraints. Constraints that have the form of inequalities are transformed to equalities by the addition of slack and surplus variables. The RMS Package solves the minimization problem described by equations 64–66; for maximization problems, a simple transformation is done internally by multiplying the objective function ($\boldsymbol{c}^t\boldsymbol{x}$) by -1.

The RMS Package solves the ground-water management problem by using the simplex algorithm, which iteratively solves for the optimal solution, $\boldsymbol{x}^*$. The mathematical details of the simplex algorithm as implemented in GWM are described in Appendix 2. Although the simplex method is reliable and stable, numerical conditions can arise in which the algorithm will cycle among points and will not converge. Such conditions are rare for well-posed linear problems, but to prevent indefinite cycling, the user must specify the maximum number of iterations that is allowed by the algorithm (input variable LPITMAX). A recommended value for LPITMAX is ten times the number of constraints, although experience suggests that the number of iterations to convergence is typically less than two times the number of functional constraints (Ahlfeld and Mulligan, 2000); functional constraints include all constraints that are not simple bounds on the decision variables.

There are four possible outcomes to the solution of each linear formulation. The first outcome is a single optimal solution for the elements of the decision-variable vector $\boldsymbol{x}$ that minimizes (or maximizes) the objective function and meets all of the constraints. Specifically, this solution consists of a set of optimal withdrawal and injection rates for each flow-rate decision variable Qw_n and a set of optimal flow rates for each external variable Ex_m. The second possible outcome is one in which there is no set of decision variables that simultaneously satisfies all constraints; in this case, the formulation is said to be infeasible. The third outcome is one in which the formulation is determined to be unbounded and the optimal objective function is either positive or negative infinity. The last possible outcome is one in which there are multiple optimal solutions to the formulation, in which the values of the decision variables will be different but the objective-function values will be identical. Ahlfeld and Mulligan (2000) describe some of the ways in which infeasible, unbounded, and multiple-optima problems can arise in ground-water management problems, as well as ways to prevent these outcomes.

One of the benefits of using the simplex method to solve a linear program is that specific sensitivity-analysis information can be determined easily by using the information in the final iteration of the algorithm. Such information includes the sensitivity of the optimal solution to the objective-function coefficients and the right-hand-side values of the constraints. Sensitivity analysis is implemented in GWM for linear problems and reported along with the optimal solution. See the "Output Files" section of this report for additional information about management-model results.

Nonlinear Formulations

Nonlinearities can arise in the ground-water management formulation (eqs. 42–59) indirectly as a result of two common characteristics of ground-water flow models. The first is the presence of layers in which transmissivity is a function of head (LAYCON equals 1 or 3). As noted previously (p. 4), water-table conditions cause a nonlinear relation between the position of the water table and withdrawal or injection stresses. The second characteristic is the presence of head-dependent boundary conditions such as streams, drains, evapotranspiration, and so forth. These boundary conditions can create nonlinear relations between ground-water heads and flow rates to or from the simulated boundary; an example of a nonlinear relation between ground-water heads and a simulated head-dependent boundary condition is illustrated in figure 1 for seepage between an aquifer and stream. Other GWF Process packages may also induce nonlinear responses.

The RMS Package provides a solution approach designed to address many of the nonlinear features that can arise in ground-water management problems. For purposes of describing this approach, a subset of the general management formulation will be considered. This formulation excludes binary variables and consists of the objective function described by equation 60 subject to constraints 43, 44, 47–55, and the streamflow and streamflow-depletion constraints described by equations 56–59.

The approach for solving nonlinear formulations is based on the sequential linear programming (SLP) algorithm and is referred to as the SLP approach. It is based on repeated linearization of the nonlinear features in the management problem, and is implemented by recalculating the response matrix for each sequential linear program. The first-order Taylor series expansion for head given by equation 61 is assumed to be accurate for each sequential linear program, but in contrast to the linear case, the vector of base flow rates changes at each iteration. The head at any location is estimated by

$$h_{i,j,k,t}(\boldsymbol{Qw}) = h^{v}_{i,j,k,t}(\boldsymbol{Qw}^{v}) + \sum_{n=1}^{N} \frac{\partial h^{v}_{i,j,k,t}}{\partial Qw^{v}_{n}}(\boldsymbol{Qw}^{v})\,(Qw_n - Qw^{v}_{n}), \qquad (67)$$

where the superscript v represents an iteration level, so that $h^{v}_{i,j,k,t}$ is the head obtained when the vector (set) of withdrawal and injection rates $\boldsymbol{Qw}^{v}$ is applied. For values of $\boldsymbol{Qw}$ that are close to $\boldsymbol{Qw}^{v}$, the error in this approximation is small, and the values of $h_{i,j,k,t}$ predicted by this equation are relatively accurate.

A first-order Taylor series expansion also is used to establish functional relations between streamflow and the managed withdrawal or injection stresses. These functional relations, which were described in a general form by equation 4, are necessary to rewrite the streamflow constraints (eqs. 56 and 57) in terms of the flow-rate decision variables. The first-order Taylor series approximation of streamflow for each sequential linear program is

$$Qsf_r(\boldsymbol{Qw}) = Qsf_r^v(\boldsymbol{Qw}^v) + \sum_{n=1}^{N} \frac{\partial Qsf_r^v}{\partial Qw_n^v}(\boldsymbol{Qw}^v)\,(Qw_n - Qw_n^v), \tag{68}$$

where

$Qsf_r(\boldsymbol{Qw})$ is streamflow at the *r*th stream reach and stress period for a new vector (that is, a new set) of withdrawal and injection flow rates, $\boldsymbol{Qw}$, having individual elements Qw_n;

$Qsf_r^v(\boldsymbol{Qw}^v)$ is streamflow at the *r*th stream reach and stress period for the vector of withdrawal and injection flow rates at iteration level v, $\boldsymbol{Qw}^v$, having individual elements Qw_n^v;

$\frac{\partial Qsf_r^v}{\partial Qw_n^v}(\boldsymbol{Qw}^v)$ is the change in streamflow at the *r*th stream reach and stress period for a change in withdrawal or injection flow rate for the *n*th flow-rate decision variable, evaluated for the set of withdrawal and injection rates $\boldsymbol{Qw}^v$; and

N is the total number of flow-rate decision variables.

The partial derivatives in equation 68, ($\partial Qsf_r^v/\partial Qw_n^v$), are the response coefficients for the response of streamflow to stresses at each withdrawal or injection site. Equation 68 is substituted for each streamflow term in constraints 56 and 57; similar substitutions are made for the streamflow-depletion constraints described by equations 58 and 59.

At each iteration of the SLP algorithm, a linear program is constructed on the basis of the first-order Taylor series approximation and the associated response coefficients, and the formulation is solved by using the simplex method described previously. Because the head and streamflow responses may be nonlinear, the response coefficients in equations 67 and 68, $\partial h_{i,j,k,t}^v/\partial Qw_n^v$ and $\partial Qsf_r^v/\partial Qw_n^v$, may no longer be constant. Therefore, the response coefficients for heads and streamflows must be recalculated at each iteration v. This calculation uses a new vector of base-condition flow rates that is derived from the optimal flow rates obtained from the linear-program solution from the prior iteration.

The sequential process is continued until two convergence criteria are met. The first requires that the change in flow-rate variable values from the prior iteration to the current iteration be less than a fraction of the magnitude of the flow-rate variables at the current iteration:

$$\left\| \boldsymbol{Qw}^{v+1} - \boldsymbol{Qw}^v \right\| \le \varepsilon_1 (1 + \left\| \boldsymbol{Qw}^{v+1} \right\|) \;. \tag{69A}$$

The infinity norm is used in this expression and ε_1 is specified by the user as input variable SLPVCRIT. Note that 1 is added to the norm on the right-hand side of equation 69*A*. While it is likely that in most cases the norm of decision variables will be much larger than 1, there may be cases where the norm of the decision variables is 0 or nearly 0. In these cases, the addition of 1 ensures that a reasonable convergence criterion is used. The input variable ε_1 can be considered a measure of the number of correct figures desired in the solution. For example, a value of 10^{-5} indicates that the solution will be correct to five significant digits or to the fifth decimal place, if the norm of the decision variables is less than 1 (Gill and others, 1981).

The second criterion requires that the change in objective function value, Z, be less than a specified fraction of the magnitude of the objective function value

$$|Z^{v+1} - Z^{v}| \le \varepsilon_2(1 + |Z^{v+1}|) . \qquad (69B)$$

The fraction, ε_2, is specified by the user as input variable SLPZCRIT.

The SLP algorithm can be summarized as:

Step 0: Set $v = 0$ and set $\boldsymbol{Qw}^0$;

Step 1: Compute response coefficients from base vector of rates $\boldsymbol{Qw}^v$;

Step 2: Assemble and solve the linear program; solution vector is assigned to $\boldsymbol{Qw}^{v+1}$;

Step 3: If convergence tests (eqs. 69*A* and 69*B*) are met, stop; else,

Step 4: Set $v = v + 1$ and go to Step 1.

Response coefficients are calculated for each iteration v by use of a forward-difference equation similar to that in equation 62. For heads, this equation is

$$\frac{\Delta h^v_{i,j,k,t}}{\Delta Qw^v_n} = \frac{h^v_{i,j,k,t}(\boldsymbol{Qw}^v_{\Delta n}) - h^v_{i,j,k,t}(\boldsymbol{Qw}^v)}{Qw^v_{\Delta n}} , \qquad (70)$$

where $Qw^v_{\Delta n}$ is the perturbation value for the *n*th flow-rate decision variable for iteration v, and $h_{i,j,k,t}(\boldsymbol{Qw}^v_{\Delta n})$ is the head at constraint location i, j, k and stress period t computed by using a vector of withdrawal and injection stress rates $\boldsymbol{Qw}^v_{\Delta n}$ that differs from the previous vector of stress rates $\boldsymbol{Qw}^v$ only in the *n*th element, which is changed by an amount $Qw^v_{\Delta n}$. An equation similar to 70 also is written for the streamflow-response coefficients $\partial Qsf^v_r / \partial Qw^v_n$.

In the first iteration ($v = 0$), response coefficients are calculated on the basis of a set of base-condition withdrawal and injection rates ($\boldsymbol{Qw}^0$) and initial perturbation values $(Qw_{\Delta n})_{initial}$ defined by the user in the same way as they are defined for linear formulations. Because of the nonlinearity of the problem and the need to solve the problem iteratively, the specified base-condition withdrawal and injection rates should be as close as possible to the expected optimal solution.

The same issues that affect the accuracy and precision of the response coefficients for linear formulations also apply for nonlinear formulations. However, because of the nonlinearity of the systems discussed here, the user must be careful to choose initial perturbation values small enough to ensure that the response coefficients for heads and streamflows calculated by the forward-difference equations are accurate. If the perturbation value used is too large, then the derivative approximation may be poor, which can lead to convergence problems in the sequential linearization process. With small perturbation values, heads determined by the GWF Process must be calculated with high precision to ensure adequate precision in the response matrix.

Because of the importance of the perturbation value to the success of the SLP algorithm, several features are provided to adjust the perturbation value automatically during the course of the algorithm. At each iteration, the perturbation values are calculated by using

$$Qw_{\Delta n}^{v} = \delta^{v} Qw_{n}^{u}, \tag{71}$$

where δ^{v} is the perturbation variable for iteration v and Qw_{n}^{u} is the specified upper bound on the withdrawal or injection rate at the *n*th well site. The initial value of the perturbation variable is set with δ^{0}, as described for linear problems. Additional input variables are available to cause the perturbation variable to change in subsequent iterations. The following formula (Minihane, 2002) is used to compute the perturbation variable at each iteration:

$$\delta^{v} = \frac{\delta_{initial} - \delta_{minimum}}{(\delta_{scale})^{v}} + \delta_{minimum}, \tag{72}$$

where v is the iteration level, $\delta_{initial}$ is an initial perturbation variable for the *n*th well site, $\delta_{minimum}$ is the minimum perturbation variable for the *n*th well site, and δ_{scale} is a scaling factor that determines the rate of decrease in the perturbation variable. Note that the superscript on δ_{scale} indicates exponentiation, so that when δ_{scale} is set to a value larger than 1, the perturbation variable decreases as the sequential process proceeds and approaches the minimum value $\delta_{minimum}$.

During each iteration of the SLP algorithm, a response matrix is calculated. As described in the section “Calculation of Response Coefficients,” it is possible that a flow-process run may fail. If a run using the perturbed stress rates does fail, then the perturbation value is changed automatically during an SLP iteration. This has no effect on the perturbation value at the next SLP iteration. That is, at the beginning of each iteration, δ^{v} is calculated according to equations 71 and 72 regardless of the changes that may have been made to the perturbation value at the prior iteration.

At each iteration of the SLP algorithm, the response coefficients are calculated from a different set of base conditions. If the first base run was successful but a later base run fails, then it is possible to adjust the base vector of flow-rate variables rather than terminate the algorithm. This is done by use of a relaxation parameter α, which moves the base solution closer to the prior (successful) base solution. For each of the flow-rate decision variables, a temporary base value is calculated according to

$$\hat{Q}w_{n}^{v} = (1 - \alpha)\hat{Q}w_{n}^{v} + \alpha \hat{Q}w_{n}^{v-1}, \tag{73}$$

where $\hat{Q}w_{n}$ is the temporary value of the flow-rate decision variable. The temporary base solution, which consists of values for each flow-rate decision variable calculated according to equation 73, is used for a new attempt at a successful base run. If the flow-process run fails again, a new temporary base solution is determined with the prior temporary base solution substituted for $\hat{Q}w_{n}^{v}$ in equation 73. A sequence of applications of equation 73 results in a sequence of temporary base solutions that move closer to the prior base solution $\hat{Q}w_{n}^{v-1}$.

Mixed-Binary Linear Formulations

In GWM, mixed-binary linear formulations are those that include the binary variables I_l in a linear formulation. When binary variables are specified, the RMS Package uses a branch and bound solution method to solve the formulation. The branch and bound method solves a series of linear programs. In each linear program, the binary variables are forced to an assumed binary value (0 or 1) or are allowed to vary between zero and one (that is, the binary variables are relaxed). Comparisons among the solutions of each of these linear programs leads to a solution without the necessity of checking all possible combinations of binary-variable values (Nemhauser and Wolsey, 1988; Hillier and Lieberman, 2001). The simplex algorithm is used to solve each of the linear programs. Implementation of the branch and bound algorithm is described in Appendix 2.

The branch and bound algorithm is based on repeated solution of linear programs. The user must specify the maximum number of iterations, or linear-program solutions, allowed (input variable BBITMAX). This parameter defines the computer storage space allocated for storing information related to each linear-program solution. Because a linear program is solved at each iteration of the branch and bound algorithm, the user must also specify a value of LPITMAX for a mixed-binary linear formulation. Output describing the progress of the branch and bound algorithm and the results of each linear program solved can be obtained by using the BBITPRT input variable.

Input Instructions and Output Files

This section describes the input instructions and output files for GWM. Before solving an optimization problem with GWM, the user must have already developed a ground-water flow model of the study area based on the MODFLOW GWF Process. As described in the Introduction of this report, the GWM Process can only be used with the Global and GWF Processes; it cannot be used with the Ground-Water Transport, Observation, Sensitivity, or Parameter-Estimation Processes. If, however, the GWF Process input files contain parameters defined for the Parameter-Estimation Process and the input file for the Sensitivity Process is included in the name file, then the specified parameter values will be used in the GWF Process (see “Sample Problem 2: SEAWATER”). The Global Process controls overall program operation and sets up data structures that can be used by all MODFLOW processes. The only Global Process file that needs to be modified for a GWM run is the name file; the required modifications to the file are described in detail below.

Name File

The name file contains the names of most input and output files used by MODFLOW, and determines which MODFLOW program options are activated (Harbaugh and others, 2000). The name file is read on unit 99. The name file contains one record of information for each input and output file. (A record is a line in a file.) Each record consists of three variables, which are read in free format; the length of each record must be 199 characters or less. Comment records can be used in the name file and are indicated by the # character in column one; comment records can be placed anywhere in the name file. Any text characters can follow the # character. Comment records have no effect on the run; their purpose is to allow users to provide documentation about a particular run.

Each record has the following format:

Ftype	Nunit	Fname

Explanation of the variables:

Ftype—is the file type. Ftype may be entered in all uppercase, all lowercase, or any combination thereof.

Nunit—is the FORTRAN unit to be used when reading from or writing to the file. Any legal unit number on the computer being used can be specified except units 96–99. Also, the unit number for the file must be unique; that is, it cannot be equal to any of the unit numbers used for other files specified in the name file.

Fname—is the name of the file, which is a character value. Pathnames may be specified as part of Fname.

GWM uses both the GLOBAL and LIST output files (see further discussion in the "Output Files" subsection below); therefore, both of these file types must be specified in the name file. The GLOBAL record must be the first non-comment record in the name file, and the LIST record must be the second non-comment record. A record also must be added to specify that the GWM Process is active. The record must specify Ftype **GWM** (bold text indicates a MODFLOW keyword). The file identified in this record contains information needed for the GWM run.

Example input records for the GLOBAL, LIST, and GWM file types are:

GLOBAL	1	global.gwm
LIST	6	list.gwm
GWM	55	input.gwm

GWM Process Files

Input files for the GWM Process consist of the GWM file and several supporting files. The GWM file is used to activate the GWM Process and to identify the files that will be opened for the GWM run. Four types of information about the management problem are specified in the input files: the decision variables, objective function, constraints of the management problem, and the solution and output-control parameters. With the exception of the GWM file, each of the input files is read from FORTRAN unit 99; each file is opened on unit 99 just prior to reading input from the file and closed immediately after reading the input.

For the most part, the general structure of the input formats for the GWM Process files are consistent with other MODFLOW processes; users of GWM should review the input instructions for MODFLOW given in Harbaugh and others (2000). Input for each GWM

Process file is grouped by numbered items, and each item consists of input variables. The first item in each of the input files is Item 0 (#Text), which can be used for comment lines but is optional. Some items consist of several variables, and the item can be repeated multiple times. The input data for each item must start on a new record. Each record is limited to a length of 199 characters. An input variable may include a single value or multiple values. Variables are defined after all the items are listed.

Each input variable has a data type, which can be Real, Integer, or Character. Integers are whole numbers and must not include a decimal point or exponent. Real numbers can include a decimal point and an exponent; if no decimal point is included in the entered value, then the decimal point is assumed to be at the right side of the value. Any printable character is allowed for character variables. Unlike the GWF Process, variables used by GWM that start with the letters I–N are not necessarily integers and those that start with the letters A–H and O–Z are not necessarily real numbers. Data types are specified for each input variable.

Free formatting is used for GWM input. With free format, values are not required to occupy a fixed number of columns in a record. Each value can occupy one or more columns as required to represent it; however, the values must still be included in the prescribed order. One or more spaces, or a single comma optionally combined with spaces, must separate adjacent values. Also, a numeric value of zero must be explicitly represented with 0 and not by one or more spaces when free format is used.

Units of values used in the GWM Process should be consistent with the units used in the other MODFLOW data-input files.

Description of Selected Conventions, Options, and Variables in GWM

As an aid in the preparation of GWM input files, some of the conventions, options, and variables of GWM are described below. Program variables are shown in plain upper-case text and file names are shown in bold upper-case text.

Sign conventions for flow-rate and external decision variables and their coefficients: As described previously, all flow-rate and external decision variables in GWM are treated as positive values, whether they represent a withdrawal, injection, export, or import of water. A flow-rate decision variable is defined with input variable FTYPE in the Decision Variables (**DECVAR**) file: If FTYPE is W, the variable is used for withdrawal; if FTYPE is I, the variable is used for injection. Because the user specifies whether a variable is for withdrawal or injection, GWM will know how to treat the variable internally. Therefore, when specifying the minimum or maximum flow rate at a site (input variables FVMIN and FVMAX), as well as the reference flow rate for the variable (input variable FVREF), values greater than 0 should be used for both withdrawal and injection variables. Likewise, GWM will know whether an external decision variable is an import or export of water by the definition of input variable ETYPE in the **DECVAR** file: If ETYPE is IM, the variable is a source (import) of water; if ETYPE is EX, the variable is a sink (export) of water. Therefore, the user should specify values greater than 0 for the minimum (EVMIN) and maximum (EVMAX) flow rates for either type of external decision variable.

When specifying the objective-function coefficients for the decision variables with input variables FVOBJC, EVOBJC, and BVOBJC in the Objective Function (**OBJFNC**) file, the user must specify the sign of each coefficient to achieve the desired objective-function form. For example, if the objective is to maximize economic benefit of the water withdrawn from or imported to a basin, and if there is a benefit of $1 per unit of water withdrawn at a particular flow-rate site but a cost of $1 per unit of water imported from an external variable, then the user would specify a positive coefficient for the withdrawal site and a negative coefficient for the external variable. The same reasoning applies to linear-summation constraints specified in the **SUMCON** file. For example, if the formulation requires that the difference in volumetric flow rates between an import Im_1 and export Ex_1 be greater than or equal to a specified value b, then the user would define a constraint with positive coefficient on the import variable and a negative coefficient on the export variable, that is, $Im_1 - Ex_1 \geq b$.

Lower bounds on flow-rate and external decision variables: Two approaches are used to define lower bounds on flow-rate and external decision variables, depending on whether or not the decision variables have been associated with binary variables. If a flow-rate or external decision variable has been associated with a binary variable (that is, eqs. 12 and 13 apply), then nonzero lower bounds can be defined for the variable in the Decision-Variables Constraints (**VARCON**) file by using input variables FVMIN for flow-rate decision variables and EVMIN for external decision variables.

If a flow-rate or external decision variable has not been associated with a binary variable (that is, eqs. 10 and 11 apply), then a lower bound of 0 must be specified for the variable by use of FVMIN and EVMIN in the **VARCON** file. The user has the option, however, of specifying a nonzero lower bound for a flow-rate or external decision variable not associated with a binary variable by use of linear-summation constraints. In this case, the user would specify FVMIN or EVMIN equal to 0 and then define a linear-summation constraint in the **SUMCON** file. For example, a minimum withdrawal rate of 1,000 ft^3/d for hypothetical flow-rate decision variable Q1 could be specified in the **SUMCON** file by use of equation 15; specifically, $Q1 \geq 1000$.

Background stresses and reference flow rates: As described in the definition of flow-rate decision variables (p. 8), the user can specify withdrawal or injection flow rates at unmanaged wells in a GWM run. These flow rates are referred to as background stresses, and are specified by use of either the WEL or MNW Packages of the GWF Process of MODFLOW. Flow rates specified in these packages will not be modified during the GWM run.

Reference flow rates must be specified for each flow-rate decision variable when either drawdown constraints (eqs. 32 and 33) or streamflow-depletion constraints (eqs. 40 and 41) are used in a GWM run. Reference flow rates are specified by use of input variable FVREF in the Decision-Variables Constraints (**VARCON**) file. Note, however, that a user could implicitly define a reference flow rate as a background stress by use of an unmanaged well in either the WEL or MNW Packages. In this case, the user would set FVREF equal to 0 for each decision variable for which a reference flow rate has been defined in the WEL or MNW Packages.

Variables related to calculation of response coefficients: The user must specify a number of input variables that are used by GWM to calculate the response coefficients; most of these variables are specified in the Solution and Output-Control Parameters (**SOLN**) file. The first of these variables are the perturbation variables (δ^0 for linear formulations and $\delta_{initial}$, $\delta_{minimum}$, and δ_{scale} for nonlinear formulations). For linear formulations, the perturbation variable δ^0 in equation 63 must be specified with input variable DELTA. The perturbation variable DELTA can be a positive or negative value: a positive value implies an increase in flow rate (referred to as a forward-difference calculation), whereas a negative value implies a decrease in flow rate (referred to as a backward-difference calculation). Moreover, DELTA is not limited to values between +1.0 and -1.0. The user may need to experiment with different values of DELTA in a series of GWM runs to determine the value of DELTA that is most appropriate for a particular problem. Guidelines for the selection of DELTA are discussed in the "Accuracy and Precision of the Response Coefficients" section of this report.

If the user selects the SLP solution type in the **SOLN** file (that is, the optimization formulation is solved using sequential linear programming), then the three perturbation variables $\delta_{initial}$, $\delta_{minimum}$, and δ_{scale} in equation 72 must be specified with input variables DINIT, DMIN, and DSC, respectively. The selection of DINIT ($\delta_{initial}$) follows the same reasoning as the selection of DELTA in the linear case (see "Accuracy and Precision of the Response Coefficients" section of this report). DMIN ($\delta_{minimum}$) should be selected such that the final perturbation value is large enough to produce several significant digits in the response matrix, but small enough to secure an accurate approximation for the nonlinear response. An initial estimate for DMIN of 0.5 percent of the expected optimal solution value is suggested. The scaling factor DSC (δ_{scale}) is normally selected to be larger than 1.0 so that the perturbation parameter decreases with increasing iterations. A value of 5.0 has been shown to give good results (Minihane, 2002). As with the selection of DELTA, the user may need to experiment with different values of DINIT, DMIN, and DSC to determine the set of variables that works best for a particular GWM problem.

All solution types require specification of an upper bound on the withdrawal or injection rate at each flow-rate decision variable (variable Qw_n^u) for calculation of the perturbation values $Qw_{\Delta n}$ for linear formulations (eq. 63) or $Qw_{\Delta n}^v$ for nonlinear formulations (eq. 71). Upper bounds on flow-rate decision variables are specified with input variable FVMAX in the Decision-Variables Constraints (**VARCON**) file.

All solution types also require specification of the base-condition withdrawal or injection flow rates for the flow-rate decision variables that are used for calculation of the response coefficients (that is, $\boldsymbol{Qw}^0$). For convenience, two options are provided for specifying the base-condition flow rates. The particular option that is selected by the user is defined by use of input variable IBASE in the **SOLN** file: if IBASE is set to 0, then input variable FVREF in the **VARCON** file will be used to specify the base-condition flow rate for each decision variable; if IBASE is set to 1, then input variable FVBASE in the **SOLN** file will be used to specify the base-condition flow rate for each decision variable.

Testing of response-coefficient precision: Users of GWM should be aware of the precision of the response matrix. As an aid in determining the precision of the response matrix that is generated, GWM output includes the average number of significant digits in the computed response matrix. The number of significant digits in a response coefficient is estimated by dividing the change in system state caused by the perturbation by the head convergence criterion used by the flow process. The number of digits to the left of the decimal point in the resulting ratio is used as a measure of the number of significant digits. This ratio is computed by using the GWF Process variable HCLOSE to represent the head-convergence criterion. For this ratio to be meaningful, the value of HCLOSE must be properly specified. When using either the Preconditioned Conjugate-Gradient (PCG), Direct Solver (DE4), or other flow-process solution packages that have multiple convergence criteria, the user should ensure that HCLOSE is assigned a meaningful value.

Because heads will tend to respond very little to pumping from flow-rate variables that are far away, the corresponding response coefficients will be small and may have poor precision. As long as other flow-rate variables produce significant response with adequate precision, however, the response matrix should be sufficiently precise to achieve a solution. The RMS Package tests each column of the response matrix as it is computed. At least one response coefficient in the column must have sufficient precision to predict a meaningful response. In other words, each flow-rate variable must have a numerically significant effect on at least one constraint. The test is conducted by requiring that the number of significant digits in the ratio of system state to HCLOSE be greater than or equal to the user-defined variable NSIGDIG for at least one entry in the column. If a column fails this test, then GWM can automatically recalculate the response-matrix column with a new perturbation value.

Correction of a failed GWF Process run: The GWF process may fail during a base-condition run or during one of the perturbation runs. This can occur for several reasons, such as the GWF Process solution algorithm fails to converge, a cell included in a constraint dewaters, or GWM detects that the simulated perturbed state value is inadequate or imprecise. If the flow process fails during a base-condition run for solution types NS, MPS, or LP, then the program will terminate; this will occur if the base values of the flow-rate decision variables supplied by the user do not produce a successful run. In this case, the user must modify the base-condition stresses and (or) GWF solution parameters to improve model convergence and (or) stability. For solution type SLP, however, the response coefficients are calculated from a different set of base-condition withdrawal or injection flow rates at each iteration of the SLP algorithm. As described in the section on solution of nonlinear formulations, if the first base-condition run was successful but a later base run fails, it is possible to adjust the base vector of flow-rate decision variables (rather than terminate the algorithm) by use of a relaxation parameter α (eq. 73), which is specified with input variable AFACT in the **SOLN** file. AFACT is restricted to a value of between 0 and 1; a value of 0.5 is suggested.

The flow process also may fail during a perturbation run. This might occur because of a poor choice for the perturbation value calculated in equations 63 or 71. In this case, GWM can automatically adjust the perturbation value and re-execute the flow process. Two user-specified input variables control adjustment of the perturbation value if the flow process fails during a perturbation run. The first parameter, PGFACT, is the factor by which the perturbation value will be adjusted; PGFACT is restricted to values between 0 and 1. Depending on the type of failure, GWM will either increase or decrease the perturbation value. A suggested value of PGFACT is 0.5. Use of this value will result in a doubling or halving of the perturbation value at each subsequent perturbation attempt. The perturbation value for each flow-rate decision variable is adjusted independently, so that any adjustments made for one decision variable do not affect the perturbation value for other variables. The second parameter, NPGNMX, is the maximum number of perturbation-adjustment attempts that will be made for a given flow-rate decision variable. If this value is exceeded for any of the decision variables, then the algorithm will terminate.

For nonlinear formulations in which the SLP algorithm is used, the simplex algorithm may determine that the linear program is infeasible or unbounded at any of the SLP iterations. This does not necessarily mean, however, that the original problem is either infeasible or unbounded. Rather, it may be that the particular set of response coefficients computed at that iteration produces an infeasible or unbounded constraint set. When a linear program fails during an SLP iteration, the RMS Package repeats the iteration, calculating the response coefficients with a different perturbation value. This is repeated up to NINFMX times, where NINFMX is user-specified. If no solution to each of the NINFMX linear programs can be found, then the RMS Package concludes that the original problem is infeasible or unbounded and halts the algorithm.

GWM File

The following items are read for each GWM run. The items must be listed in the order shown below. Values in bold are keywords that can be specified in either uppercase or lowercase letters. Input instructions for the files identified after each keyword are described in following subsections. Each GWM run requires specification of a **DECVAR**, **OBJFNC**, **VARCON**, and **SOLN** file type; the **SUMCON**, **HEDCON**, and **STRMCON** file types are optional. GWM automatically selects file units, so the user does not specify file units for these files.

Input items:

0. #Text
1. Each management problem must include a file that provides information about the decision variables and is read from the file whose name is specified by Fname:

   DECVAR Fname

2. Each management problem must include a file that provides information about the objective function and is read from the file whose name is specified by Fname:

   OBJFNC Fname

3. Each management problem must include a file that provides information on the lower and upper bounds specified for the flow-rate and external decision variables and is read from the file whose name is specified by Fname:

   VARCON Fname

4. Each management problem may also include up to three additional files that provide information about the other types of constraints of the management problem that are allowed in GWM (summation constraints, head constraints, and streamflow constraints). These files are specified by the following records, which can be listed in any order:

   SUMCON Fname

   HEDCON Fname

   STRMCON Fname

5. Each management problem must include a file that provides information about the solution and output-control parameters and is read from the file whose name is specified by Fname:

   SOLN Fname

Explanation of variables:

Text—is a character variable (199 characters) that starts in column 2. Any characters can be included in Text. The “#” character must be in column 1. Lines beginning with “#” are restricted to these first lines of the file. Text is printed when the file is read. Item 0 can be repeated multiple times.

Fname—is a character variable that specifies the name of an existing file. Pathnames may be specified as part of Fname.

Decision Variables (**DECVAR**) File

This file is used to define the decision variables of the management model. There are three types of decision variables. The primary decision variables are the flow rates (either withdrawal or injection) at each managed well site. A single well site may have more than one flow-rate decision variable associated with it; moreover, a single flow-rate decision variable can extend over one or more model cells and can be active during one or more stress periods. The second types of decision variables are external flow rates representing imported or exported water to the model domain. External variables do not have a direct effect on the system state variables and are not assigned to a specific location in the model. The third types of decision variables are binary variables used to define the status of each flow site or external variable as active (for example, the site is constructed) or inactive (the site is not constructed). Binary variables have a value of 0 (inactive site) or 1 (active site). One or more flow-rate and external decision variables are associated with each binary variable.

Input items:

0. #Text
1. IPRN
2. NFVAR NEVAR NBVAR

3a. The following records are read for each of NFVAR decision variables:

FVNAME NC LAY ROW COL FTYPE FSTAT WSP

3b. If NC > 1 in record 3a, then the following record is read NC times and the values of LAY, ROW, and COL read in record 3a are ignored:

RATIO LAY ROW COL

4. The following record is read for each of NEVAR external decision variables:

EVNAME ETYPE ESP

5. The following record is read for each of NBVAR binary decision variables:

BVNAME NDV BVLIST

Explanation of the variables:

Text—is a character variable (199 characters) that starts in column 2. Any characters can be included in Text. The "#" character must be in column 1. Lines beginning with "#" are restricted to these first lines of the file. Text is printed when the file is read. Item 0 can be repeated multiple times.

IPRN—is an integer variable that describes the type of input echo that is written to the GLOBAL file. IPRN must be specified as either 0 or 1. When IPRN equals 0, a minimum amount of information about the decision variables is written to the GLOBAL output file; when IPRN equals 1, detailed information about the decision variables is written to the GLOBAL output file.

NFVAR—is an integer variable equal to the number of flow-rate decision variables. NFVAR must be greater than 0. Only one flow-rate decision variable can be defined for a particular well site and set of stress periods, with the exception that both a withdrawal variable (FTYPE=W) and an injection variable (FTYPE=I) can be defined for the site.

NEVAR—is an integer variable equal to the number of external decision variables. NEVAR must be greater than or equal to 0.

NBVAR—is an integer variable equal to the number of binary variables. NBVAR must be greater than or equal to 0. If NBVAR is 0, binary variables are not included in the management formulation.

FVNAME—is a character variable (maximum length of 10 characters) that is a unique name designated for the flow-rate decision variable. No spaces are allowed in the name. The end of the name is designated by a blank space.

NC—is an integer variable equal to the number of model cells over which the flow rate for decision-variable FVNAME is distributed. NC must be greater than or equal to 1. If NC equals 1, then all of the water withdrawn or injected at decision variable FVNAME is applied at the single model cell LAY, ROW, COL. If NC is greater than 1, then the flow rate calculated for decision variable FVNAME is distributed over the NC cells specified in record 3b.

LAY, ROW, COL—are integer variables equal to the layer, row, and column number of the model cell to which flow for decision-variable FVNAME will be assigned.

FTYPE—is a character variable that indicates whether the decision variable is a withdrawal or injection site. If FTYPE is W, the site is used for withdrawal; if FTYPE is I, the site is used for injection. If either withdrawal or injection is allowed at the site, two decision variables must be defined for the site, one for withdrawal (that is, with FTYPE = W) and one for injection (FTYPE = I).

FSTAT—is a character variable that indicates whether the decision variable will be considered in the management problem. If FSTAT is Y, the decision variable is available; if FSTAT is N, the decision variable is unavailable. If the decision variable is unavailable, then no withdrawal or injection will be calculated at the decision-variable location. For linear-optimization problems, FSTAT can be used to remove a well from the candidate set of decision variables without having to recalculate the response matrix (in this case, IRM = 0; see instructions for the Solution and Output-Control Parameters File below).

WSP—is a character string (up to 120 characters long) that indicates the stress periods associated with decision variable FVNAME. A single flow rate will be determined by GWM for all the stress periods included in WSP. The string must not contain any blank spaces. Multiple stress periods are listed using colons (:) or hyphens (-). For example,

1 indicates that stress period 1 is the only stress period associated with the decision variable;

1:3 indicates that the flow rate is the same for stress periods 1 and 3; and

1–12 indicates that the flow rate is the same for stress periods 1 through 12.

RATIO—is a real variable. RATIO is the fraction of the total flow rate for decision variable FVNAME that is distributed to cell LAY, ROW, COL. The sum of RATIO must equal 1.0 for all of the NC cells specified for FVNAME; if the sum does not equal 1.0, GWM calculates the fraction for each cell by dividing the RATIO value specified for each cell by the sum of the RATIO values specified for all cells within FVNAME.

EVNAME—is a character variable (maximum length of 10 characters) that is a unique name designated for the external decision variable. No spaces are allowed in the name. The end of the name is designated by a blank space.

ETYPE—is a character variable that indicates whether the external variable is a source (import) or sink (export) of water. If ETYPE is IM, the variable is a source (import) of water; if ETYPE is EX, the variable is a sink (export) of water. Both types of external variables can be used in a management problem.

ESP—is a character string (up to 120 characters long) that indicates the stress periods associated with external variable EVNAME. A single flow rate will be determined by GWM for all the stress periods included in ESP. The string must not contain any blank spaces. Multiple stress periods are listed using colons (:) or hyphens (-). For example,

1 indicates that stress period 1 is the only stress period associated with the decision variable;

1:3 indicates that the flow rate for the external variable is the same for stress periods 1 and 3; and

1-12 indicates that the flow rate for the external variable is the same for stress periods 1 through 12.

BVNAME—is a character variable (maximum length of 10 characters) that is a unique name designated for the binary decision variable. The use of BVNAME, NDV, and BVLIST allows the user to associate one or more FVNAME or EVNAME decision variables with a single binary-variable identifier. For example, the user may want to define 12 decision variables that are the monthly withdrawal rates at a single well site. If any one of the 12 decision variables is selected in the optimal solution, then an installation cost associated with the binary variable for the well site must be incurred. Also see example 3 in Appendix 1.

NDV—is an integer variable equal to the number of flow-rate or external decision variables associated with BVNAME.

BVLIST—is a list of the flow-rate and external decision variables associated with binary variable BVNAME. The list is drawn from the character names of these variables, FVNAME and EVNAME, defined in records 3a and 4. Each character variable in the list must be separated by a space, and there must be a total of NDV variables listed. The list can include any combination of decision variables, irrespective of well-site locations or stress period.

Objective Function (**OBJFNC**) File

This file is used to define the type of objective function that is to be solved and the coefficients for each decision variable in the objective function. Note that it is not necessary to include all decision variables in the objective function. In other words, some management formulations will have objective functions that do not include all of the decision variables defined for the problem.

Input items:

0. #Text
1. IPRN
2. OBJTYP FNTYP
3. NFVOBJ NEVOBJ NBVOBJ
4. The following record is repeated for each of NFVOBJ flow-rate decision variables:
   FVNAME FVOBJC
5. The following record is repeated for each of NEVOBJ external decision variables:
   EVNAME EVOBJC
6. The following record is repeated for each of NBVOBJ binary decision variables:
   BVNAME BVOBJC

Explanation of the variables:

Text—is a character variable (199 characters) that starts in column 2. Any characters can be included in Text. The “#” character must be in column 1. Lines beginning with “#” are restricted to these first lines of the file. Text is printed when the file is read. Item 0 can be repeated multiple times.

IPRN—is an integer variable that describes the type of input echo that is written to the GLOBAL file. IPRN must be specified as either 0 or 1. When IPRN equals 0, a minimum amount of information about the objective function is written to the GLOBAL output file; when IPRN equals 1, detailed information about the objective function is written to the GLOBAL output file.

OBJTYP—is a character variable used to define whether the objective is to maximize or minimize the objective function. OBJTYP must be defined as either MIN (for minimize) or MAX (for maximize).

FNTYP—is a character variable used to define the type of objective function. Currently, only one type of function is available, which is WSDV for weighted sum of decision variables. The user must specify WSDV on the input record.

NFVOBJ—is an integer variable equal to the number of flow-rate decision variables in the objective function and must have a value less than or equal to NFVAR specified in the decision-variables file.

NEVOBJ—is an integer variable equal to the number of external decision variables in the objective function and must have a value less than or equal to NEVAR specified in the decision-variables file.

NBVOBJ—is an integer variable equal to the number of binary decision variables in the objective function and must have a value less than or equal to NBVAR specified in the decision-variables file.

FVNAME—is a character variable (maximum length of 10 characters) that is one of the flow-rate decision-variable names. Each of the FVNAME variables listed must be defined in the DECVAR file. A flow-rate decision-variable name can only be listed once in the OBJFNC file.

FVOBJC—is a real variable that is a coefficient on each flow-rate decision variable FVNAME. For example, FVOBJC could represent the cost per unit volume of water withdrawn or injected at the management site.

EVNAME—is a character variable (maximum length of 10 characters) that is one of the external decision-variable names. Each of the EVNAME variables listed must be defined in the DECVAR file. An external decision-variable name can only be listed once in the OBJFNC file.

EVOBJC—is a real variable that is a coefficient on each external decision variable EVNAME. For example, EVOBJC could represent the cost per unit volume of water associated with the external variable.

BVNAME—is a character variable (maximum length of 10 characters) that is one of the binary-variable names. Each of the BVNAME variables listed must be defined in the DECVAR file. A binary-variable name can only be listed once in the OBJFNC file.

BVOBJC—is a real variable that is a coefficient on each binary variable BVNAME. For example, BVOBJC could represent the cost for installation of the management site. In typical usage of binary variables, the coefficients will be positive when OBJTYP is MIN and negative when OBJTYP is MAX. This will ensure that the binary variables are only active when their associated flow-rate and external decision variables are active.

Constraint Files

Four general types of constraints can be specified in a GWM management problem: constraints on the lower and upper bounds on the decision variables themselves by use of the **VARCON** file; linear-summation constraints by use of the **SUMCON** file; hydraulic-head constraints by use of the **HEDCON** file; and streamflow constraints by use of the **STRMCON** file. A **VARCON** constraint file must be specified for each management problem, but the last three constraint type files are optional. Each of these four types of constraints is described below.

Decision-variable constraints (**VARCON**) file

The decision-variable constraints file is used to define lower and upper bounds for the flow-rate and external decision variables, and the reference flow rates to be used in the first ground-water flow run conducted by GWM. Records must be specified for all NFVAR and NEVAR decision variables defined in the DECVAR file.

Input items:

0. #Text
1. IPRN
2. The following record is read for each of NFVAR decision variables:

   FVNAME FVMIN FVMAX FVREF
3. The following record is read for each of NEVAR decision variables:

   EVNAME EVMIN EVMAX

Explanation of the variables:

Text—is a character variable (199 characters) that starts in column 2. Any characters can be included in Text. The “#” character must be in column 1. Lines beginning with “#” are restricted to these first lines of the file. Text is printed when the file is read. Item 0 can be repeated multiple times.

IPRN—is an integer variable that describes the type of input echo that is written to the GLOBAL file. IPRN must be specified as either 0 or 1. When IPRN equals 0, a minimum amount of information about the decision-variable constraints is written to the GLOBAL output file; when IPRN equals 1, detailed information about the decision-variable constraints is written to the GLOBAL output file.

FVNAME—is a character variable (maximum length of 10 characters) that is one of the flow-rate decision-variable names. Each of the FVNAME variables listed must be defined in the DECVAR file. A flow-rate decision-variable name can only be listed once in the VARCON file.

FVMIN, FVMAX—are real variables that are equal to the minimum (FVMIN) and maximum (FVMAX) flow rate allowed for the decision variable. Values greater than or equal to 0 must be specified for FVMIN and FVMAX; GWM will know whether the pumping rates are withdrawal or injection rates from the specification of FTYPE given in the Decision Variable File. FVMIN must be less than or equal to FVMAX. Note that a nonzero value of FVMIN implies that the decision variable has been associated with a binary variable in the DECVAR file. If the decision variable is not associated with a binary variable, then the nonzero value of FVMIN is ignored by GWM. The user can specify a nonzero lower bound for a flow-rate decision variable not associated with a binary variable by use of a linear-summation constraint (see description of **SUMCON** File).

FVREF—is a real variable equal to the flow rate for the decision variable that is used by GWM to calculate the reference values of the state variables. These include heads at drawdown-constraint locations if drawdown constraints are used (see input instructions for **HEDCON** file) and reference streamflows at streamflow-constraint locations if streamflow-depletion constraints are used (see input instructions for **STRMCON** file). FVREF also may be used to calculate base conditions for the calculation of the response matrix (see discussion of variable IBASE in the **SOLN** file). If no value is entered for FVREF, it is assigned a value of 0.

EVNAME—is a character variable (maximum length of 10 characters) that is one of the external decision-variable names. Each of the EVNAME variables listed must be defined in the DECVAR file. An external decision-variable name can only be listed once in the VARCON file.

EVMIN, EVMAX—are real variables that are equal to the minimum (EVMIN) and maximum (EVMAX) flow rate allowed for the external decision variable. Values greater than or equal to 0 must be specified for EVMIN and EVMAX; GWM will know whether the flow rates for the external variable are imported or exported flow rates from the specification of ETYPE given in the Decision Variable File. EVMIN must be less than or equal to EVMAX. Note that a nonzero value of EVMIN implies that the decision variable has been associated with a binary variable in the DECVAR file. If the decision variable is not associated with a binary variable, then the nonzero value of EVMIN is ignored by GWM. The user can specify a nonzero lower bound for an external decision variable not associated with a binary variable by use of a summation constraint (see description of **SUMCON** File).

Linear-summation constraints (**SUMCON**) file

The linear-summation constraints file is used to define linear relations among decision variables. Examples of the use of these constraints are given by equations 17–28 in the “Formulation of Ground-Water Management Problems” section of the report, and in the sample problems.

Input items:

0. #Text
1. IPRN
2. SMCNUM

3a. Records 3a and 3b are read for each of the SMCNUM constraints:

SMCNAME NTERMS TYPE RHS

3b. The following record is repeated NTERMS times for each of the NTERMS specified in record 3a:

GVNAME GVCOEFF

Explanation of the variables:

Text—is a character variable (199 characters) that starts in column 2. Any characters can be included in Text. The "#" character must be in column 1. Lines beginning with "#" are restricted to these first lines of the file. Text is printed when the file is read. Item 0 can be repeated multiple times.

IPRN—is an integer variable that describes the type of input echo that is written to the GLOBAL file. IPRN must be specified as either 0 or 1. When IPRN equals 0, a minimum amount of information about the summation constraints is written to the GLOBAL output file; when IPRN equals 1, detailed information about the summation constraints is written to the GLOBAL output file.

SMCNUM—is an integer variable equal to the number of summation constraints defined in the file.

SMCNAME—is a character variable (maximum length of 10 characters) that is a unique name designated for the constraint. No spaces are allowed in the name. The end of the name is designated by a blank space.

NTERMS—is an integer variable equal to the number of terms on the left-hand side of the constraint. All of the terms are added together to form the left-hand side of the constraint.

TYPE—is a character variable used to specify the type of constraint. Three options are allowed:

LE: the left-hand side of the equation is less than or equal to the right-hand side of the constraint;

GE: the left-hand side of the equation is greater than or equal to the right-hand side of the constraint;

EQ: the left-hand and right-hand sides of the constraint are equal.

RHS—is a real variable equal to the value of the right-hand side of the constraint.

GVNAME—is a character variable (maximum length of 10 characters) that is one of the decision-variable names defined in the DECVAR file for either an FVNAME, EVNAME, or BVNAME variable. Any combination of flow-rate, external, and binary decision variables may be present in a constraint. The user must ensure that the variables used are logically consistent.

GVCOEFF—is a real variable equal to the value of the coefficient in front of variable GVNAME. The user must ensure that a consistent set of units is used for all GVCOEFF and RHS terms.

Head constraints (**HEDCON**) file

The head constraints file is used to define head constraints at model cells. These include upper and lower bounds on heads, drawdowns, head differences between two cells, and gradients between two cells.

Input items:

0. #Text
1. IPRN
2. NHB NDD NDF NGD
3. The following record is read for each of the NHB constraints:
   HBNAME LAYH ROWH COLH TYPH BND NSP
4. The following record is read for each of the NDD constraints:
   DDNAME LAYD ROWD COLD TYPD BND NSP
5. The following record is read for each of the NDF constraints:
   HDIFNAME LAY1 ROW1 COL1 LAY2 ROW2 COL2 HD NSP
6. The following record is read for each of the NGD constraints:
   GRADNAME LAY1 ROW1 COL1 LAY2 ROW2 COL2 LEN GRAD NSP

Explanation of the variables:

Text—is a character variable (199 characters) that starts in column 2. Any characters can be included in Text. The “#” character must be in column 1. Lines beginning with “#” are restricted to these first lines of the file. Text is printed when the file is read. Item 0 can be repeated multiple times.

IPRN—is an integer variable that describes the type of input echo that is written to the GLOBAL file. IPRN must be specified as either 0 or 1. When IPRN equals 0, a minimum amount of information about the head constraints is written to the GLOBAL output file; when IPRN equals 1, detailed information about the head constraints is written to the GLOBAL output file.

NHB—is an integer variable equal to the number of head-bound constraints that need to be satisfied in the management model.

NDD—is an integer variable equal to the number of drawdown constraints that need to be satisfied in the management model.

NDF—is an integer variable equal to the number of head difference constraints that need to be satisfied in the management model.

NGD—is an integer variable equal to the number of gradient constraints that need to be satisfied in the management model.

Head-bound constraints:

HBNAME—is a character variable (maximum length of 10 characters) that is a unique name designated for the head-bound constraint. No spaces are allowed in the name. The end of the name is designated by a blank space.

LAYH, ROWH, COLH—are integer variables equal to the layer, row, and column number of the model cell in which the head-bound constraint is located.

TYPH—is a character variable used to specify the type of head bound. Two options are allowed:

LE: head calculated by the model must be less than or equal to the value specified by BND;

GE: head calculated by the model must be greater than or equal to the value specified by BND.

BND—is a real variable equal to the specified upper ($h^u_{i,j,k,t}$) or lower ($h^l_{i,j,k,t}$) bound on head at the model cell at the end of the stress period (fig. 2*A*).

NSP—is an integer variable that indicates the stress period during which the constraint is imposed. If the constraint is imposed over multiple stress periods, then a separate record must be provided for each stress period.

Drawdown constraints:

DDNAME—is a character variable (maximum length of 10 characters) that is a unique name designated for the drawdown constraint. No spaces are allowed in the name. The end of the name is designated by a blank space.

LAYD, ROWD, COLD—are integer variables equal to the layer, row, and column number of the model cell in which the drawdown constraint is located.

TYPD—is a character variable used to specify the type of head-drawdown bound. Two options are allowed:

LE: drawdown calculated by the model at the model cell must be less than or equal to the value specified by BND;

GE: drawdown calculated by the model must be greater than or equal to the value specified by BND.

BND—is a real variable equal to the specified upper ($dd^u_{i,j,k,t}$) or lower ($dd^l_{i,j,k,t}$) bound on drawdown at the model cell at the end of the stress period (fig. 2*B*).

NSP—as defined for Record 3.

Head-difference constraints:

HDIFNAME—is a character variable (maximum length of 10 characters) that is a unique name designated for the head-difference constraint. No spaces are allowed in the name. The end of the name is designated by a blank space.

LAY1, ROW1, COL1—are integer variables equal to the layer, row, and column number of the model cell corresponding to the first location, $(i, j, k)_1$, in the head-difference constraint (fig. 2C).

LAY2, ROW2, COL2—are integer variables equal to the layer, row, and column number of the model cell corresponding to the second location, $(i, j, k)_2$, in the head-difference constraint (fig. 2*C*). GWM requires that the head at the second head-difference location be lower than the head at the first location by an amount of at least HD.

HD—is a real variable equal to the specified difference in heads, $(hdif_{i,j,k,t})_{1,2}$, between the first and second model cells at the end of the stress period (fig. 2*C*).

NSP—as defined for Record 3.

Gradient constraints:

GRADNAME—is a character variable (maximum length of 10 characters) that is a unique name designated for the gradient constraint. No spaces are allowed in the name. The end of the name is designated by a blank space.

LAY1, ROW1, COL1—are integer variables equal to the layer, row, and column number of the model cell corresponding to the first location, $(i, j, k)_1$, in the gradient constraint (fig. 2*D*).

LAY2, ROW2, COL2—are integer variables equal to the layer, row, and column number of the model cell corresponding to the second location, $(i, j, k)_2$, in the gradient constraint (fig. 2*D*). GWM requires that the head at the second head-difference location be lower than the head at the first location.

LEN—is a real variable equal to the distance between the first and second model cells (Δx shown in fig. 2*D*).

GRAD—is a real variable equal to the specified gradient, $(grad_{i,j,k,t})_{1,2}$, between the first and second model cells at the end of the stress period (fig. 2*D*).

NSP—as defined for Record 3.

Streamflow constraints (**STRMCON**) file

The streamflow constraints file is used to define streamflow constraints when the STR1 Package of MODFLOW (Prudic, 1989) is used. Two types of streamflow constraints are allowed—constraints on the upper and lower bounds on streamflow and constraints on the upper and lower bounds on streamflow depletion.

Input items:

0. #Text
1. IPRN
2. NSF NSD
3. The following record is read for each of the NSF constraints:

   SFNAME SEG REACH TYPSF BND NSP
4. The following record is read for each of the NSD constraints:

   SDNAME SEG REACH TYPSD BND NSP

Explanation of the variables:

Text—is a character variable (199 characters) that starts in column 2. Any characters can be included in Text. The “#” character must be in column 1. Lines beginning with “#” are restricted to these first lines of the file. Text is printed when the file is read. Item 0 can be repeated multiple times.

IPRN—is an integer variable that describes the type of input echo that is written to the GLOBAL file. IPRN must be specified as either 0 or 1. When IPRN equals 0, a minimum amount of information about the streamflow constraints is written to the GLOBAL output file; when IPRN equals 1, detailed information about the streamflow constraints is written to the GLOBAL output file.

NSF—is an integer variable equal to the number of streamflow constraints that need to be satisfied in the management model.

NSD—is an integer variable equal to the number of streamflow-depletion constraints that need to be satisfied in the management model.

Streamflow constraints:

SFNAME—is a character variable (maximum length of 10 characters) that is a unique name designated for the streamflow constraint. No spaces are allowed in the name. The end of the name is designated by a blank space.

SEG, REACH—are integer variables equal to the segment and reach number, as specified in the STR1 Package, of the model cell in which the streamflow constraint is located (Prudic, 1989).

TYPSF—is a character variable used to specify the type of streamflow constraint. Two options are allowed:

LE: streamflow at the stream site must be less than or equal to the value specified by BND;

GE: streamflow at the stream site must be greater than or equal to the value specified by BND.

BND—is a real variable equal to the specified amount of streamflow allowed at the stream site at the end of the stress period.

NSP—is an integer variable that indicates the stress period during which the constraint is imposed. To impose the constraint for multiple stress periods, define additional constraints.

Streamflow-depletion constraints:

SDNAME—is a character variable (maximum length of 10 characters) that is a unique name designated for the streamflow-depletion constraint. No spaces are allowed in the name. The end of the name is designated by a blank space.

SEG, REACH—are integer variables equal to the segment and reach number of the model cell in which the streamflow-depletion constraint is located (Prudic, 1989).

TYPSD—is a character variable used to specify the type of streamflow-depletion constraint. Two options are allowed:

LE: streamflow depletion at the stream site must be less than or equal to the value specified by BND;

GE: streamflow depletion at the stream site must be greater than or equal to the value specified by BND.

BND—is a real variable equal to the specified amount of streamflow depletion allowed at the stream site at the end of the stress period.

NSP—as defined for Record 3.

Solution and Output-Control Parameters (**SOLN**) File

The solution and output-control parameters file is used to define several variables that control the solution algorithm for the optimization problem and the type and amount of output that is printed to the output files.

Input items:

0.	#Text				
1.	SOLNTYP				

If SOLNTYP is NS then:

2a.	DELTA		
2b.	NSIGDIG	NPGNMX	PGFACT
2c.	RMNAME		

Skip to record 6a.

If SOLNTYP is MPS then:

3a.	DELTA		
3b.	NSIGDIG	NPGNMX	PGFACT
3c.	MPSNAME		

Skip to record 6a.

If SOLNTYP is LP then:

4a.	IRM		
4b.	LPITMAX	BBITMAX	
4c.	DELTA		
4d.	NSIGDIG	NPGNMX	PGFACT
4e.	BBITPRT	RANGE	

The following record is read if IRM equals 0 or 1:

4f. RMNAME

Skip to record 6a.

If SOLNTYP is SLP then:

5a.	SLPITMAX	LPITMAX	BBITMAX		
5b.	SLPVCRIT	SLPZCRIT	DINIT	DMIN	DSC
5c.	NSIGDIG	NPGNMX	PGFACT	AFACT	NINFMX
5d.	SLPITPRT	BBITPRT	RANGE		
6a.	IBASE				

If IBASE equals 1, the following record is read for each of NFVAR decision variables:

6b. FVNAME FVBASE

Explanation of the variables:

Text—is a character variable (199 characters) that starts in column 2. Any characters can be included in Text. The "#" character must be in column 1. Lines beginning with "#" are restricted to these first lines of the file. Text is printed when the file is read. Item 0 can be repeated multiple times.

SOLNTYP—is a character variable. SOLNTYP must be specified as either NS, MPS, LP, or SLP:

NS: no solution to the management formulation will be found. GWM will calculate the response matrix and write it to the response-matrix file specified by RMNAME in record 2c; GWM will then stop.

MPS: no solution to the management formulation will be found. GWM will write the management formulation in MPS (Mathematical Programming System) format to file specified by MPSNAME in record 3c; GWM will then stop.

LP (with or without binary variables): the optimization formulation is solved by using linear programming, or, for problems with binary variables, linear programming and the branch and bound method. A SOLNTYP equal to LP is normally used if the flow model contains only linear features (see "Solution of Ground-Water Management Problems with GWM" for a discussion of linear and nonlinear features). Alternatively, the use of LP allows the user to force the problem to be solved as a linear problem even if nonlinear features are present. In that case, the management formulation will be solved with a single response matrix. This option should be used carefully and may lead to inaccurate results if the management problem has a significant nonlinear response.

SLP (with or without binary variables): the optimization formulation is solved using sequential (iterative) linear programming, or, for problems with binary variables, sequential linear programming and the branch and bound method.

If SOLNTYP is NS:

DELTA—is a real variable equal to the perturbation parameter δ^0 (eq. 63) used to determine the response matrix. DELTA is multiplied by FVMAX for each flow-rate decision variable to determine each perturbation value. A positive value of DELTA implies a forward-difference calculation of the response coefficient (that is, an increase in flow rate), whereas a negative value implies a backward-difference calculation (that is, a decrease in flow rate).

NSIGDIG—is an integer variable equal to a lower limit on the number of significant digits in response-matrix entries. For each entry in a column of the response matrix, the ratio of the difference in observed state to the HCLOSE variable is computed. If the largest ratio in the column has fewer than NSIGDIG significant digits, the perturbation is considered to have failed.

NPGNMX—is an integer variable equal to the maximum number of attempts to achieve a successful flow-process run. Failure may occur during either base or perturbation flow-process runs. When failure is detected, automatic resetting of flow-rate decision-variable values may produce a successful solution. NPGNMX controls the maximum number of attempts for perturbation adjustments (controlled by PGFACT) or for base adjustments (controlled by AFACT).

PGFACT—is a real variable equal to the perturbation step-length adjustment factor used during perturbation failure. PGFACT must be greater than 0 and less than 1. A value of 0.5 is suggested.

RMNAME—is the file name (or pathname) to which the response matrix will be written.

If SOLNTYP is MPS:

DELTA—as defined for item 2a.

NSIGDIG, NPGNMX, PGFACT—as defined for item 2b.

MPSNAME—is the file name (or pathname) to which the the formulation will be written in MPS format.

If SOLNTYP is LP:

IRM—is an integer variable equal to 0, 1, or 2. Its value specifies whether or not the response matrix will be calculated or read from an input file and whether or not the response matrix will be saved. A value of 0 indicates that the response matrix has been generated in a previous run and will be read from the file specified by RMNAME in record 4f; this feature can greatly reduce the time required to solve GWM if the response matrix has been previously calculated. A value of 1 indicates that the response matrix will be calculated by GWM and written to the file specified by RMNAME in record 4f. A value of 2 indicates that the response matrix will be calculated by GWM but not written to a file.

LPITMAX—is an integer variable. Its value is the maximum number of iterations allowed for the linear program solver; this limit prevents the solver from iterating indefinitely if it does not converge to a solution. If the linear solver is being used and the value of LPITMAX is reached, the program will be terminated, and the output file will indicate that the maximum number of iterations has been exceeded. A typical value for LPITMAX is ten times the number of constraints.

BBITMAX—is an integer variable. BBITMAX is only relevant if the management problem contains one or more binary variables, otherwise, its value is ignored. BBITMAX is the maximum number of iterations allowed for the branch and bound program solver. Each iteration consists of one solution of the linear program. If the value of BBITMAX is reached, the program will be terminated, and the output file will indicate that the maximum number of iterations has been exceeded.

DELTA—as defined for item 2a.

NSIGDIG, NPGNMX, PGFACT—as defined for item 2b.

BBITPRT—is an integer variable that specifies whether output describing the details of the branch and bound algorithm for solving mixed binary problems will be written to the GLOBAL output file. A value of 1 indicates that this output will be created and a value of 0 indicates that it will not. For problems with many binary variables, this file can be very large. If the management problem contains no binary variables, this value will be ignored.

RANGE—is an integer variable that indicates the status of the range analysis. A value of 1 indicates that range analysis should be done and the results written to the GLOBAL file. A value of 0 indicates no range analysis. Range analysis is described in Appendix 2. Range analysis is based on the assumption that the optimization problem is strictly linear with continuous variables. If binary variables or nonlinear responses are significant in the problem, then the range analysis may be inaccurate.

RMNAME—is the file name (or pathname) from which the response matrix will be read if IRM equals 0 and is the file name (or pathname) to which the response matrix will be written if IRM equals 1.

If SOLNTYP is SLP:

SLPITMAX—is an integer variable. Its value is the maximum number of iterations allowed for the sequential linear-programming algorithm. If the value of SLPITMAX is reached, the program will be terminated, and the output file will indicate that the maximum number of iterations has been exceeded.

LPITMAX—as defined for item 4b.

BBITMAX—as defined for item 4b.

SLPVCRIT—is a real variable. Its value is the convergence criterion ε_1 (eq. 69*A*), which is the first of two termination rules when the sequential linear programming algorithm is used. This rule is satisfied when the change in the values of all flow-rate decision variables from the previous iteration to the current iteration is less than a fraction, ε_1, of the magnitude of the flow-rate decision variables at the current iteration.

SLPZCRIT—is a real variable. Its value is the convergence criterion ε_2 (eq. 69*B*), which is the second of two termination rules when the sequential linear programming algorithm is used. This rule is satisfied when the change in the value of the objective function, Z, is less than a specified fraction, ε_2, of the value of the objective function.

DINIT, DMIN, DSC—are real variables that control the value of the perturbation variable (eq. 72) used to compute response coefficients. DINIT is the perturbation variable used for the first iteration, DMIN is the minimum perturbation variable used, and DSC is a parameter that controls the rate of change of the perturbation parameter. DINIT and DMIN must have the same sign. Positive values of DINIT and DMIN imply a forward-difference calculation of the response coefficient (that is, an increase in flow rate), whereas negative values imply a backward-difference calculation (that is, a decrease in flow rate). DSC must always be positive.

NSIGDIG, NPGNMX, PGFACT—as defined for item 2b.

AFACT—is a real variable equal to the relaxation parameter (α in eq. 73) used to determine a temporary base solution when a base run fails. AFACT controls the interpolation between the current base solution and the most recent successful base solution. AFACT must be greater than 0 and less than 1. A value close to 0 implies that the temporary base solution will be close to the current base solution, whereas a value close to 1.0 implies that the temporary base solution will be close to the prior base solution. A value of 0.5 is suggested.

NINFMX—is an integer variable that specifies the maximum number of consecutive infeasible iterations that will be accepted by the SLP algorithm before the algorithm terminates.

SLPITPRT—is an integer variable that specifies whether output describing the details of the sequential-iteration algorithm will be written to the GLOBAL output file. A value of 1 indicates that this output will be created and a value of 0 indicates that it will not.

BBITPRT—as defined in item 4e.

RANGE—as defined in item 4e.

IBASE—is an integer variable equal to 0 or 1 that indicates the source for the values of the flow-rate decision variables that will be used as the base run. For problems solved using the SLP algorithm, these values are the starting point for the iterative algorithm. A value of IBASE equal to 0 indicates that the reference flow rates (FVREF) specified for each flow-rate decision variable in file VARCON will be used in the base run (and record 6b is not necessary). A value of IBASE equal to 1 indicates that the flow rates specified for each decision variable by FVBASE in record 6b will be used to calculate the base run.

FVNAME—is a character variable (maximum length of 10 characters) that is one of the flow-rate decision-variable names. Each of the FVNAME variables listed must be defined in the DECVAR file. A flow-rate decision-variable name can only be listed once in the SOLN file.

FVBASE—is a real variable equal to the rate for the flow-rate decision variable. These values are used by GWM to calculate the base run. If the SLP solution algorithm is used, these values are the starting point for the iterative algorithm.

Output Files

Two output files are always produced by a GWM run—a GLOBAL file and a LIST file. Two additional files, one to hold response matrixes and one to hold formulations written in MPS format, also are produced if specified by the user (see instructions for variables RMNAME and MPSNAME in SOLN file). The GLOBAL and LIST files are used in a manner that is similar to their use in MODFLOW applications that include the parameter-estimation or sensitivity processes (Harbaugh and others, 2000, p. 6–7). Most of the output from a particular run of the GWF Process is written to the LIST file. Input data that are written to the LIST file include array-allocation information, hydraulic properties of simulated aquifers, and initial and boundary conditions. Output that results from the GWF Process includes calculated heads, drawdowns, and volumetric-budget terms. The LIST file is erased and generated anew each time a ground-water flow run is required.

The GLOBAL file contains information that applies to the GWM run as a whole. The first information that is written includes the names and types of MODFLOW files that are opened from the NAME file, information about the spatial and temporal discretization of the model that is read from the MODFLOW discretization (DIS) file, and input for the selected MODFLOW solver package. The remainder of the GLOBAL file consists of input to, information about, and results of the GWM run. The first part of the GWM section consists of data that are echoed from the input files; the amount of input data that is printed depends on the values of IPRN specified by the user in the GWM input files. Information about the size of the management problem, which consists of the number of decision variables and the number of constraints, also is printed.

The next part of the GWM section ("Solution Algorithm") provides information about the solution process for the management problem. First, results of the call to the GWF Process for the base-condition run are reported, including the status of each of the constraints. The status of each constraint is indicated as either "satisfied," "not met," or "near-binding." For output purposes, GWM considers a constraint to be near-binding if the left and right sides of the constraint are in agreement to at least five digits. Next, information about the flow-process runs that are required to generate the response matrix is reported, including the perturbation value used at each candidate well and information about the number of significant digits in the response matrix. If the problem is nonlinear or includes binary variables (and if either of the input variables SLPITPRT or BBITPRT has been set to 1), then this section of the output also will provide detailed information on the progress of iterations necessary to solve the problem. If the problem contains binary variables, then the output will include the solution to the management problem calculated for each iteration (subproblem) of the branch and bound algorithm. The solutions are designated as relaxed, current best solution, feasible, or infeasible. For each solution, the name of the flow-rate or external decision variable and its value for the current solution are reported, as well as the status of each binary variable.

The last part of the GWM section ("Ground-Water Management Solution") provides information on the solution of the management problem, including the value of the objective function, the optimal values for each of the decision variables, and the binding constraints. Although linear programs often have many constraints, only a subset of them typically control the optimal solution. Those constraints that restrict the value of the objective function are said to be binding, because they prevent the decision variables from taking on values that further improve the objective function, and, therefore, bind the solution. At the optimal solution, the inequality constraints that are met as equalities are binding. Conversely, nonbinding constraints do not affect the optimal values of the decision variables and could be removed from the formulation without changing the solution.

Each constraint has associated with it a shadow price (or dual variable) that provides information on the effect of small changes in the value of the right-hand-side constraint coefficient on the value of the objective function. Shadow prices represent the marginal increase in benefit that results from relaxing each constraint coefficient. When the right-hand-side value of a binding constraint increases by a unit amount, the objective function will change by an amount given by the shadow price. This implies that the optimal solution is quite sensitive to constraints that have large shadow prices. Conversely, non-binding constraints have zero shadow prices, because small changes in the right-hand side of a non-binding constraint will have no effect on the objective function. Shadow prices are reported in the GLOBAL file along with the list of binding constraints.

Shadow prices represent the local sensitivity of the optimal solution to the right-hand side of the constraints and are valid as long as changes in the constraints do not alter the basis at the optimal solution. For management problems solved by the RMS Package of GWM, the basis consists of the set of flow-rate and external decision variables that have values between their upper and lower bounds at the optimal solution, as well as the set of slack variables associated with the nonbinding constraints. Range analysis is used to determine the range of values over which a shadow price is valid. The range analysis is reported after the list of binding constraints if the input variable RANGE in the SOLN file is set to 1. The range analysis calculates the interval over which the right-hand-side value of the constraint can vary without changing the basis of the optimal solution, if all other parameters of the management model remain unchanged. The analysis also determines the decision variables and (or) constraints that enter and leave the basis when the right-hand-side value is increased or decreased beyond its range.

A range analysis on the objective-function coefficients is also done and reported if requested by the user. In this analysis, the interval over which the coefficients can vary without changing the basis of the optimal solution is determined. Although the optimal basis remains unchanged, if the objective-function coefficients stay within their specified ranges, the values of the objective function and shadow prices are functions of the coefficients and therefore will be affected by any changes. The coefficient range analysis also determines the decision variables and (or) constraints that enter and leave the basis when the objective-function coefficient of interest is increased or decreased beyond its range. In addition to reporting the objective-function coefficient range-analysis information, the reduced cost associated with each decision variable is provided. The reduced cost of each decision variable that is not part of the basis at the optimal solution (nonbasic variables) is the amount by which the value of the objective function would be penalized if that decision variable were to become basic. For example, if a nonbasic decision variable has a value of zero and were to be brought into the optimal solution at a nonzero value, the objective function would be worsened—increased for a minimization problem and decreased for a maximization problem—by an amount equal to the reduced cost of the variable. The reduced cost of each of the decision variables in the optimal basis is zero.

Because range analysis assumes continuous variables and linear constraints, it is strictly valid for linear, non-binary problems only. Range analysis may be reasonably accurate for mildly nonlinear problems. If binary variables are present, then range analysis can be reported; however, it should be used with caution. The reported range analysis is based on the final linear program solved during the branch and bound algorithm and does not include the changes in binary variables that might result when right-hand-side and objective-function coefficients are changed. The most complete range analysis, when binary variables are present, can be obtained by resolving the problem as a non-binary, linear program with the inactive flow variables removed from the problem.

Additional details concerning sensitivity and range analyses can be found in Appendix 2 and Ahlfeld and Mulligan (2000).

The last step in a successful GWM run is a final run of the GWF Process using the optimal flow rates determined by the solution algorithm. The LIST file will contain output from the GWF Process using these flow rates. The GLOBAL output file will contain the final status of the constraints. A successful run should indicate that all constraints are either satisfied or near-binding. A distinction is made between binding constraints and near-binding constraints. When solving the linear program, which contains an approximation to the GWF Process, certain constraints will be binding, that is, satisfied as strict equalities. However, when the GWF Process is run with the optimal flow rates, these same constraints may not be exactly binding. This can result from nonlinear responses in the GWF Process and precision limitations in the computation of heads. In addition, some constraints that are not strictly binding in the linear program, may be very close to binding in the GWF Process. As a result, there may be a difference in the set of constraints listed as binding in the linear program output and those listed as near-binding in the final GWF Process output.

Sample Problems

Three sample problems are provided to demonstrate how to set up files for GWM, some of the output generated by GWM, and how GWM may be used to solve typical ground-water management problems. It is assumed that the reader is familiar with the MODFLOW code (Harbaugh and others, 2000), including terminology that is used in MODFLOW. Selected input and output files are listed at the end of each sample problem.

Sample Problem 1: DEWATER

This sample problem represents a steady-state dewatering problem for a construction site. The objective of the ground-water management problem is to minimize the cost of withdrawing ground water to lower heads to an elevation of 50 ft so that footings can be installed in the area shown in figure 3. Wells at the construction site will be pumped for 1,000 days. The aquifer at the site is confined and is simulated by a single model layer that is 3,000 ft long and 2,000 ft wide. The model grid consists of 20 rows and 30 columns, and each grid cell is 100 ft by 100 ft (fig. 3). The model uses no-flow boundary conditions along the north and south boundaries of the aquifer and constant heads of 80 ft and 60 ft along the east and west boundaries of the aquifer, respectively. The transmissivity of the aquifer is 50 ft^2/d.

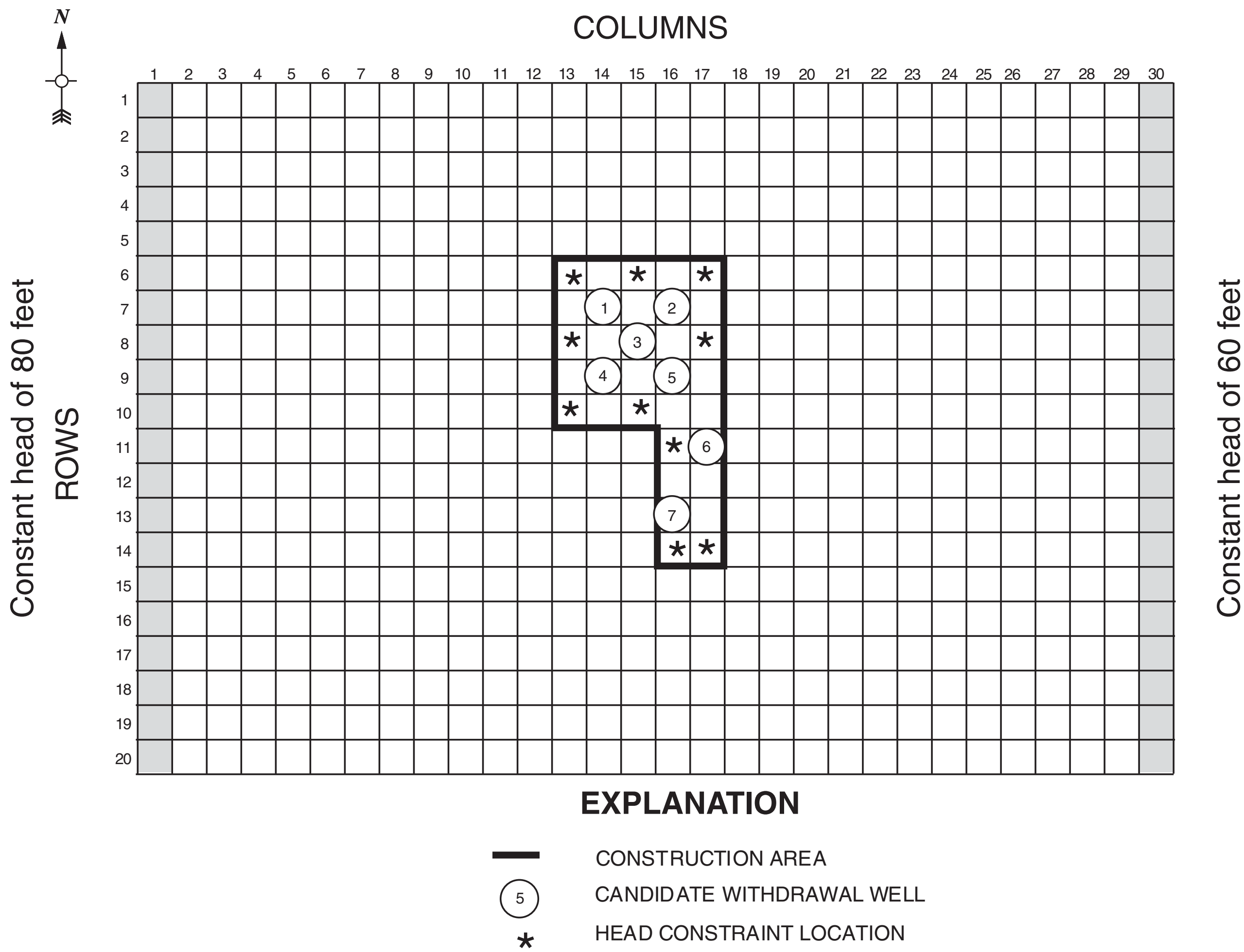

Figure 3. Model grid for DEWATER sample problem.

The MODFLOW input files consist of a NAME file, a DIS-Package file, a BAS6-Package file, a BCF6-Package file, and a PCG-Package file. In the DIS file, the number of stress periods (NPER) is set to 1, the time units are specified as days (ITMUNI = 4), the length of the single stress period is 1,000 days (PERLEN = 1.000E+3), and a single time step is used for the stress period (NSTP = 1). A very small head-change convergence criterion of 1.0E-8 ft is specified in the PCG solution package.

The management problem is solved in two ways. In the first formulation, only the operational costs of pumping are considered; this formulation results in a linear problem. The GWM input files that are necessary for the linear formulation are: DECVAR, OBJFNC, VARCON, HEDCON, and SOLN. In the second formulation, both the operational and construction costs of the wells are considered; in addition, there is a restriction on the minimum number of wells that must be constructed. The construction costs and constraint on the minimum number of wells that must be constructed add binary variables to the problem; the formulation is therefore a mixed-binary linear problem. The GWM input files that are necessary for the mixed-binary linear problem include all of those necessary for the linear formulation, as well as a SUMCON file to specify the minimum number of wells that must be constructed. Input files for these two formulations are listed at the end of the sample problem.

Linear Formulation

Seven candidate well locations are selected as possible locations of withdrawal. These seven flow-rate decision variables (named Q1, Q2, and so forth) are specified in the DECVAR file with input variable NFVAR, and each decision variable extends over a single cell (NC=1). The minimum and maximum pumping rates at each well, which are specified in the VARCON file, are 0 and 20,000 ft^3/d, respectively.

The linear problem is solved using two approaches. In the first approach, the operational costs of pumping are considered to be directly proportional to the amount of water withdrawn (in units of cubic feet per day), so the objective is to minimize total withdrawal from the seven wells, with unitless objective-function coefficients of 1.0:

$$\text{Minimize } Q1 + Q2 + Q3 + Q4 + Q5 + Q6 + Q7\,. \tag{74}$$

Note that equation 74 is multiplied by the length of the stress period (1,000 days) by GWM to determine the value of the objective function in units of cubic feet.

The 50-ft head criterion can be imposed by defining 10 upper-bound head constraints (constraint type NHB) in the HEDCON file at the cell locations shown in figure 3. The problem solution type specified in the SOLN file is LP (linear program). A perturbation-parameter (DELTA) value of 0.5 is specified. DELTA is multiplied by the maximum withdrawal rate at each well (input variable FVMAX in file VARCON) to determine the initial perturbation rate at each well. In this example, because the maximum withdrawal rate for each well is 20,000 ft^3/day, the initial perturbation value for each well is -10,000 ft^3/d. Variable IBASE in the SOLN file is specified as 0, which means that the base-condition withdrawal rates for each well are specified by FVREF in the VARCON file; these rates are specified as 0 ft^3/d for each well.

The value of the objective function at the optimal solution for this approach is 2.8657×10^6 ft^3 of water withdrawn. Four wells were selected for pumping in the optimal solution: well Q1 pumps at a rate of 1,077 ft^3/d, well Q2 at 78.2 ft^3/d, well Q4 at 769.0 ft^3/d, and well Q7 at 941.1 ft^3/d. Wells Q3, Q5, and Q6 are inactive in the optimal solution (that is, they have pumping rates of 0 ft^3/d). Four of the head constraints are binding at the optimal solution, those located in cells (1, 6, 13), (1, 6, 17), (1, 10, 13), and (1, 14, 17). The GLOBAL output file for this GWM run is listed at the end of the sample problem.

In the second approach, the actual costs of pumping during construction are included. These costs, which are independent of the depth to water (that is, the lift), are \$0.02/ft^3 of water withdrawn:

$$\begin{aligned}\text{Minimize } &\left(\frac{\$0.02}{ft^3}\right)Q1 + \left(\frac{\$0.02}{ft^3}\right)Q2 + \left(\frac{\$0.02}{ft^3}\right)Q3 + \left(\frac{\$0.02}{ft^3}\right)Q4 + \\ &\left(\frac{\$0.02}{ft^3}\right)Q5 + \left(\frac{\$0.02}{ft^3}\right)Q6 + \left(\frac{\$0.02}{ft^3}\right)Q7\,.\end{aligned} \tag{75}$$

Again, GWM multiplies each term in the objective function by the length of the stress period (1,000 days).

The value of the objective function at the optimal solution for the second approach is \$57,313 (output from this formulation is not included in this report). The same four wells are selected for pumping as were selected in the first approach, and the pumping rates at each of the wells are the same as for the first approach. The reason that the solutions for the two approaches are the same is that the solution to minimization (or maximization) problems that differ only by a constant multiple in the objective function will be the same. In this case, the two approaches differ only by the multiple \$0.02/ft^3 in the objective function, so the solutions are the same except for the value (and the units) of the objective function (2.8657x10^6 ft^3 compared to \$57,313). Note that this would not be the case if the cost of withdrawing water differed among the seven wells.

Mixed-Binary Linear Formulation

In this second formulation, the costs of both installing and operating the wells will be considered. This change requires that seven binary variables be defined for the problem (named BV1, BV2, and so forth), one of which is associated with each flow-rate decision variable. The operational costs of the system are \$0.02/ft^3; installation costs are \$2,000 per well. The objective function of the mixed-binary linear problem is

Minimize

$$\left(\frac{\$0.02}{ft^3}\right)Q1 + \left(\frac{\$0.02}{ft^3}\right)Q2 + \left(\frac{\$0.02}{ft^3}\right)Q3 + \left(\frac{\$0.02}{ft^3}\right)Q4 + \left(\frac{\$0.02}{ft^3}\right)Q5 + \left(\frac{\$0.02}{ft^3}\right)Q6 \tag{76}$$
$$+ \left(\frac{\$0.02}{ft^3}\right)Q7 + (\$2{,}000)BV1 + (\$2{,}000)BV2 + (\$2{,}000)BV3$$
$$+ (\$2{,}000)BV4 + (\$2{,}000)BV5 + (\$2{,}000)BV6 + (\$2{,}000)BV7 \quad .$$

To ensure reliability of the system, a minimum of at least three wells must be installed, and the minimum withdrawal rate that is allowed at any well is 100 ft^3/d (maximum pumping rates are still 20,000 ft^3/d). A linear-summation constraint is added to the formulation that states that a minimum of at least three wells must be installed (see eq. 26):

$$BV1 + BV2 + BV3 + BV4 + BV5 + BV6 + BV7 \geq 3\,. \tag{77}$$

This constraint is specified in the SUMCON file. No changes to the HEDCON or SOLN files used in the linear formulation are required for this formulation.

The optimal solution was determined to be \$63,598 and only three wells were selected for pumping: well Q1 pumps at a rate of 1,242 ft^3/d, well Q4 at 694.1 ft^3/d, and well Q7 at 943.3 ft^3/d. Note that well Q2, which pumped at a rate of only 78.2 ft^3/d in the linear formulations, was not selected for pumping in this formulation because of the relatively high installation cost for the well in comparison to the amount of water produced. The installation cost for the three wells is \$6,000 and the operational cost for the 1,000-day period is \$57,598, which is close to the value of \$57,313 determined for the linear formulation.

Selected Input and Output Files for Sample Problem

Name File (dewater.nam):

```
GLOBAL 9   dewater.glo
LIST  10   dewater.lst
DIS   11   ..\data\dewater.dis
BAS6  12   ..\data\dewater.ba6
BCF6  13   ..\data\dewater.bc6
PCG   14   ..\data\dewater.pcg
GWM   15   ..\data\dewater.gwm
```

GWM File (dewater.gwm) for linear formulation:

```
#DEWATER Sample Problem, GWM file
#February 20, 2005
DECVAR  ..\data\dewater.decvar
OBJFNC  ..\data\dewater.objfnc
VARCON  ..\data\dewater.varcon
HEDCON  ..\data\dewater.hedcon
SOLN    ..\data\dewater.soln
```

Decision variable (DECVAR) File (dewater.decvar) for linear formulation:

```
#DEWATER Sample Problem, DECVAR file
#February 20, 2005
 1                           #1-IPRN
 7 0 0                       #2-NFVAR  NEVAR  NBVAR
 Q1  1   1  7  14  W  Y  1   #3a-FVNAME NC LAY ROW COL FTYPE ...
 Q2  1   1  7  16  W  Y  1
 Q3  1   1  8  15  W  Y  1
 Q4  1   1  9  14  W  Y  1
 Q5  1   1  9  16  W  Y  1
 Q6  1   1 11  17  W  Y  1
 Q7  1   1 13  16  W  Y  1
```

Objective Function (OBJFNC) File (dewater.objfnc) for linear formulation:

```
#DEWATER Sample Problem, OBJFNC file
#February 20, 2005
 1              #1-IPRN
 MIN  WSDV      #2-OBJTYP  FNTYP
 7  0  0        #3-NFVOBJ  NEVOBJ  NBVOBJ
 Q1  1.0        #4-FVNAME  FVOBJC
 Q2  1.0
 Q3  1.0
 Q4  1.0
 Q5  1.0
 Q6  1.0
 Q7  1.0
```

Decision-variable constraints (VARCON) File (dewater.varcon) for linear formulation:

```
#DEWATER Sample Problem, VARCON file
#February 20, 2005
  1                             #1-IPRN
 Q1 0.0d2  2.0d4  0.0d2         #2-FVNAME  FVMIN  FVMAX  FVREF
 Q2 0.0d2  2.0d4  0.0d2
 Q3 0.0d2  2.0d4  0.0d2
 Q4 0.0d2  2.0d4  0.0d2
 Q5 0.0d2  2.0d4  0.0d2
 Q6 0.0d2  2.0d4  0.0d2
 Q7 0.0d2  2.0d4  0.0d2
```

Head constraints (HEDCON) File (dewater.hedcon) for linear formulation:

```
#DEWATER Sample Problem, HEDCON file
#February 20, 2005
 1                              #1-IPRN
 10 0  0  0                     #2-NHB NDD NDF NGD
 b-01  1  6 13 le 50.0 1        #3-HBNAME LAYH ROWH COLH TYPH BND NSP
 b-02  1  6 15 le 50.0 1
 b-03  1  6 17 le 50.0 1
 b-04  1  8 13 le 50.0 1
 b-05  1  8 17 le 50.0 1
 b-06  1 10 13 le 50.0 1
 b-07  1 10 15 le 50.0 1
 b-08  1 11 16 le 50.0 1
 b-09  1 14 16 le 50.0 1
 b-10  1 14 17 le 50.0 1
```

Solution and output control file (SOLN) File (dewater.soln) for linear formulation:

```
#DEWATER Sample Problem, SOLN file
#February 20, 2005
 LP                #1-SOLNTYP
2                  #4a-IRM
 1000  2000        #4b-LPITMAX  BBITMAX
 0.5               #4c-DELTA
 1  10  0.5        #4d-NSIGDIG  NPGNMX  PGFACT
 1  1              #4e-BBITPRT  RANGE
 0                 #6a-IBASE
```

GWM File (dewatermb.gwm) for mixed-binary linear formulation:

```
#DEWATER Sample Problem, GWM file
#February 20, 2005
DECVAR  ..\data\dewatermb.decvar
OBJFNC  ..\data\dewatermb.objfnc
VARCON  ..\data\dewatermb.varcon
HEDCON  ..\data\dewater.hedcon
SUMCON  ..\data\dewatermb.sumcon
SOLN    ..\data\dewatermb.soln
```

Decision variable (DECVAR) File (dewatermb.decvar) for mixed-binary linear formulation:

```
#DEWATER Sample Problem, DECVAR file
#February 20, 2005
 1                              #1-IPRN
 7 0 7                          #2-NFVAR  NEVAR  NBVAR
 Q1  1   1  7  14  W  Y  1      #3a-FVNAME NC LAY ROW COL FTYPE ...
 Q2  1   1  7  16  W  Y  1
 Q3  1   1  8  15  W  Y  1
 Q4  1   1  9  14  W  Y  1
 Q5  1   1  9  16  W  Y  1
 Q6  1   1 11  17  W  Y  1
 Q7  1   1 13  16  W  Y  1
 BV1 1  Q1                      #5-BVNAME NDV BVLIST
 BV2 1  Q2
 BV3 1  Q3
 BV4 1  Q4
 BV5 1  Q5
 BV6 1  Q6
 BV7 1  Q7
```

Objective Function (OBJFNC) File (dewatermb.objfnc) for mixed-binary linear formulation:

```
#DEWATER Sample Problem, OBJFNC file
#February 20, 2005
 1              #1-IPRN
 MIN  WSDV      #2-OBJTYP  FNTYP
 7  0  7        #3-NFVOBJ  NEVOBJ  NBVOBJ
 Q1  0.02       #4-FVNAME  FVOBJC
 Q2  0.02
 Q3  0.02
 Q4  0.02
 Q5  0.02
 Q6  0.02
 Q7  0.02
 BV1 2000.0    #6-BVNAME  BVOBJC
 BV2 2000.0
 BV3 2000.0
 BV4 2000.0
 BV5 2000.0
 BV6 2000.0
 BV7 2000.0
```

Decision-variable constraints (VARCON) File (dewatermb.varcon) for mixed-binary linear formulation:

```
#DEWATER Sample Problem, VARCON file
#February 20, 2005
  1                         #1-IPRN
 Q1 1.0d2  2.0d4  0.0d2     #2-FVNAME  FVMIN  FVMAX  FVREF
 Q2 1.0d2  2.0d4  0.0d2
 Q3 1.0d2  2.0d4  0.0d2
 Q4 1.0d2  2.0d4  0.0d2
 Q5 1.0d2  2.0d4  0.0d2
 Q6 1.0d2  2.0d4  0.0d2
 Q7 1.0d2  2.0d4  0.0d2
```

Linear-summation constraint file (SUMCON) File (dewatermb.sumcon) for mixed-binary linear formulation:

```
#DEWATER Sample Problem, SUMCON file
#February 20, 2005
 1                           #1-IPRN
 1                           #2-SMCNUM
 Binlower  7 ge 3.0          #3a-SMCNAME NTERMS TYPE RHS
 BV1 1                       #3b-GVNAME  GVCOEFF
 BV2 1
 BV3 1
 BV4 1
 BV5 1
 BV6 1
 BV7 1
```

GLOBAL File (dewater.glo) for linear formulation with objective-function coefficients equal to 1.0:

```
                                  MODFLOW-2000
 U.S. GEOLOGICAL SURVEY MODULAR FINITE-DIFFERENCE GROUND-WATER FLOW MODEL
              VERSION MF2K_GWM 1.0.0 022005,FROM MF2K V1.13.00


This model run produced both GLOBAL and LIST files. This is the GLOBAL file.

 GLOBAL LISTING FILE: dewater.glo
                        UNIT    9

 OPENING dewater.lst
 FILE TYPE:LIST   UNIT   10   STATUS:REPLACE
 FORMAT:FORMATTED              ACCESS:SEQUENTIAL

 OPENING ..\data\dewater.dis
 FILE TYPE:DIS   UNIT   11   STATUS:OLD
 FORMAT:FORMATTED              ACCESS:SEQUENTIAL

 OPENING ..\data\dewater.ba6
 FILE TYPE:BAS6   UNIT   12   STATUS:OLD
 FORMAT:FORMATTED              ACCESS:SEQUENTIAL

 OPENING ..\data\dewater.bc6
 FILE TYPE:BCF6   UNIT   13   STATUS:OLD
 FORMAT:FORMATTED              ACCESS:SEQUENTIAL
```

```
OPENING ..\data\dewater.pcg
FILE TYPE:PCG   UNIT   14   STATUS:OLD
FORMAT:FORMATTED              ACCESS:SEQUENTIAL

OPENING ..\data\dewater.gwm
FILE TYPE:GWM   UNIT   15   STATUS:OLD
FORMAT:FORMATTED              ACCESS:SEQUENTIAL

THE FREE FORMAT OPTION HAS BEEN SELECTED

DISCRETIZATION INPUT DATA READ FROM UNIT   11
#DEWATER Sample Problem
#February 20, 2005
   1 LAYERS        20 ROWS        30 COLUMNS
   1 STRESS PERIOD(S) IN SIMULATION
MODEL TIME UNIT IS DAYS
MODEL LENGTH UNIT IS UNDEFINED
THE GROUND-WATER TRANSPORT PROCESS IS INACTIVE

THE OBSERVATION PROCESS IS INACTIVE
THE SENSITIVITY PROCESS IS INACTIVE
THE PARAMETER-ESTIMATION PROCESS IS INACTIVE

MODE: FORWARD

 Confining bed flag for each layer:
  0

      5450  ELEMENTS OF GX ARRAY USED OUT OF       5450
       600  ELEMENTS OF GZ ARRAY USED OUT OF        600
       600  ELEMENTS OF IG ARRAY USED OUT OF        600

                    DELR =   100.000

                    DELC =   100.000

TOP ELEVATION OF LAYER 1 =   100.000

  MODEL LAYER BOTTOM EL. =   100.000     FOR LAYER   1


STRESS PERIOD     LENGTH       TIME STEPS     MULTIPLIER FOR DELT SS FLAG
-------------------------------------------------------------------------
       1        1000.000         1                    1.000         SS

STEADY-STATE SIMULATION




PCG2 -- CONJUGATE GRADIENT SOLUTION PACKAGE, VERSION 2.4, 12/29/98
MAXIMUM OF     50 CALLS OF SOLUTION ROUTINE
MAXIMUM OF      5 INTERNAL ITERATIONS PER CALL TO SOLUTION ROUTINE
MATRIX PRECONDITIONING TYPE :    1
      1700 ELEMENTS IN X ARRAY ARE USED BY PCG
      1750 ELEMENTS IN IX ARRAY ARE USED BY PCG
      2400 ELEMENTS IN Z ARRAY ARE USED BY PCG

      1700  ELEMENTS OF X ARRAY USED OUT OF       1700
      2400  ELEMENTS OF Z ARRAY USED OUT OF       2400
      1750  ELEMENTS OF IX ARRAY USED OUT OF       1750
```

```
                              SOLUTION BY THE CONJUGATE-GRADIENT METHOD
                             ------------------------------------------
                    MAXIMUM NUMBER OF CALLS TO PCG ROUTINE =       50
                        MAXIMUM ITERATIONS PER CALL TO PCG =        5
                               MATRIX PRECONDITIONING TYPE =        1
       RELAXATION FACTOR (ONLY USED WITH PRECOND. TYPE 1) =    0.10000E+01
 PARAMETER OF POLYNOMIAL PRECOND. = 2 (2) OR IS CALCULATED :        0
                        HEAD CHANGE CRITERION FOR CLOSURE =    0.10000E-07
                    RESIDUAL CHANGE CRITERION FOR CLOSURE =    0.10000E+01
            PCG HEAD AND RESIDUAL CHANGE PRINTOUT INTERVAL =        1
     PRINTING FROM SOLVER IS LIMITED(1) OR SUPPRESSED (>1) =        0
                                        DAMPING PARAMETER =    0.10000E+01


GWM1 -- GROUND-WATER MANAGEMENT PROCESS, VERSION 1.0.0 022005
INPUT READ FROM UNIT  15


----------------------------------------------------------------------
               Reading GWM Input
----------------------------------------------------------------------
 #DEWATER Sample Problem, GWM file
 #February 20, 2005

 OPENING DECISION-VARIABLE FILE ON UNIT   99:
..\data\dewater.decvar
 #DEWATER Sample Problem, DECVAR file
 #February 20, 2005

 NO. OF FLOW-RATE DECISION VARIABLES (NFVAR)       7
 NO. OF EXTERNAL DECISION VARIABLES (NEVAR):       0
 BINARY VARIABLES ARE NOT ACTIVE.

 FLOW-RATE VARIABLES:
                                                   FRACTION
  NUMBER     NAME       TYPE      LAY   ROW   COL   OF FLOW
 ----------------------------------------------------------
    1      Q1         WITHDRAWAL     1     7    14    1.0000
 AVAILABLE IN STRESS PERIODS: 1

    2      Q2         WITHDRAWAL     1     7    16    1.0000
 AVAILABLE IN STRESS PERIODS: 1

    3      Q3         WITHDRAWAL     1     8    15    1.0000
 AVAILABLE IN STRESS PERIODS: 1

    4      Q4         WITHDRAWAL     1     9    14    1.0000
 AVAILABLE IN STRESS PERIODS: 1

    5      Q5         WITHDRAWAL     1     9    16    1.0000
 AVAILABLE IN STRESS PERIODS: 1

    6      Q6         WITHDRAWAL     1    11    17    1.0000
 AVAILABLE IN STRESS PERIODS: 1

    7      Q7         WITHDRAWAL     1    13    16    1.0000
 AVAILABLE IN STRESS PERIODS: 1
```

```
     490 BYTES OF MEMORY ALLOCATED TO STORE DATA FOR DECISION VARIABLES

CLOSING DECISION-VARIABLE FILE


OPENING OBJECTIVE-FUNCTION FILE ON UNIT   99:
..\data\dewater.objfnc
#DEWATER Sample Problem, OBJFNC file
#February 20, 2005

OBJECTIVE TYPE: MIN   FUNCTION TYPE: WSDV

NO. OF FLOW-RATE DECISION VARIABLES IN OBJECTIVE FUNCTION (NFVOBJ):    7
NO. OF EXTERNAL DECISION VARIABLES IN OBJECTIVE FUNCTION (NEVOBJ):     0
NO. OF BINARY DECISION VARIABLES IN OBJECTIVE FUNCTION (NBVOBJ):       0

OBJECTIVE FUNCTION:

MIN      +  1.00E+00 Q1     +  1.00E+00 Q2       +  1.00E+00 Q3
         +  1.00E+00 Q4     +  1.00E+00 Q5       +  1.00E+00 Q6
         +  1.00E+00 Q7

      28 BYTES OF MEMORY ALLOCATED TO STORE DATA FOR OBJECTIVE-FUNCTION

CLOSING OBJECTIVE-FUNCTION FILE


OPENING DECISION-VARIABLE CONSTRAINTS FILE
ON UNIT   99:
..\data\dewater.varcon
#DEWATER Sample Problem, VARCON file
#February 20, 2005

FLOW RATE VARIABLES:
                      MINIMUM        MAXIMUM        REFERENCE
 NUMBER   NAME        FLOW RATE      FLOW RATE      FLOW RATE
----------------------------------------------------------------
   1      Q1          0.000E+00      2.000E+04      0.000E+00
   2      Q2          0.000E+00      2.000E+04      0.000E+00
   3      Q3          0.000E+00      2.000E+04      0.000E+00
   4      Q4          0.000E+00      2.000E+04      0.000E+00
   5      Q5          0.000E+00      2.000E+04      0.000E+00
   6      Q6          0.000E+00      2.000E+04      0.000E+00
   7      Q7          0.000E+00      2.000E+04      0.000E+00

CLOSING DECISION-VARIABLE CONSTRAINTS FILE


OPENING HEAD CONSTRAINTS FILE
ON UNIT   99:
..\data\dewater.hedcon
#DEWATER Sample Problem, HEDCON file
#February 20, 2005
```

```
HEAD CONSTRAINTS:
                                                  RIGHT-HAND    STRESS
NUMBER  NAME        LAY  ROW  COL  TYPE            SIDE       PERIOD
---------------------------------------------------------------
    1   b-01          1    6   13   <     5.0000E+01         1
    2   b-02          1    6   15   <     5.0000E+01         1
    3   b-03          1    6   17   <     5.0000E+01         1
    4   b-04          1    8   13   <     5.0000E+01         1
    5   b-05          1    8   17   <     5.0000E+01         1
    6   b-06          1   10   13   <     5.0000E+01         1
    7   b-07          1   10   15   <     5.0000E+01         1
    8   b-08          1   11   16   <     5.0000E+01         1
    9   b-09          1   14   16   <     5.0000E+01         1
   10   b-10          1   14   17   <     5.0000E+01         1

     660 BYTES OF MEMORY ALLOCATED TO STORE DATA FOR HEAD CONSTRAINTS

CLOSING HEAD CONSTRAINTS FILE


OPENING SOLUTION FILE ON UNIT   99:
..\data\dewater.soln
#DEWATER Sample Problem, SOLN file
#February 20, 2005

SOLNTYP IS LP: GWM WILL COMPLETE A SINGLE ITERATION OF THE LINEAR PROBLEM.

IRM EQUALS 2: RESPONSE MATRIX WILL BE CALCULATED BY GWM
BUT NOT WRITTEN TO THE RESPONSE FILE.

MAXIMUM NUMBER OF LP ITERATIONS:                1000
MAXIMUM NUMBER OF BRANCH AND BOUND ITER:        2000

PERTURBATION VALUE:                           0.50D+00

MAXIMUM NUMBER OF PERTURBATION ATTEMPTS:          10
PERTURBATION ADJUSTMENT FACTOR (PGFACT):     0.50000

OUTPUT FROM BRANCH-AND-BOUND ALGORITHM WILL NOT BE PRINTED.

BASE PUMPING RATES TAKEN FROM FVREF SPECIFIED IN VARCON INPUT FILE

    PROBLEM SIZE

 NUMBER OF VARIABLES (INCLUDING SLACKS)           17
 NUMBER OF CONSTRAINT EQUATIONS                   10


    6268 BYTES OF MEMORY ALLOCATED FOR RESPONSE MATRIX ALGORITHM

CLOSING SOLUTION AND OUTPUT FILE
```

```
---------------------------------------------------------------
               Solution Algorithm
---------------------------------------------------------------
  Begin Solution Algorithm
    Running Base Flow Process Simulation
      Status of Simulation-Based Constraints
      Constraint Type        Name     Status     Distance To RHS
      ---------------        ----     ------     ---------------
      Head Bound             b-01     Not Met         2.1724E+01
      Head Bound             b-02     Not Met         2.0345E+01
      Head Bound             b-03     Not Met         1.8966E+01
      Head Bound             b-04     Not Met         2.1724E+01
      Head Bound             b-05     Not Met         1.8966E+01
      Head Bound             b-06     Not Met         2.1724E+01
      Head Bound             b-07     Not Met         2.0345E+01
      Head Bound             b-08     Not Met         1.9655E+01
      Head Bound             b-09     Not Met         1.9655E+01
      Head Bound             b-10     Not Met         1.8966E+01

    Calculating Response Matrix
      Perturb Flow Variable    1
       By Perturbation Value: -1.000000E+04
      Perturb Flow Variable    2
       By Perturbation Value: -1.000000E+04
      Perturb Flow Variable    3
       By Perturbation Value: -1.000000E+04
      Perturb Flow Variable    4
       By Perturbation Value: -1.000000E+04
      Perturb Flow Variable    5
       By Perturbation Value: -1.000000E+04
      Perturb Flow Variable    6
       By Perturbation Value: -1.000000E+04
      Perturb Flow Variable    7
       By Perturbation Value: -1.000000E+04

      Average Number of Significant Digits in Matrix  1.011429E+01

    Solving Linear Program
  Optimal Solution Found
```

```
--------------------------------------------------------------------
              Ground-Water Management Solution
--------------------------------------------------------------------

       OPTIMAL SOLUTION FOUND

       OPTIMAL RATES FOR EACH FLOW VARIABLE
       --------------------------------------
Variable          Withdrawal          Injection           Contribution
Name              Rate                Rate                To Objective
----------        --------------      ------------        ------------
 Q1               1.077390E+03                            1.077390E+06
 Q2               7.823877E+01                            7.823877E+04
 Q3               0.000000E+00                            0.000000E+00
 Q4               7.689506E+02                            7.689506E+05
 Q5               0.000000E+00                            0.000000E+00
 Q6               0.000000E+00                            0.000000E+00
 Q7               9.410751E+02                            9.410751E+05
                  ------------        ------------        ------------
TOTALS            2.865655E+03        0.000000E+00        2.865655E+06


       OBJECTIVE FUNCTION VALUE                           2.865655E+06

       BINDING CONSTRAINTS
Constraint Type       Name     Status      Shadow Price
---------------       ----     ------      ------------
Head Bound            b-01     Binding     -2.7273E+04
Head Bound            b-03     Binding     -3.2593E+04
Head Bound            b-06     Binding     -3.1185E+04
Head Bound            b-10     Binding     -5.1544E+04

Binding constraint and range analysis values are determined from the linear
program and based on the response matrix approximation of the flow-process.


       RANGE ANALYSIS

       Constraint Ranges

Lower/Upper Bound are the values of the RHS beyond which basis will change.
    Leaving is the variable which will leave the basis.
    Entering is the variable which will enter the basis.
    If the entering or leaving variable is a constraint name,
      then the constraint slack variable is active
```

```
Constraint               Original      Lower/Upper
Name        Slack        RHS           Bound          Entering      Leaving
----------  ----------  ----------    ----------     ----------    -------
b-01        0.0000E+00  5.0000E+01    4.9477E+01     b-03          Q2
                                      5.3228E+01     b-01          b-04

b-02        2.0745E+00  5.0000E+01    4.7926E+01     b-01          b-02
                                      Infinity       ----- No Change -----

b-03        0.0000E+00  5.0000E+01    4.3065E+01     b-01          Q1
                                      5.0317E+01     b-03          Q2

b-04        2.0528E+00  5.0000E+01    4.7947E+01     b-01          b-04
                                      Infinity       ----- No Change -----

b-05        1.1167E+00  5.0000E+01    4.8883E+01     Q3            b-05
                                      Infinity       ----- No Change -----

b-06        0.0000E+00  5.0000E+01    4.7939E+01     b-03          Q2
                                      5.2635E+01     b-06          Q4

b-07        2.6182E+00  5.0000E+01    4.7382E+01     Q3            b-07
                                      Infinity       ----- No Change -----

b-08        1.8584E+00  5.0000E+01    4.8142E+01     Q6            b-08
                                      Infinity       ----- No Change -----

b-09        1.0158E+00  5.0000E+01    4.8984E+01     b-10          b-09
                                      Infinity       ----- No Change -----

b-10        0.0000E+00  5.0000E+01    4.7205E+01     b-03          Q2
                                      5.0850E+01     b-10          b-09


       Objective-Function Coefficient Ranges

Lower/Upper Bound are the values of the coefficients beyond which basis
will change.
    Leaving is the variable which will leave the basis.
    Entering is the variable which will enter the basis.
    If the entering or leaving variable is a constraint name,
      then the constraint slack variable is active
    Basic variables are shown with zero reduced cost

Variable    Reduced      Original      Lower/Upper
Name        Cost         Coefficient   Bound          Entering      Leaving
----------  ----------    ----------    ----------       ----------  -------
Q1          0.0000E+00  1.0000E+03    9.1368E+02     b-01          Q2
                                      1.0669E+03     Q3            Q2

Q2          0.0000E+00  1.0000E+03    8.6811E+02     b-03          Q1
                                      1.0438E+03     Q3            Q2

Q3          1.5770E+01  1.0000E+03    9.8423E+02     Q3            Q2
                                      Infinity       ----- No Change -----

Q4          0.0000E+00  1.0000E+03    8.9312E+02     b-06          Q2
                                      1.0471E+03     Q3            Q2
```

```
Q5          4.4085E+01  1.0000E+03    9.5592E+02    Q5            Q2
                                      Infinity      ----- No Change -----

Q6          7.4018E+01  1.0000E+03    9.2598E+02    Q6            Q2
                                      Infinity       ----- No Change -----

Q7          0.0000E+00  1.0000E+03    6.7387E+02    b-10          Q2
                                      1.1286E+03    Q6            Q2


----------------------------------------------------------------------
               Final Flow Process Simulation
----------------------------------------------------------------------
      Status of Simulation-Based Constraints
        Using Optimal Flow Rate Variable Values
      Constraint Type        Name     Status     Distance To RHS
      ---------------        ----     ------     ---------------
      Head Bound             b-01     Near-Binding    3.4739E-07
      Head Bound             b-02     Satisfied       2.0745E+00
      Head Bound             b-03     Near-Binding    2.8216E-07
      Head Bound             b-04     Satisfied       2.0528E+00
      Head Bound             b-05     Satisfied       1.1167E+00
      Head Bound             b-06     Near-Binding    2.6732E-07
      Head Bound             b-07     Satisfied       2.6182E+00
      Head Bound             b-08     Satisfied       1.8584E+00
      Head Bound             b-09     Satisfied       1.0158E+00
      Head Bound             b-10     Near-Binding    1.8573E-07

  Because of precision limitations and possible nonlinear behavior,
    the status of binding constraints computed directly by the flow process
     may differ slightly from those computed using the linear program.
```

Sample Problem 2: SEAWATER

In this sample problem, seawater intrusion is to be controlled in a coastal area where ground water is pumped for water supply. Because of the contrast in water density between freshwater and saltwater, a density-dependent ground-water flow model is needed to rigorously address the problem of seawater intrusion caused by ground-water pumping; specifically, to adequately protect the coastal aquifers from the intrusion of salty water. As a first approximation, however, a constant-density, MODFLOW ground-water flow model has been developed that uses hydraulic controls at the coast as a surrogate for density effects. Such an approach has been used by Reichard (1995), Nishikawa (1998), and Reichard and others (2003).

The ground-water flow system consists of an upper unconfined aquifer with a uniform bottom elevation of 10 ft below local sea level and a lower confined aquifer with a bottom elevation of 20 ft below local sea level. The flow system extends 6,000 ft landward from the coast to a mountainous area underlain by impermeable rocks. The flow system is simulated by two model layers separated by a confining unit with a very low hydraulic conductivity. The width of the modeled area parallel to the coast is 4,000 ft. The model grid consists of 20 rows and 30 columns, and each grid cell is 200 ft by 200 ft (fig. 4). The hydraulic conductivity of each aquifer is homogeneous and isotropic. The hydraulic conductivity of the unconfined aquifer is 5 ft/d; the transmissivity of the confined aquifer is 800 ft^2/d. The confining unit is modeled implicitly by use of a vertical conductance between the layers equal to 0.05 d^{-1}. An existing production well is placed in the lower aquifer at model cell (2, 13, 20) and withdraws at a rate of 5,000 ft^3/d.

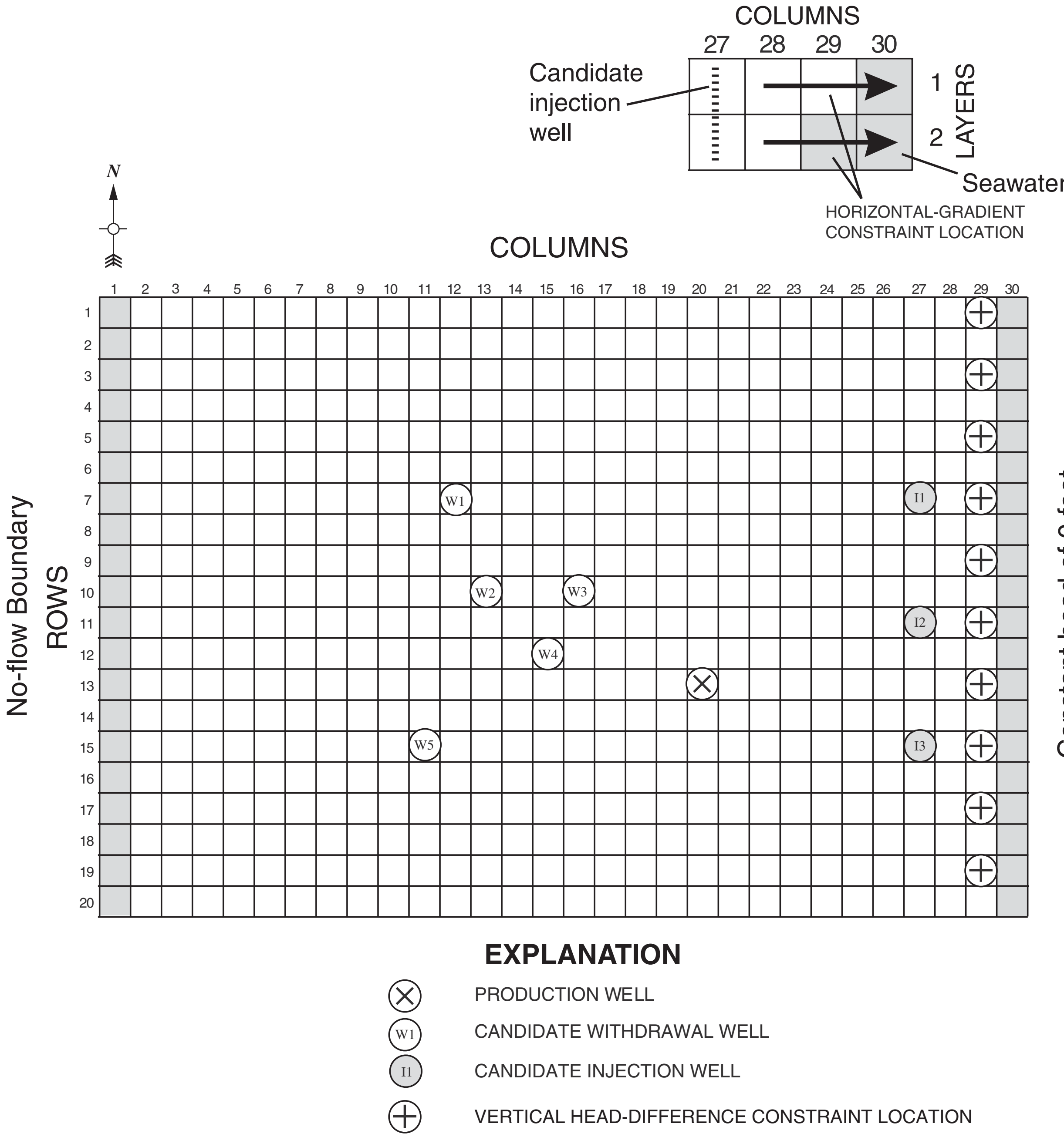

Figure 4. Model grid for SEAWATER sample problem.

Seawater at the coast is modeled by constant heads of 0 ft in column 30 of the top layer of the model and in columns 29 and 30 of the bottom layer of the model (fig. 4 inset). The model uses no-flow boundary conditions at the contact between the bottom of the lower aquifer and the impermeable rocks and along the western, northern, and southern boundaries of each layer. The upper aquifer is recharged at a uniform rate of 0.002 ft/d. As an example of the use of the Sensitivity Process to input model parameters, recharge is specified in the model by use of the RECH and SEN input files.

In the absence of pumping at the existing production well, there is a uniform gradient toward the coast of 2.67 ft/400 ft (0.00668 ft/ft) from column 28 to column 30 in the upper aquifer and of 2.62 ft/400 ft (0.00655 ft/ft) from column 28 to column 30 in the lower aquifer. There is also a downward head difference everywhere along the coast of 0.11 ft from the unconfined aquifer to the lower confined aquifer in column 29. With pumping at the production well, however, the gradients are reduced to a minimum value of 2.33 ft/400 ft (0.00583 ft/ft) from column 28 to column 30 in the upper aquifer and 2.28 ft/400 ft (0.0057 ft/ft) from column 28 to column 30 in the lower aquifer. Also, the downward head difference everywhere along the coast is lowered to about 0.10 ft.

The MODFLOW input files consist of a NAME file, a DIS-Package file, a BAS6-Package file, a BCF6-Package file, a WEL-Package file, a RCH-Package file, a SEN-Process file, and a PCG-Package file. Because the model simulates steady-state flow, the simulation time of the management problem is arbitrarily set to 1.0 day (NPER = 1, ITMUNI=4, and PERLEN = 1.0) in the discretization file. A very small head-change convergence criterion of 1.0E-8 ft is used.

Nonlinear Formulation

Water managers would like to develop additional ground-water supplies from the two aquifers at the five candidate withdrawal locations shown in figure 4 (W1, W2, and so forth), and simultaneously maintain a seaward hydraulic gradient of 1.50 ft/400 ft (0.00375 ft/ft) along the coast and a minimum downward gradient from the upper to the lower aquifer of 0.05 ft in column 29 of the modeled area. Three candidate injection wells (I1, I2, and I3) near the coastline have been proposed to help control the resulting seawater intrusion. Withdrawals from wells W1 through W4 would be from the unconfined aquifer (layer 1), whereas withdrawals from well W5 would be distributed evenly between the upper and lower aquifers (layers 1 and 2). Injection at each of the three candidate injection wells also would be distributed evenly between the upper and lower aquifers. The maximum rate of withdrawal or injection at any of the wells is 10,000 ft^3/d and the minimum rate of withdrawal or injection is 0.

The objective of the ground-water management problem is to maximize the sum of withdrawals from the five candidate withdrawal wells minus the amount of water that is reinjected to control seawater intrusion at the three candidate injection wells:

$$\text{Maximize } W1 + W2 + W3 + W4 + W5 - I1 - I2 - I3\,. \tag{78}$$

GWM multiplies each term in the objective function by the length of the single, steady-state stress period (1 day).

A linear-summation constraint is added to ensure (1) that the sum of withdrawals is greater than or equal to the sum of injections and (2) that the value of the objective function is positive:

$$W1 + W2 + W3 + W4 + W5 \geq I1 + I2 + I3\,. \tag{79}$$

Three types of head constraints are used to control hydraulic conditions along the coast. First, water-level mounding at the three candidate injection wells must be less than or equal to land surface at the three locations (5 ft above local sea level). Second, 10 vertical head-difference constraints are used at locations shown on figure 4 to maintain a downward hydraulic gradient from the upper aquifer to the lower aquifer along column 29 of the model. Finally, 20 horizontal head-gradient constraints are used to maintain a seaward hydraulic gradient of at least 1.50 ft/400 ft (0.00375 ft/ft) from column 28 to column 30 in each aquifer; these constraints are located in the odd-numbered rows (1, 3, 5, and so forth) and are shown schematically by the inset illustration on figure 4.

Although the objective function and many of the constraints of the problem are linear, the head constraints in the water-table aquifer are nonlinear. Therefore, sequential linear programming is selected in the SOLN file to solve the nonlinear formulation (SOLNTYP is set to SLP). An initial perturbation value of -5 percent (DINIT equal to -0.05) of the maximum withdrawal or injection rate of each candidate well is specified, which results in an initial perturbation withdrawal rate of 500 ft^3/d at each withdrawal well and -500 ft^3/d at each injection well. A minimum perturbation value of -0.005 percent (DMIN equal to -5.0×10^{-5}) is selected, which results in an asymptotic perturbation withdrawal rate of 0.5 ft^3/d at each withdrawal well and -0.5 ft^3/d at each injection well. Convergence criteria on the withdrawal/injection rates, ε_1, of 1.0 ft^3/d (SLPVCRIT equal to 1.0) and on the value of the objective function, ε_2, of 0.001 ft^3 (SLPZCRIT equal to 0.001) are specified. Base withdrawal and injection rates are set to 0 by specifying IBASE = 0 in the SOLN file and FVREF = 0.0d0 in the VARCON file for each withdrawal and injection well.

The GWM input files necessary for the problem formulation are DECVAR, OBJFNC, VARCON, SUMCON, HEDCON, and SOLN. These files are listed at the end of the sample problem.

Three iterations of the sequential linear-programming algorithm were required to satisfy the two convergence criteria ε_1 and ε_2 (see GLOBAL file output at end of sample problem). The value of the objective function at the optimal solution (that is, net withdrawal) is 14,088 ft^3 of water withdrawn each day. Nonzero withdrawals were calculated for three of the candidate withdrawal wells: W1 (10,000 ft^3/d), W2 (3,301 ft^3/d), and W5 (1,678 ft^3/d); therefore, total withdrawals are 14,978 ft^3/d. All three injection wells were selected, with a total daily injection rate into the three wells of 890 ft^3/d. Five of the hydraulic-gradient constraints were binding, and all of them were located in the bottom layer of the model (rows 5, 7, 13, 17, and 19). The “Solution Algorithm,” “Ground-Water Management Solution,” and “Final Flow Process Simulation” sections of the GLOBAL output file for this GWM run are listed at the end of the sample problem.

The optimal solution is highly dependent on the value of the hydraulic-gradient constraints specified at the coast; as the required gradient constraint is increased, the amount of net withdrawal decreases. For example, at a specified gradient of 1.7 ft/400 ft (0.00425 ft/ft), net withdrawal decreases to 10,666 ft^3/d, and for a specified gradient of 2.0 ft/400 ft (0.005 ft/ft), net withdrawal decreases to 544 ft^3/d.

Selected Input and Output Files for Sample Problem

Name File (seawater.nam):

```
GLOBAL 9  seawater.glo
LIST  10  seawater.lst
DIS   11  ..\data\seawater.dis
BAS6  12  ..\data\seawater.ba6
BCF6  13  ..\data\seawater.bc6
WEL   14  ..\data\seawater.wel
RCH   15  ..\data\seawater.rch
PCG   16  ..\data\seawater.pcg
SEN   17  ..\data\seawater.sen
GWM   18  ..\data\seawater.gwm
```

GWM File (seawater.gwm):

```
#SEAWATER Sample Problem, GWM file
#February 20, 2005
DECVAR  ..\data\seawater.decvar
OBJFNC  ..\data\seawater.objfnc
VARCON  ..\data\seawater.varcon
SUMCON  ..\data\seawater.sumcon
HEDCON  ..\data\seawater.hedcon
SOLN    ..\data\seawater.soln
```

Decision variable (DECVAR) File (seawater.decvar):

```
#SEAWATER Sample Problem, DECVAR file
#February 20, 2005
 1                            #1-IPRN
 8 0 0                        #2-NFVAR  NEVAR  NBVAR
 W1  1  1  7  12  W  Y  1     #3a-FVNAME NC LAY ROW COL FTYPE ...
 W2  1  1 10  13  W  Y  1
 W3  1  1 10  16  W  Y  1
 W4  1  1 12  15  W  Y  1
 W5  2  0  0   0  W  Y  1
     0.500 1 15 11            #3b-RATIO LAY ROW COL
     0.500 2 15 11
 I1  2  0  0   0  I  Y  1
     0.500 1 7 27
     0.500 2 7 27
 I2  2  0  0   0  I  Y  1
     0.500 1 11 27
     0.500 2 11 27
 I3  2  0  0   0  I  Y  1
     0.500 1 15 27
     0.500 2 15 27
```

Objective Function (OBJFNC) File (seawater.objfnc):

```
#SEAWATER Sample Problem, OBJFNC file
#February 20, 2005
 1          #1-IPRN
 MAX  WSDV  #2-OBJTYP FNTYP
 8  0  0    #3-NFVOBJ NEVOBJ NBVOBJ
 W1 1.00    #4-FVNAME FVOBJC
 W2 1.00
 W3 1.00
 W4 1.00
 W5 1.00
 I1 -1.00
 I2 -1.00
 I3 -1.00
```

Decision-variable constraints (VARCON) File (seawater.varcon):

```
#SEAWATER Sample Problem, VARCON file
#February 20, 2005
 1                         #1-IPRN
 W1 0.0d0  1.0d4  0.0d0    #2-FVNAME FVMIN FVMAX FVREF
 W2 0.0d0  1.0d4  0.0d0
 W3 0.0d0  1.0d4  0.0d0
 W4 0.0d0  1.0d4  0.0d0
 W5 0.0d0  1.0d4  0.0d0
 I1 0.0d0  1.0d4  0.0d0
 I2 0.0d0  1.0d4  0.0d0
 I3 0.0d0  1.0d4  0.0d0
```

Linear-summation constraint (SUMCON) File (seawater.sumcon):

```
#SEAWATER Sample Problem, SUMCON file
#February 20, 2005
 1              #1-IPRN
 1              #2-SMCNUM
 rech-a 8 ge 0. #3a-SMCNAME NTERMS TYPE RHS
  W1  1.0       #3b-GVNAME GVCOEFF
  W2  1.0
  W3  1.0
  W4  1.0
  W5  1.0
  I1 -1.0
  I2 -1.0
  I3 -1.0
```

Head constraints (HEDCON) File (seawater.hedcon):

```
#SEAWATER Sample Problem, HEDCON file
#February 20, 2005
 1                              #1-IPRN
 3 0  10  20                    #2-NHB NDD NDF NGD
 m-01  1  7 27  le 5. 1         #3-HBNAME LAYH ROWH COLH TYPH ...
 m-02  1 11 27  le 5. 1
 m-03  1 15 27  le 5. 1
 hd-01 1 1 29  2 1 29 0.05 1    #5-HDIFNAME LAY1 ROW1 COL1 ...
 hd-02 1 3 29  2 3 29 0.05 1
 hd-03 1 5 29  2 5 29 0.05 1
 hd-04 1 7 29  2 7 29 0.05 1
 hd-05 1 9 29  2 9 29 0.05 1
 hd-06 1 11 29  2 11 29 0.05 1
 hd-07 1 13 29  2 13 29 0.05 1
 hd-08 1 15 29  2 15 29 0.05 1
 hd-09 1 17 29  2 17 29 0.05 1
 hd-10 1 19 29  2 19 29 0.05 1
 gd-01 1 1 28 1 1 30 400. 0.00375 1 #6-GRADNAME (L,R,C)1 ...
 gd-02 2 1 28 2 1 30 400. 0.00375 1
 gd-03 1 3 28 1 3 30 400. 0.00375 1
 gd-04 2 3 28 2 3 30 400. 0.00375 1
 gd-05 1 5 28 1 5 30 400. 0.00375 1
 gd-06 2 5 28 2 5 30 400. 0.00375 1
 gd-07 1 7 28 1 7 30 400. 0.00375 1
 gd-08 2 7 28 2 7 30 400. 0.00375 1
 gd-09 1 9 28 1 9 30 400. 0.00375 1
 gd-10 2 9 28 2 9 30 400. 0.00375 1
 gd-11 1 11 28 1 11 30 400. 0.00375 1
 gd-12 2 11 28 2 11 30 400. 0.00375 1
 gd-13 1 13 28 1 13 30 400. 0.00375 1
 gd-14 2 13 28 2 13 30 400. 0.00375 1
 gd-15 1 15 28 1 15 30 400. 0.00375 1
 gd-16 2 15 28 2 15 30 400. 0.00375 1
 gd-17 1 17 28 1 17 30 400. 0.00375 1
 gd-18 2 17 28 2 17 30 400. 0.00375 1
 gd-19 1 19 28 1 19 30 400. 0.00375 1
 gd-20 2 19 28 2 19 30 400. 0.00375 1
```

Solution and output control file (SOLN) File (seawater.soln):

```
#SEAWATER Sample Problem, SOLN file
#February 20, 2005
 SLP                           #1-SOLNTYP
 10 1000  0                    #5a-SLPITMAX LPITMAX BBITMAX
1.0  0.001   -0.05 -0.00005 5  #5b-SLPVCRIT SLPZCRIT DINIT DMIN DSC
1  10  0.5  0.5   5           #5c-NSIGDIG NPGNMX PGFACT AFACT NINFMX
1 0 1                          #5d-SLPITPRT BBITPRT RANGE
 0                             #6a-IBASE
```

Selected output from the GLOBAL File (seawater.glo):

```
---------------------------------------------------------------
               Solution Algorithm
---------------------------------------------------------------
  Begin Solution Algorithm
    Running Base Flow Process Simulation


     0 Well parameters


     1 Recharge parameters

 PARAMETER NAME:REC-SP1      TYPE:RCH    CLUSTERS:   1
 Parameter value from package file is:    0.0000
 This value has been changed to:         2.00000E-03, as read from
 the Sensitivity Process file
               MULTIPLIER ARRAY: NONE    ZONE ARRAY: ALL

    1 PARAMETER HAS BEEN DEFINED IN ALL PACKAGES.
 (SPACE IS ALLOCATED FOR  500 PARAMETERS.)
      Status of Simulation-Based Constraints
      Constraint Type        Name     Status     Distance To RHS
      ---------------        ----     ------     ---------------
      Head Bound             m-01     Satisfied       3.7103E-01
      Head Bound             m-02     Satisfied       4.8707E-01
      Head Bound             m-03     Satisfied       5.1690E-01
      Head Difference        hd-01    Satisfied       5.3533E-02
      Head Difference        hd-02    Satisfied       5.3335E-02
      Head Difference        hd-03    Satisfied       5.2875E-02
      Head Difference        hd-04    Satisfied       5.2172E-02
      Head Difference        hd-05    Satisfied       5.1312E-02
      Head Difference        hd-06    Satisfied       5.0499E-02
      Head Difference        hd-07    Satisfied       5.0027E-02
      Head Difference        hd-08    Satisfied       5.0048E-02
      Head Difference        hd-09    Satisfied       5.0374E-02
      Head Difference        hd-10    Satisfied       5.0681E-02
      Head Gradient          gd-01    Satisfied       9.4952E-01
      Head Gradient          gd-02    Satisfied       8.9861E-01
      Head Gradient          gd-03    Satisfied       9.4284E-01
      Head Gradient          gd-04    Satisfied       8.9200E-01
      Head Gradient          gd-05    Satisfied       9.2728E-01
      Head Gradient          gd-06    Satisfied       8.7661E-01
      Head Gradient          gd-07    Satisfied       9.0348E-01
      Head Gradient          gd-08    Satisfied       8.5307E-01
      Head Gradient          gd-09    Satisfied       8.7427E-01
      Head Gradient          gd-10    Satisfied       8.2417E-01
      Head Gradient          gd-11    Satisfied       8.4655E-01
      Head Gradient          gd-12    Satisfied       7.9675E-01
      Head Gradient          gd-13    Satisfied       8.3044E-01
      Head Gradient          gd-14    Satisfied       7.8080E-01
      Head Gradient          gd-15    Satisfied       8.3121E-01
      Head Gradient          gd-16    Satisfied       7.8156E-01
      Head Gradient          gd-17    Satisfied       8.4240E-01
      Head Gradient          gd-18    Satisfied       7.9262E-01
      Head Gradient          gd-19    Satisfied       8.5287E-01
      Head Gradient          gd-20    Satisfied       8.0298E-01
```

```
  Calculating Response Matrix
    Perturb Flow Variable    1
     By Perturbation Value:  5.000000E+02
    Perturb Flow Variable    2
     By Perturbation Value:  5.000000E+02
    Perturb Flow Variable    3
     By Perturbation Value:  5.000000E+02
    Perturb Flow Variable    4
     By Perturbation Value:  5.000000E+02
    Perturb Flow Variable    5
     By Perturbation Value:  5.000000E+02
    Perturb Flow Variable    6
     By Perturbation Value: -5.000000E+02
    Perturb Flow Variable    7
     By Perturbation Value: -5.000000E+02
    Perturb Flow Variable    8
     By Perturbation Value: -5.000000E+02

    Average Number of Significant Digits in Matrix  6.393939E+00

  Solving Linear Program
  Optimal Solution Found
  Objective Value -1.412443E+04
  Maximum Relative Change in Flow Variable  9.999000E-01
SLP Algorithm: End Iteration    1
SLP Algorithm: Begin Iteration     2
  Running Base Flow Process Simulation
    Status of Simulation-Based Constraints
    Constraint Type        Name     Status     Distance To RHS
    ---------------        ----     ------     ---------------
    Head Bound             m-01     Satisfied       2.0549E+00
    Head Bound             m-02     Satisfied       1.8048E+00
    Head Bound             m-03     Satisfied       1.9387E+00
    Head Difference        hd-01    Satisfied       2.7829E-02
    Head Difference        hd-02    Satisfied       2.7734E-02
    Head Difference        hd-03    Satisfied       2.7586E-02
    Head Difference        hd-04    Satisfied       2.7594E-02
    Head Difference        hd-05    Satisfied       2.7646E-02
    Head Difference        hd-06    Satisfied       2.9108E-02
    Head Difference        hd-07    Satisfied       2.7611E-02
    Head Difference        hd-08    Satisfied       2.8253E-02
    Head Difference        hd-09    Satisfied       2.7601E-02
    Head Difference        hd-10    Satisfied       2.7596E-02
    Head Gradient          gd-01    Satisfied       4.9407E-02
    Head Gradient          gd-02    Satisfied       6.4056E-03
    Head Gradient          gd-03    Satisfied       4.5939E-02
    Head Gradient          gd-04    Satisfied       2.9631E-03
    Head Gradient          gd-05    Satisfied       4.0503E-02
    Head Gradient          gd-06    Not Met         2.4294E-03
    Head Gradient          gd-07    Satisfied       4.0970E-02
    Head Gradient          gd-08    Not Met         2.3047E-03
    Head Gradient          gd-09    Satisfied       4.2385E-02
    Head Gradient          gd-10    Not Met         5.2509E-04
    Head Gradient          gd-11    Satisfied       9.7447E-02
    Head Gradient          gd-12    Satisfied       5.1164E-02
    Head Gradient          gd-13    Satisfied       4.0931E-02
    Head Gradient          gd-14    Not Met         1.9472E-03
    Head Gradient          gd-15    Satisfied       6.5654E-02
    Head Gradient          gd-16    Satisfied       2.0929E-02
    Head Gradient          gd-17    Satisfied       4.0929E-02
    Head Gradient          gd-18    Not Met         1.9924E-03
```

```
    Head Gradient          gd-19    Satisfied       4.0902E-02
    Head Gradient          gd-20    Not Met         2.0380E-03

  Calculating Response Matrix
    Perturb Flow Variable    1
     By Perturbation Value:  1.004000E+02
    Perturb Flow Variable    2
     By Perturbation Value:  1.004000E+02
    Perturb Flow Variable    3
     By Perturbation Value:  1.004000E+02
    Perturb Flow Variable    4
     By Perturbation Value:  1.004000E+02
    Perturb Flow Variable    5
     By Perturbation Value:  1.004000E+02
    Perturb Flow Variable    6
     By Perturbation Value: -1.004000E+02
    Perturb Flow Variable    7
     By Perturbation Value: -1.004000E+02
    Perturb Flow Variable    8
     By Perturbation Value: -1.004000E+02

    Average Number of Significant Digits in Matrix  5.742424E+00

  Solving Linear Program
  Optimal Solution Found
  Objective Value -1.408782E+04
  Maximum Relative Change in Flow Variable  7.213245E-03
SLP Algorithm: End Iteration    2
SLP Algorithm: Begin Iteration     3
  Running Base Flow Process Simulation
    Status of Simulation-Based Constraints
    Constraint Type        Name     Status     Distance To RHS
    ---------------        ----     ------     ---------------
    Head Bound             m-01     Satisfied       2.0505E+00
    Head Bound             m-02     Satisfied       1.8023E+00
    Head Bound             m-03     Satisfied       1.9355E+00
    Head Difference        hd-01    Satisfied       2.7897E-02
    Head Difference        hd-02    Satisfied       2.7802E-02
    Head Difference        hd-03    Satisfied       2.7653E-02
    Head Difference        hd-04    Satisfied       2.7658E-02
    Head Difference        hd-05    Satisfied       2.7706E-02
    Head Difference        hd-06    Satisfied       2.9156E-02
    Head Difference        hd-07    Satisfied       2.7665E-02
    Head Difference        hd-08    Satisfied       2.8304E-02
    Head Difference        hd-09    Satisfied       2.7656E-02
    Head Difference        hd-10    Satisfied       2.7652E-02
    Head Gradient          gd-01    Satisfied       5.1911E-02
    Head Gradient          gd-02    Satisfied       8.8922E-03
    Head Gradient          gd-03    Satisfied       4.8431E-02
    Head Gradient          gd-04    Satisfied       5.4380E-03
    Head Gradient          gd-05    Satisfied       4.2950E-02
    Head Gradient          gd-06    Near-Binding    1.4160E-08
    Head Gradient          gd-07    Satisfied       4.3288E-02
    Head Gradient          gd-08    Near-Binding    2.4126E-08
    Head Gradient          gd-09    Satisfied       4.4562E-02
    Head Gradient          gd-10    Satisfied       1.6376E-03
    Head Gradient          gd-11    Satisfied       9.9181E-02
    Head Gradient          gd-12    Satisfied       5.2901E-02
    Head Gradient          gd-13    Satisfied       4.2891E-02
    Head Gradient          gd-14    Near-Binding    2.8028E-08
    Head Gradient          gd-15    Satisfied       6.7484E-02
```

```
      Head Gradient          gd-16    Satisfied       2.2754E-02
      Head Gradient          gd-17    Satisfied       4.2935E-02
      Head Gradient          gd-18    Near-Binding    1.4009E-08
      Head Gradient          gd-19    Satisfied       4.2954E-02
      Head Gradient          gd-20    Near-Binding    8.0249E-09

    Calculating Response Matrix
      Perturb Flow Variable    1
       By Perturbation Value:  2.048000E+01
      Perturb Flow Variable    2
       By Perturbation Value:  2.048000E+01
      Perturb Flow Variable    3
       By Perturbation Value:  2.048000E+01
      Perturb Flow Variable    4
       By Perturbation Value:  2.048000E+01
      Perturb Flow Variable    5
       By Perturbation Value:  2.048000E+01
      Perturb Flow Variable    6
       By Perturbation Value: -2.048000E+01
      Perturb Flow Variable    7
       By Perturbation Value: -2.048000E+01
      Perturb Flow Variable    8
       By Perturbation Value: -2.048000E+01

      Average Number of Significant Digits in Matrix  5.234848E+00

    Solving Linear Program
    Optimal Solution Found
    Objective Value -1.408782E+04
    Maximum Relative Change in Flow Variable  4.443988E-08
  SLP Algorithm: End Iteration    3
  Iterations have converged

----------------------------------------------------------------------
               Ground-Water Management Solution
----------------------------------------------------------------------

       OPTIMAL SOLUTION FOUND

       OPTIMAL RATES FOR EACH FLOW VARIABLE
       ---------------------------------------
Variable           Withdrawal          Injection           Contribution
Name               Rate                Rate                To Objective
----------         --------------      ------------        ------------
 W1                1.000000E+04                            1.000000E+04
 W2                3.300918E+03                            3.300918E+03
 W3                0.000000E+00                            0.000000E+00
 W4                0.000000E+00                            0.000000E+00
 W5                1.677461E+03                            1.677461E+03
 I1                                    6.293964E+01       -6.293964E+01
 I2                                    5.321156E+02       -5.321156E+02
 I3                                    2.955083E+02       -2.955083E+02
                   ------------        ------------        ------------
TOTALS             1.497838E+04        8.905636E+02        1.408782E+04
```

```
       OBJECTIVE FUNCTION VALUE                            1.408782E+04

       BINDING CONSTRAINTS
Constraint Type        Name     Status      Shadow Price
---------------        ----     ------      ------------
Head Gradient          gd-06    Binding     -5.7711E+03
Head Gradient          gd-08    Binding     -9.5887E+02
Head Gradient          gd-14    Binding     -8.0729E+03
Head Gradient          gd-18    Binding     -1.4635E+02
Head Gradient          gd-20    Binding     -2.1473E+03
Maximum Flow Rate      W1       Binding    Not Available

Binding constraint and range analysis values are determined from the linear
program and based on the response matrix approximation of the flow-process.

       RANGE ANALYSIS


       Constraint Ranges

Lower/Upper Bound are the values of the RHS beyond which basis will change.
    Leaving is the variable which will leave the basis.
    Entering is the variable which will enter the basis.
    If the entering or leaving variable is a constraint name,
      then the constraint slack variable is active

Constraint             Original      Lower/Upper
Name        Slack      RHS           Bound          Entering      Leaving
----------  ---------- ----------    ----------     ----------    --------
m-01        2.0505E+00  5.0000E+00   2.9495E+00     gd-06         m-01
                                     Infinity       ----- No Change -----

m-02        1.8023E+00  5.0000E+00   3.1977E+00     gd-18         m-02
                                     Infinity       ----- No Change -----

m-03        1.9355E+00  5.0000E+00   3.0645E+00     gd-20         m-03
                                     Infinity       ----- No Change -----

hd-01       2.7897E-02  5.0000E-02   -Infinity      ----- No Change -----
                                     7.7897E-02     gd-06         hd-01

hd-02       2.7802E-02  5.0000E-02   -Infinity      ----- No Change -----
                                     7.7802E-02     gd-06         hd-02

hd-03       2.7653E-02  5.0000E-02   -Infinity      ----- No Change -----
                                     7.7653E-02     gd-06         hd-03

hd-04       2.7658E-02  5.0000E-02   -Infinity      ----- No Change -----
                                     7.7658E-02     gd-08         hd-04

hd-05       2.7706E-02  5.0000E-02   -Infinity      ----- No Change -----
                                     7.7706E-02     gd-20         hd-05

hd-06       2.9156E-02  5.0000E-02   -Infinity      ----- No Change -----
                                     7.9156E-02     gd-20         hd-06

hd-07       2.7665E-02  5.0000E-02   -Infinity      ----- No Change -----
                                     7.7665E-02     gd-14         hd-07

hd-08       2.8304E-02  5.0000E-02   -Infinity      ----- No Change -----
                                     7.8304E-02     gd-18         hd-08
```

```
hd-09      2.7656E-02  5.0000E-02   -Infinity    ----- No Change -----
                                    7.7656E-02   gd-18        hd-09

hd-10      2.7652E-02  5.0000E-02   -Infinity    ----- No Change -----
                                    7.7652E-02   gd-20        hd-10

gd-01      5.1911E-02  1.5000E+00   -Infinity    ----- No Change -----
                                    1.5519E+00   gd-06        gd-01

gd-02      8.8922E-03  1.5000E+00   -Infinity    ----- No Change -----
                                    1.5089E+00   gd-06        gd-02

gd-03      4.8431E-02  1.5000E+00   -Infinity    ----- No Change -----
                                    1.5484E+00   gd-06        gd-03

gd-04      5.4380E-03  1.5000E+00   -Infinity    ----- No Change -----
                                    1.5054E+00   gd-06        gd-04

gd-05      4.2950E-02  1.5000E+00   -Infinity    ----- No Change -----
                                    1.5429E+00   gd-06        gd-05

gd-06      0.0000E+00  1.5000E+00   1.4962E+00   gd-06        gd-04
                                    1.5100E+00   gd-18        I1

gd-07      4.3288E-02  1.5000E+00   -Infinity    ----- No Change -----
                                    1.5433E+00   gd-08        gd-07

gd-08      0.0000E+00  1.5000E+00   1.4935E+00   gd-20        gd-10
                                    1.5133E+00   gd-06        gd-04

gd-09      4.4562E-02  1.5000E+00   -Infinity    ----- No Change -----
                                    1.5446E+00   gd-20        gd-09

gd-10      1.6375E-03  1.5000E+00   -Infinity    ----- No Change -----
                                    1.5016E+00   gd-20        gd-10

gd-11      9.9181E-02  1.5000E+00   -Infinity    ----- No Change -----
                                    1.5992E+00   gd-20        gd-11

gd-12      5.2901E-02  1.5000E+00   -Infinity    ----- No Change -----
                                    1.5529E+00   gd-20        gd-12

gd-13      4.2891E-02  1.5000E+00   -Infinity    ----- No Change -----
                                    1.5429E+00   gd-14        gd-13

gd-14      0.0000E+00  1.5000E+00   1.4984E+00   gd-20        gd-10
                                    1.5301E+00   gd-18        I1

gd-15      6.7484E-02  1.5000E+00   -Infinity    ----- No Change -----
                                    1.5675E+00   gd-18        gd-15

gd-16      2.2754E-02  1.5000E+00   -Infinity    ----- No Change -----
                                    1.5228E+00   gd-1         gd-16

gd-17      4.2935E-02  1.5000E+00   -Infinity    ----- No Change -----
                                    1.5429E+00   gd-18        gd-17

gd-18      0.0000E+00  1.5000E+00   1.4935E+00   gd-18        gd-16
                                    1.5015E+00   gd-20        gd-10
```

```
gd-19       4.2954E-02  1.5000E+00    -Infinity     ----- No Change -----
                                      1.5430E+00    gd-20         gd-19

gd-20       0.0000E+00  1.5000E+00    1.4975E+00    gd-20         gd-10
                                      1.5094E+00    gd-18         gd-16

rech-a      1.4088E+04  0.0000E+00    -Infinity     ----- No Change -----
                                      1.4088E+04    -- Degenerate Basis -


       Objective-Function Coefficient Ranges

Lower/Upper Bound are the values of the coefficients beyond which basis
will change.
    Leaving is the variable which will leave the basis.
    Entering is the variable which will enter the basis.
    If the entering or leaving variable is a constraint name,
      then the constraint slack variable is active
    Basic variables are shown with zero reduced cost



Variable   Reduced      Original      Lower/Upper
Name       Cost         Coefficient   Bound         Entering      Leaving
---------  ----------   ----------    ----------    ----------    -------
W1         3.1892E-03   1.0000E+00    9.9681E-01    W1            W5
                                      Infinity      ----- No Change -----

W2         0.0000E+00   1.0000E+00    9.9801E-01    gd-18         gd-10
                                      1.0020E+00    W1            W5

W3         -3.4953E-03  1.0000E+00    Infinity      ----- No Change -----
                                      1.0035E+00    W3            W2

W4         -3.5766E-03  1.0000E+00    Infinity      ----- No Change -----
                                      1.0036E+00    W4            W5

W5         0.0000E+00   1.0000E+00    9.9466E-01    W1            W5
                                      1.0018E+00    gd-18         gd-10

I1         0.0000E+00   -1.0000E+00   -1.9137E+00   gd-06         I1
                                      -9.4052E-01   gd-18         gd-10

I2         0.0000E+00   -1.0000E+00   -1.0083E+00   gd-18         gd-10
                                      -7.8592E-01   W3            W2

I3         0.0000E+00   -1.0000E+00   -1.1114E+00   gd-20         gd-16
                                      -9.9345E-01   gd-18         gd-10
```

```
----------------------------------------------------------------------
               Final Flow Process Simulation
----------------------------------------------------------------------
      Status of Simulation-Based Constraints
        Using Optimal Flow Rate Variable Values
      Constraint Type        Name     Status     Distance To RHS
      ---------------        ----     ------     ---------------
      Head Bound             m-01     Satisfied       2.0505E+00
      Head Bound             m-02     Satisfied       1.8023E+00
      Head Bound             m-03     Satisfied       1.9355E+00
      Head Difference        hd-01    Satisfied       2.7897E-02
      Head Difference        hd-02    Satisfied       2.7802E-02
      Head Difference        hd-03    Satisfied       2.7653E-02
      Head Difference        hd-04    Satisfied       2.7658E-02
      Head Difference        hd-05    Satisfied       2.7706E-02
      Head Difference        hd-06    Satisfied       2.9156E-02
      Head Difference        hd-07    Satisfied       2.7665E-02
      Head Difference        hd-08    Satisfied       2.8304E-02
      Head Difference        hd-09    Satisfied       2.7656E-02
      Head Difference        hd-10    Satisfied       2.7652E-02
      Head Gradient          gd-01    Satisfied       5.1911E-02
      Head Gradient          gd-02    Satisfied       8.8922E-03
      Head Gradient          gd-03    Satisfied       4.8431E-02
      Head Gradient          gd-04    Satisfied       5.4380E-03
      Head Gradient          gd-05    Satisfied       4.2950E-02
      Head Gradient          gd-06    Near-Binding    1.4530E-08
      Head Gradient          gd-07    Satisfied       4.3288E-02
      Head Gradient          gd-08    Near-Binding    1.3689E-08
      Head Gradient          gd-09    Satisfied       4.4562E-02
      Head Gradient          gd-10    Satisfied       1.6376E-03
      Head Gradient          gd-11    Satisfied       9.9181E-02
      Head Gradient          gd-12    Satisfied       5.2901E-02
      Head Gradient          gd-13    Satisfied       4.2891E-02
      Head Gradient          gd-14    Near-Binding    1.2701E-08
      Head Gradient          gd-15    Satisfied       6.7484E-02
      Head Gradient          gd-16    Satisfied       2.2754E-02
      Head Gradient          gd-17    Satisfied       4.2935E-02
      Head Gradient          gd-18    Near-Binding    1.2746E-08
      Head Gradient          gd-19    Satisfied       4.2954E-02
      Head Gradient          gd-20    Near-Binding    1.2570E-08

  Because of precision limitations and possible nonlinear behavior,
    the status of binding constraints computed directly by the flow process
    may differ slightly from those computed using the linear program.
```

Sample Problem 3: SUPPLY

This sample problem represents a transient water-supply problem in which total ground-water withdrawals over a 3-year period are limited by the amount of streamflow depletion allowed in two streams that are in hydraulic connection with the aquifer (fig. 5). The 3-year period is divided into 12 seasons (winter, spring, summer, and fall of each year), each of which is represented by a single stress period. The aquifer is confined and the area of interest is 6,000 ft long by 5,000 ft wide. The model consists of a single layer with 25 rows and 30 columns; each model cell is 200 ft by 200 ft. The aquifer is homogenous and isotropic with a transmissivity of 5,000 ft^2/d and a storage coefficient of 0.05. The modeled area is bounded on the east and west by no-flow conditions and on the north and south by constant heads that decrease in elevation from west to east (fig. 5). The aquifer is recharged at a rate of 0.0005 ft/d in the winter, 0.002 ft/d in the spring, 0 ft/d in the summer, and 0.001 ft/d in the fall. The two streams are simulated with the MODFLOW Stream (STR1) Package (Prudic, 1989) by use of three stream segments. Both streams are 20 ft wide and have a streambed conductance of 20,000 ft^2/d. The mainstem has a slope of 0.0025, whereas the tributary stream has a slope of 0.0010.

The MODFLOW input files consist of a NAME file, a DIS-Package file, a BAS6-Package file, a BCF6-Package file, a RCH-Package file, a STR1-Package file, an OC-Package file, and a PCG-Package file.

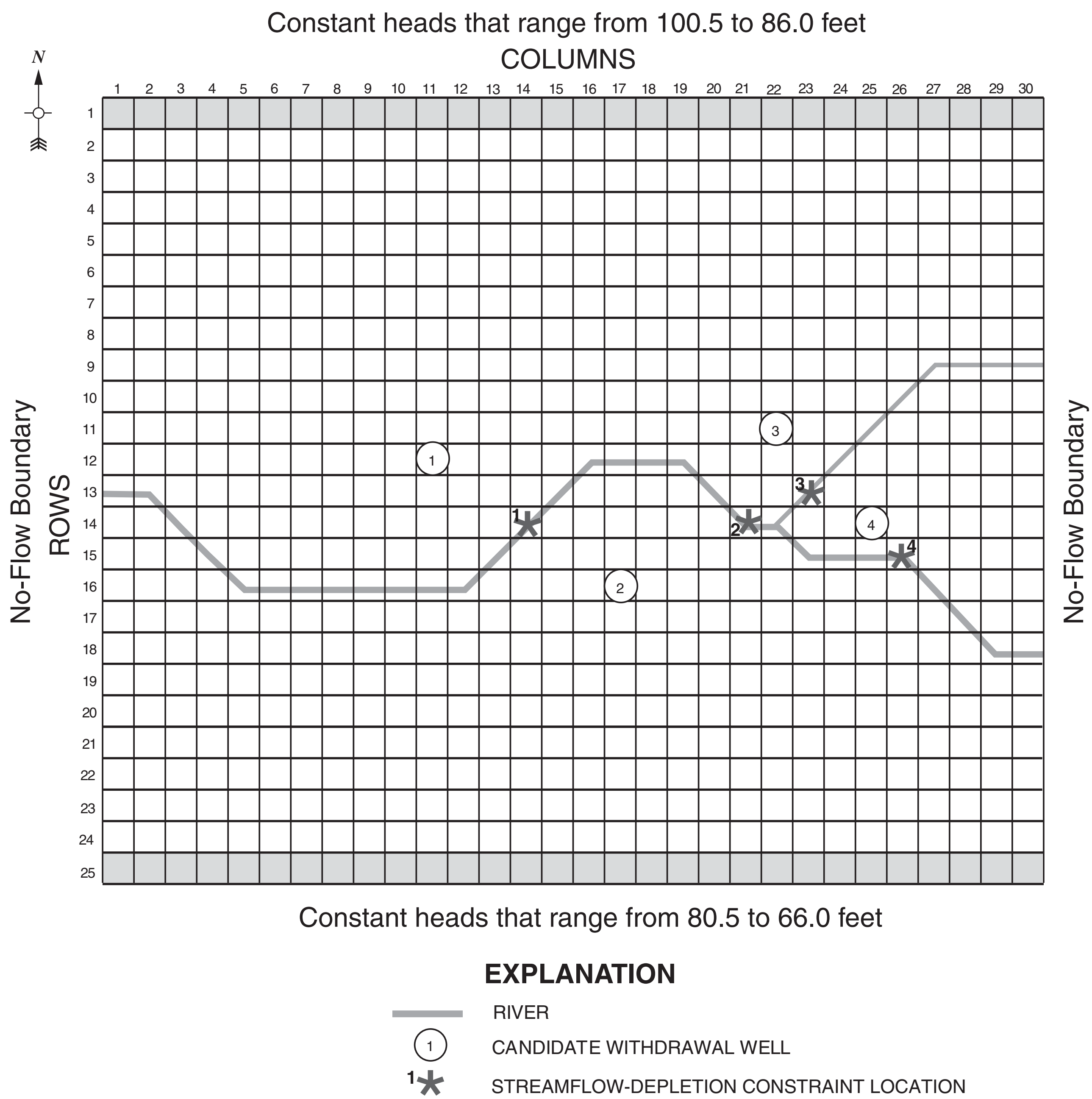

Figure 5. Model grid for SUPPLY sample problem.

Nonlinear Formulation

The management objective is to maximize the value of ground water withdrawn from four wells over the 3-year period, while simultaneously limiting streamflow depletions at four streamflow-constraint locations along the two streams (fig. 5). Because it may be advantageous to have some of the wells pump at different rates during each season, multiple decision variables are defined at well sites 2 and 4. Well 2 is allowed to have one withdrawal rate during the winter (decision variable Q2a, stress periods 1, 5, and 9), another during spring (Q2b, stress periods 2, 6, and 10), another during summer (Q2c, stress periods 3, 7, and 11), and another during fall (Q2d, stress periods 4, 8, and 12). Well 4 only withdraws during the spring and fall, and is allowed to have one withdrawal rate during the spring (Q4a, stress periods 2, 6, and 10) and a second withdrawal rate during the fall (Q4b, stress periods 4, 8, and 12). Well 1 must have a constant withdrawal rate for all 12 stress periods and well 3 is allowed to withdraw only during the second year (stress periods 5-8). The lower and upper bounds on withdrawals at each well are 0 and 50,000 ft^3/d, respectively. There is a net benefit from pumping at each well of \$1 per 1,000 ft^3 of water withdrawn (that is, \$0.001/ft^3).

Importation of water from an external source is allowed in each of the last four stress periods. These external variables are named Im9, Im10, Im11, and Im12. The maximum amount of water that can be imported each season is 1,000,000 ft^3/d. The cost to import the water is \$0.0012/ft^3. Because this is a cost rather than a benefit, it appears in the objective function with a negative coefficient.

The objective function of the problem is, therefore,

$$\begin{aligned}
&\text{Maximize}\\
&\left(\frac{\$0.001}{ft^3}\right)Q1 + \left(\frac{\$0.001}{ft^3}\right)Q2a + \left(\frac{\$0.001}{ft^3}\right)Q2b + \left(\frac{\$0.001}{ft^3}\right)Q2c + \left(\frac{\$0.001}{ft^3}\right)Q2d \\
&+ \left(\frac{\$0.001}{ft^3}\right)Q3 + \left(\frac{\$0.001}{ft^3}\right)Q4a + \left(\frac{\$0.001}{ft^3}\right)Q4b - \left(\frac{\$0.0012}{ft^3}\right)Im9 \\
&- \left(\frac{\$0.0012}{ft^3}\right)Im10 - \left(\frac{\$0.0012}{ft^3}\right)Im11 - \left(\frac{\$0.0012}{ft^3}\right)Im12 \quad ,
\end{aligned} \tag{80}$$

and the units of the objective function are dollars (that is, dollars per cubic foot multiplied by cubic feet per day multiplied by days of pumping at each well).

There are constraints that specify the upper and lower bounds on the total amounts of water that must be withdrawn each season to meet water-supply demands. The upper bound on the total withdrawals needed each season is 80,000 ft^3/d, whereas the lower bound on total withdrawals is 30,000 ft^3/d during the first year, 25,000 ft^3/d during the second year, and 45,000 ft^3/d during the third year. These water-supply demands are specified using linear-summation constraints that include both the flow-rate and external variables.

Allowed streamflow depletions at constraint locations 1 and 3 (fig. 5) are 15,000 ft^3/d during each season, those at constraint location 2 are 20,000 ft^3/d during the summer and fall, and those at constraint location 4 are 30,000 ft^3/d during the summer and fall.

Although the aquifer is confined and the objective function is linear, the streamflow-depletion constraints are nonlinear because of the presence of the head-dependent boundary conditions at the streams. Nonlinearities arise at these boundaries in two ways. First, there may be a nonlinear relation between pumping at the four well sites and seepage rates across the streambeds (see fig. 1); second, the iterative method that is used to calculate the stage of each stream reach can produce nonlinear relations between pumping rates and streamflow (Prudic, 1989, p. 10-11). Therefore, sequential linear programming is selected in the SOLN file to solve the nonlinear formulation (SOLNTYP is set to SLP). An initial perturbation value of 20 percent (DINIT equal to 0.2) of the maximum withdrawal rate of each candidate well is specified, which results in an initial perturbation withdrawal rate of 10,000 ft^3/d at each well. A minimum perturbation value of 0.002 percent (DMIN equal to 2.0×10^{-5}) is selected, which results in an asymptotic perturbation withdrawal rate of 1.0 ft^3/d. Convergence criteria on the withdrawal/injection rates, ε_1, of 1.0×10^{-5} ft^3/d (SLPVCRIT equal to 1.0×10^{-5}) and on the value of the objective function, ε_2, of \$$1.0 \times 10^{-4}$ (SLPZCRIT equal to 1.0×10^{-4}) are specified. Base withdrawal rates are set to 0 ft^3/d by specifying IBASE = 0 in the SOLN file and FVREF = 0.0d0 in the VARCON file for each well.

The GWM input files necessary for the problem formulation are: DECVAR, OBJFNC, VARCON, SUMCON, STRMCON, and SOLN. These files are listed at the end of the sample problem.

Three iterations of the sequential linear programming algorithm were required to satisfy the two convergence criteria ε_1 and ε_2 . The value of the objective function at the optimal solution is \$53,022, which is the value of the water withdrawn over the 3-year period. All eight decision variables are active in the solution, although the rate calculated for well 3 (decision variable Q3) is inconsequential. Withdrawal rates at the other three well sites range from 3,483 ft^3/d at well 1 (decision variable Q1) to a maximum of 50,000 ft^3/d at well 2 during the winter (decision variable Q2a). Two of the four external variables are active in the solution. Six of the streamflow-depletion constraints are binding: at constraint site 1 in winter of the third year, at constraint site 2 in summer and fall of the third year, at constraint site 3 in spring of the third year, and at constraint site 4 in fall of the third and fourth years. The “Solution Algorithm,” “Ground-Water Management Solution,” and “Final Flow Process Simulation” sections of the GLOBAL output file for this GWM run are listed at the end of the sample problem.

Selected Input and Output Files for Sample Problem

Name File (supply.nam):

```
GLOBAL   9   supply.glo
LIST    10   supply.lst
DIS     11   ..\data\supply.dis
BAS6    12   ..\data\supply.ba6
BCF6    13   ..\data\supply.bc6
RCH     15   ..\data\supply.rch
STR     16   ..\data\supply.str
OC      17   ..\data\supply.oc
PCG     18   ..\data\supply.pcg
GWM     19   ..\data\supply.gwm
```

GWM File (supply.gwm):

```
#SUPPLY Sample Problem, GWM file
#February 20, 2005
DECVAR  ..\data\supply.decvar
OBJFNC  ..\data\supply.objfnc
VARCON  ..\data\supply.varcon
SUMCON  ..\data\supply.sumcon
STRMCON ..\data\supply.strmcon
SOLN    ..\data\supply.soln
```

Decision Variable (DECVAR) File (supply.decvar):

```
#SUPPLY Sample Problem, DECVAR file
#February 20, 2005
 1                              #1-IPRN
 8 4 0                          #2-NFVAR NEVAR NBVAR
 Q1  1  1  12  11  W  Y  1-12     #3a-FVNAME NC LAY ROW COL ...
 Q2a 1  1  16  17  W  Y  1:5:9
 Q2b 1  1  16  17  W  Y  2:6:10
 Q2c 1  1  16  17  W  Y  3:7:11
 Q2d 1  1  16  17  W  Y  4:8:12
 Q3  1  1  11  22  W  Y  5-8
 Q4a 1  1  14  25  W  Y  2:6:10
 Q4b 1  1  14  25  W  Y  4:8:12
 Im9         IM     9           #4-EVNAME ETYPE ESP
 Im10        IM    10
 Im11        IM    11
 Im12        IM    12
```

Objective Function (OBJFNC) File (supply.objfnc):

```
#SUPPLY Sample Problem, OBJFNC file
#February 20, 2005
 1              #1-IPRN
 MAX  WSDV      #2-OBJTYP FNTYP
 8  4  0        #3-NFVOBJ NEVOBJ  NBVOBJ
 Q1  0.001      #4-FVNAME  FVOBJC
 Q2a 0.001
 Q2b 0.001
 Q2c 0.001
 Q2d 0.001
 Q3  0.001
 Q4a 0.001
 Q4b 0.001
 Im9    -0.0012  #5-EVNAME  EVOBJC
 Im10   -0.0012
 Im11   -0.0012
 Im12   -0.0012
```

Decision-variable constraints (VARCON) File (supply.varcon):

```
#SUPPLY Sample Problem, VARCON file
#February 20, 2005
 1                           #1-IPRN
 Q1  0.0  5.0d4  0.0D2       #2-FVNAME FVMIN FVMAX FVREF
 Q2a 0.0  5.0d4  0.0D2
 Q2b 0.0  5.0d4  0.0D2
 Q2c 0.0  5.0d4  0.0D2
 Q2d 0.0  5.0d4  0.0D2
 Q3  0.0  5.0d4  0.0D2
 Q4a 0.0  5.0d4  0.0D2
 Q4b 0.0  5.0d4  0.0D2
 Im9  0.0  1.0d6             #3-EVNAME EVMIN EVMAX
 Im10 0.0  1.0d6
 Im11 0.0  1.0d6
 Im12 0.0  1.0d6
```

Part of the linear-summation constraints (SUMCON) File (supply.sumcon):

```
#SUPPLY Sample Problem, SUMCON file
#February 20, 2005
 1                      #1-IPRN
 24                     #2-SMCNUM
 p01u 2 le 80000.       #3a-SMCNAME NTERMS TYPE RHS
  Q1  1.0               #3b-GVNAME GVCOEFF
  Q2a 1.0
 p01l 2 ge 30000.
  Q1  1.0
  Q2a 1.0
 p02u 3 le 80000.
  Q1  1.0
  Q2b 1.0
  Q4a 1.0
 p02l 3 ge 30000.
  Q1  1.0
  Q2b 1.0
  Q4a 1.0
**12 constraints deleted here**
 p09u 3 le 80000.
  Q1  1.0
  Q2a 1.0
  Im9 1.0
 p09l 3 ge 45000.
  Q1  1.0
  Q2a 1.0
  Im9 1.0
 p10u 4 le 80000.
  Q1  1.0
  Q2b 1.0
  Q4a 1.0
  Im10 1.0
 p10l 4 ge 45000.
  Q1  1.0
  Q2b 1.0
  Q4a 1.0
  Im10 1.0
 p11u 3 le 80000.
  Q1  1.0
  Q2c 1.0
  Im11 1.0
 p11l 3 ge 45000.
  Q1  1.0
  Q2c 1.0
  Im11 1.0
 p12u 4 le 80000.
  Q1  1.0
  Q2d 1.0
  Q4b 1.0
  Im12 1.0
 p12l 4 ge 45000.
  Q1  1.0
  Q2d 1.0
  Q4b 1.0
  Im12 1.0
```

Streamflow constraints (STRMCON) File (supply.strmcon):

```
#SUPPLY Sample Problem, STRMCON file
#February 20, 2005
 1                            #1-IPRN
 0 36                         #2-NSF NSD
 S01.1 1 14 le 15000. 1       #4-SDNAME SEG REACH TYPSD BND NSP
 S01.2 1 14 le 15000. 2
 S01.3 1 14 le 15000. 3
 S01.4 1 14 le 15000. 4
 S01.5 1 14 le 15000. 5
 S01.6 1 14 le 15000. 6
 S01.7 1 14 le 15000. 7
 S01.8 1 14 le 15000. 8
 S01.9 1 14 le 15000. 9
 S01.10 1 14 le 15000. 10
 S01.11 1 14 le 15000. 11
 S01.12 1 14 le 15000. 12
 S02.3 1 21 le 20000. 3
 S02.4 1 21 le 20000. 4
 S02.7 1 21 le 20000. 7
 S02.8 1 21 le 20000. 8
 S02.11 1 21 le 20000. 11
 S02.12 1 21 le 20000. 12
 S03.1 2  8 le 15000. 1
 S03.2 2  8 le 15000. 2
 S03.3 2  8 le 15000. 3
 S03.4 2  8 le 15000. 4
 S03.5 2  8 le 15000. 5
 S03.6 2  8 le 15000. 6
 S03.7 2  8 le 15000. 7
 S03.8 2  8 le 15000. 8
 S03.9 2  8 le 15000. 9
 S03.10 2  8 le 15000. 10
 S03.11 2  8 le 15000. 11
 S03.12 2  8 le 15000. 12
 S04.3 3  5 le 30000. 3
 S04.4 3  5 le 30000. 4
 S04.7 3  5 le 30000. 7
 S04.8 3  5 le 30000. 8
 S04.11 3  5 le 30000. 11
 S04.12 3  5 le 30000. 12
```

Solution and output control file (SOLN) File (supply.soln):

```
#SUPPLY Sample Problem, SOLN file
#February 20, 2005
 SLP                               #1-SOLNTYP
 50 10000   2000                   #5a-SLPITMAX LPITMAX BBITMAX
0.00001  0.0001  0.2  0.00002 2 #5b-SLPVCRIT SLPZCRIT DINIT DMIN DSC
1   10  0.5  0.5   5          #5c-NSIGDIG NPGNMX PGFACT AFACT NINFMX
 1 1 0                             #5d-SLPITPRT BBITPRT RANGE
 0                                 #6a-IBASE
```

```
Selected output from the GLOBAL File (supply.glo):
---------------------------------------------------------------
               Solution Algorithm
---------------------------------------------------------------
  Begin Solution Algorithm
    Running Flow Process Simulation
      for both Reference and Base


     0 Recharge parameters


     0 Stream parameters
      Status of Simulation-Based Constraints
      Constraint Type        Name     Status     Distance To RHS
      ---------------        ----     ------     ---------------
      Stream Depletion       S01.1    Satisfied       1.5000E+04
      Stream Depletion       S01.2    Satisfied       1.5000E+04
      Stream Depletion       S01.3    Satisfied       1.5000E+04
      Stream Depletion       S01.4    Satisfied       1.5000E+04
      Stream Depletion       S01.5    Satisfied       1.5000E+04
      Stream Depletion       S01.6    Satisfied       1.5000E+04
      Stream Depletion       S01.7    Satisfied       1.5000E+04
      Stream Depletion       S01.8    Satisfied       1.5000E+04
      Stream Depletion       S01.9    Satisfied       1.5000E+04
      Stream Depletion       S01.10   Satisfied       1.5000E+04
      Stream Depletion       S01.11   Satisfied       1.5000E+04
      Stream Depletion       S01.12   Satisfied       1.5000E+04
      Stream Depletion       S02.3    Satisfied       2.0000E+04
      Stream Depletion       S02.4    Satisfied       2.0000E+04
      Stream Depletion       S02.7    Satisfied       2.0000E+04
      Stream Depletion       S02.8    Satisfied       2.0000E+04
      Stream Depletion       S02.11   Satisfied       2.0000E+04
      Stream Depletion       S02.12   Satisfied       2.0000E+04
      Stream Depletion       S03.1    Satisfied       1.5000E+04
      Stream Depletion       S03.2    Satisfied       1.5000E+04
      Stream Depletion       S03.3    Satisfied       1.5000E+04
      Stream Depletion       S03.4    Satisfied       1.5000E+04
      Stream Depletion       S03.5    Satisfied       1.5000E+04
      Stream Depletion       S03.6    Satisfied       1.5000E+04
      Stream Depletion       S03.7    Satisfied       1.5000E+04
      Stream Depletion       S03.8    Satisfied       1.5000E+04
      Stream Depletion       S03.9    Satisfied       1.5000E+04
      Stream Depletion       S03.10   Satisfied       1.5000E+04
      Stream Depletion       S03.11   Satisfied       1.5000E+04
      Stream Depletion       S03.12   Satisfied       1.5000E+04
      Stream Depletion       S04.3    Satisfied       3.0000E+04
      Stream Depletion       S04.4    Satisfied       3.0000E+04
      Stream Depletion       S04.7    Satisfied       3.0000E+04
      Stream Depletion       S04.8    Satisfied       3.0000E+04
      Stream Depletion       S04.11   Satisfied       3.0000E+04
      Stream Depletion       S04.12   Satisfied       3.0000E+04

    Calculating Response Matrix
      Perturb Flow Variable    1
       By Perturbation Value: -1.000000E+04
      Perturb Flow Variable    2
       By Perturbation Value: -1.000000E+04
      Perturb Flow Variable    3
       By Perturbation Value: -1.000000E+04
      Perturb Flow Variable    4
```

```
       By Perturbation Value: -1.000000E+04
      Perturb Flow Variable    5
       By Perturbation Value: -1.000000E+04
      Perturb Flow Variable    6
       By Perturbation Value: -1.000000E+04
      Perturb Flow Variable    7
       By Perturbation Value: -1.000000E+04
      Perturb Flow Variable    8
       By Perturbation Value: -1.000000E+04

      Average Number of Significant Digits in Matrix  7.019841E+00

    Solving Linear Program
    Optimal Solution Found
    Objective Value -5.302102E+04
    Maximum Relative Change in Flow Variable  9.999800E-01
    Max Relative External Variable Change  9.999316E-01
  SLP Algorithm: End Iteration    1
  SLP Algorithm: Begin Iteration     2
    Running Base Flow Process Simulation
      Status of Simulation-Based Constraints
      Constraint Type        Name     Status     Distance To RHS
      ---------------        ----     ------     ---------------
      Stream Depletion       S01.1    Satisfied       4.4820E+02
      Stream Depletion       S01.2    Satisfied       5.3103E+03
      Stream Depletion       S01.3    Satisfied       5.8133E+03
      Stream Depletion       S01.4    Satisfied       5.8671E+03
      Stream Depletion       S01.5    Near-Binding    5.7007E-02
      Stream Depletion       S01.6    Satisfied       5.2871E+03
      Stream Depletion       S01.7    Satisfied       5.8121E+03
      Stream Depletion       S01.8    Satisfied       5.8670E+03
      Stream Depletion       S01.9    Near-Binding    6.2400E-02
      Stream Depletion       S01.10   Satisfied       5.2871E+03
      Stream Depletion       S01.11   Satisfied       5.8121E+03
      Stream Depletion       S01.12   Satisfied       5.8670E+03
      Stream Depletion       S02.3    Satisfied       3.2657E+00
      Stream Depletion       S02.4    Near-Binding    2.5023E-01
      Stream Depletion       S02.7    Satisfied       1.3145E+00
      Stream Depletion       S02.8    Near-Binding    1.0742E-01
      Stream Depletion       S02.11   Satisfied       1.3145E+00
      Stream Depletion       S02.12   Near-Binding    1.0741E-01
      Stream Depletion       S03.1    Satisfied       1.4120E+04
      Stream Depletion       S03.2    Satisfied       2.5641E+00
      Stream Depletion       S03.3    Satisfied       1.4209E+04
      Stream Depletion       S03.4    Satisfied       1.2081E+04
      Stream Depletion       S03.5    Satisfied       1.4032E+04
      Stream Depletion       S03.6    Not Met         1.1239E+00
      Stream Depletion       S03.7    Satisfied       1.4209E+04
      Stream Depletion       S03.8    Satisfied       1.2081E+04
      Stream Depletion       S03.9    Satisfied       1.4032E+04
      Stream Depletion       S03.10   Not Met         1.1240E+00
      Stream Depletion       S03.11   Satisfied       1.4209E+04
      Stream Depletion       S03.12   Satisfied       1.2081E+04
      Stream Depletion       S04.3    Satisfied       6.4478E+03
      Stream Depletion       S04.4    Near-Binding    8.1515E-02
      Stream Depletion       S04.7    Satisfied       6.4453E+03
      Stream Depletion       S04.8    Near-Binding    1.2603E-01
      Stream Depletion       S04.11   Satisfied       6.4453E+03
      Stream Depletion       S04.12   Near-Binding    1.2603E-01
```

```
  Calculating Response Matrix
    Perturb Flow Variable    1
     By Perturbation Value: -5.000500E+03
    Perturb Flow Variable    2
     By Perturbation Value: -5.000500E+03
    Perturb Flow Variable    3
     By Perturbation Value: -5.000500E+03
    Perturb Flow Variable    4
     By Perturbation Value: -5.000500E+03
    Perturb Flow Variable    5
     By Perturbation Value: -5.000500E+03
    Perturb Flow Variable    6
     By Perturbation Value: -5.000500E+03
    Perturb Flow Variable    7
     By Perturbation Value: -5.000500E+03
    Perturb Flow Variable    8
     By Perturbation Value: -5.000500E+03

    Average Number of Significant Digits in Matrix  6.698413E+00

  Solving Linear Program
  Optimal Solution Found
  Objective Value -5.302168E+04
  Maximum Relative Change in Flow Variable  8.326215E-05
  Max Relative External Variable Change  1.369594E-04
SLP Algorithm: End Iteration    2
SLP Algorithm: Begin Iteration     3
  Running Base Flow Process Simulation
    Status of Simulation-Based Constraints
    Constraint Type        Name     Status     Distance To RHS
    ---------------        ----     ------     ---------------
    Stream Depletion       S01.1    Satisfied       4.4826E+02
    Stream Depletion       S01.2    Satisfied       5.3093E+03
    Stream Depletion       S01.3    Satisfied       5.8128E+03
    Stream Depletion       S01.4    Satisfied       5.8671E+03
    Stream Depletion       S01.5    Near-Binding    5.3802E-03
    Stream Depletion       S01.6    Satisfied       5.2861E+03
    Stream Depletion       S01.7    Satisfied       5.8116E+03
    Stream Depletion       S01.8    Satisfied       5.8670E+03
    Stream Depletion       S01.9    Near-Binding    1.2965E-05
    Stream Depletion       S01.10   Satisfied       5.2861E+03
    Stream Depletion       S01.11   Satisfied       5.8116E+03
    Stream Depletion       S01.12   Satisfied       5.8670E+03
    Stream Depletion       S02.3    Satisfied       1.9513E+00
    Stream Depletion       S02.4    Near-Binding    1.4319E-01
    Stream Depletion       S02.7    Near-Binding    1.5495E-04
    Stream Depletion       S02.8    Near-Binding    3.8380E-04
    Stream Depletion       S02.11   Near-Binding    1.2510E-04
    Stream Depletion       S02.12   Near-Binding    3.8262E-04
    Stream Depletion       S03.1    Satisfied       1.4120E+04
    Stream Depletion       S03.2    Satisfied       3.6890E+00
    Stream Depletion       S03.3    Satisfied       1.4209E+04
    Stream Depletion       S03.4    Satisfied       1.2081E+04
    Stream Depletion       S03.5    Satisfied       1.4032E+04
    Stream Depletion       S03.6    Near-Binding    1.0418E-03
    Stream Depletion       S03.7    Satisfied       1.4209E+04
    Stream Depletion       S03.8    Satisfied       1.2081E+04
    Stream Depletion       S03.9    Satisfied       1.4032E+04
    Stream Depletion       S03.10   Near-Binding    9.9406E-04
    Stream Depletion       S03.11   Satisfied       1.4209E+04
    Stream Depletion       S03.12   Satisfied       1.2081E+04
```

```
      Stream Depletion       S04.3    Satisfied       6.4462E+03
      Stream Depletion       S04.4    Near-Binding    2.0796E-01
      Stream Depletion       S04.7    Satisfied       6.4438E+03
      Stream Depletion       S04.8    Near-Binding    4.1881E-04
      Stream Depletion       S04.11   Satisfied       6.4438E+03
      Stream Depletion       S04.12   Near-Binding    4.1881E-04

    Calculating Response Matrix
      Perturb Flow Variable    1
       By Perturbation Value: -2.500750E+03
      Perturb Flow Variable    2
       By Perturbation Value: -2.500750E+03
      Perturb Flow Variable    3
       By Perturbation Value: -2.500750E+03
      Perturb Flow Variable    4
       By Perturbation Value: -2.500750E+03
      Perturb Flow Variable    5
       By Perturbation Value: -2.500750E+03
      Perturb Flow Variable    6
       By Perturbation Value: -2.500750E+03
      Perturb Flow Variable    7
       By Perturbation Value: -2.500750E+03
      Perturb Flow Variable    8
       By Perturbation Value: -2.500750E+03

      Average Number of Significant Digits in Matrix  6.400794E+00

    Solving Linear Program
    Optimal Solution Found
    Objective Value -5.302168E+04
    Maximum Relative Change in Flow Variable  7.157547E-08
    Max Relative External Variable Change  3.980410E-08
  SLP Algorithm: End Iteration    3
  Iterations have converged

----------------------------------------------------------------------
               Ground-Water Management Solution
----------------------------------------------------------------------

       OPTIMAL SOLUTION FOUND

       OPTIMAL RATES FOR EACH FLOW VARIABLE
       ---------------------------------------
Variable           Withdrawal          Injection           Contribution
Name               Rate                Rate                To Objective
----------         --------------      ------------        ------------
 Q1                3.483296E+03                            3.814209E+03
 Q2a               5.000000E+04                            1.380000E+04
 Q2b               2.755393E+04                            7.522222E+03
 Q2c               2.690226E+04                            7.344317E+03
 Q2d               2.673937E+04                            7.299849E+03
 Q3                2.829550E-06                            1.032786E-06
 Q4a               4.896278E+04                            1.336684E+04
 Q4b               8.068633E+03                            2.202737E+03
                   ------------        ------------        ------------
TOTALS             1.917103E+05        0.000000E+00        5.535017E+04
```

```
       OPTIMAL RATES FOR EACH EXTERNAL VARIABLE
       ---------------------------------------
Variable           Export              Import              Contribution
Name               Rate                Rate                To Objective
----------         --------------      ------------        ------------
 Im9                                   0.000000E+00        0.000000E+00
 Im10                                  0.000000E+00        0.000000E+00
 Im11                                  1.461444E+04       -1.595897E+03
 Im12                                  6.708699E+03       -7.325900E+02
                   ------------        ------------        ------------
TOTALS             0.000000E+00        2.132314E+04       -2.328487E+03


       OBJECTIVE FUNCTION VALUE                            5.302168E+04

       BINDING CONSTRAINTS
Constraint Type        Name     Status      Shadow Price
---------------        ----     ------      ------------
Stream Depletion       S01.9    Binding     -3.2407E-01
Stream Depletion       S02.11   Binding     -5.9427E-01
Stream Depletion       S02.12   Binding     -1.1097E-01
Stream Depletion       S03.10   Binding     -5.3048E-02
Stream Depletion       S04.8    Binding     -1.2527E-01
Stream Depletion       S04.12   Binding     -3.0472E-01
Summation              p06u     Binding      2.5658E-01
Summation              p11l     Binding     -1.0920E-01
Summation              p12l     Binding     -1.0920E-01
Maximum Flow Rate      Q2a      Binding    Not Available

  Binding constraint values are determined from the linear program
    and based on the response matrix approximation of the flow-process.

       RANGE ANALYSIS NOT REPORTED

----------------------------------------------------------------------
               Final Flow Process Simulation
----------------------------------------------------------------------
      Status of Simulation-Based Constraints
        Using Optimal Flow Rate Variable Values
      Constraint Type        Name     Status     Distance To RHS
      ---------------        ----     ------     ---------------
      Stream Depletion       S01.1    Satisfied       4.4826E+02
      Stream Depletion       S01.2    Satisfied       5.3093E+03
      Stream Depletion       S01.3    Satisfied       5.8128E+03
      Stream Depletion       S01.4    Satisfied       5.8671E+03
      Stream Depletion       S01.5    Near-Binding    5.3803E-03
      Stream Depletion       S01.6    Satisfied       5.2861E+03
      Stream Depletion       S01.7    Satisfied       5.8116E+03
      Stream Depletion       S01.8    Satisfied       5.8670E+03
      Stream Depletion       S01.9    Near-Binding    1.2851E-05
      Stream Depletion       S01.10   Satisfied       5.2861E+03
      Stream Depletion       S01.11   Satisfied       5.8116E+03
      Stream Depletion       S01.12   Satisfied       5.8670E+03
      Stream Depletion       S02.3    Satisfied       1.9514E+00
      Stream Depletion       S02.4    Near-Binding    1.4320E-01
      Stream Depletion       S02.7    Near-Binding    2.4777E-04
      Stream Depletion       S02.8    Near-Binding    3.8769E-04
      Stream Depletion       S02.11   Near-Binding    2.1793E-04
      Stream Depletion       S02.12   Near-Binding    3.8650E-04
      Stream Depletion       S03.1    Satisfied       1.4120E+04
      Stream Depletion       S03.2    Satisfied       3.6880E+00
```

```
    Stream Depletion        S03.3    Satisfied       1.4209E+04
    Stream Depletion        S03.4    Satisfied       1.2081E+04
    Stream Depletion        S03.5    Satisfied       1.4032E+04
    Stream Depletion        S03.6    Near-Binding    4.3092E-05
    Stream Depletion        S03.7    Satisfied       1.4209E+04
    Stream Depletion        S03.8    Satisfied       1.2081E+04
    Stream Depletion        S03.9    Satisfied       1.4032E+04
    Stream Depletion        S03.10   Near-Binding    9.0869E-05
    Stream Depletion        S03.11   Satisfied       1.4209E+04
    Stream Depletion        S03.12   Satisfied       1.2081E+04
    Stream Depletion        S04.3    Satisfied       6.4462E+03
    Stream Depletion        S04.4    Near-Binding    2.0796E-01
    Stream Depletion        S04.7    Satisfied       6.4438E+03
    Stream Depletion        S04.8    Near-Binding    4.2327E-04
    Stream Depletion        S04.11   Satisfied       6.4438E+03
    Stream Depletion        S04.12   Near-Binding    4.2327E-04

Because of precision limitations and possible nonlinear behavior,
  the status of binding constraints computed directly by the flow process
   may differ slightly from those computed using the linear program.
```

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Appendix Contents

Figures

Tables

APPENDIX 1: Examples of Decision Variables that can be Defined in GWM

This section provides examples of how decision variables can be defined in a GWM problem. Included in the examples are discussions of how characteristics of the decision variables are then specified by input variables in the decision variable (DECVAR) input file to GWM.

Example 1: Flow-rate decision variables for a steady-state model: Figure 1-1*A* shows a small model grid that consists of 3 layers, 6 rows, and 4 columns; the model consists of a single stress period. Four flow-rate decision variables have been defined within the model grid. Decision variable Q1 is a withdrawal well located at model cell (1, 6, 4). Decision variable Q2 is a withdrawal well that extends over two adjacent vertical cells (1, 6, 2) and (2, 6, 2); transmissivity of the aquifer is such that it can be assumed that each cell contributes 50 percent to the well's discharge. Decision variable Q3 is a withdrawal well that extends over two vertical cells that are not adjacent, (1, 4, 4) and (3, 4, 4); it can be assumed that 75 percent of the well's discharge is from the upper layer and 25 percent of the well's discharge is from the lower layer. Decision variable Q4 is a recharge basin extending over five cells in the top layer of the model (row 1, columns 2-4 and row 2, columns 3-4); recharge is distributed evenly to the five cells.

Records 3a and 3b of the DECVAR file would have the following information for these four decision variables:

3a.	FVNAME	NC	LAY	ROW	COL	FTYPE	FSTAT	WSP
3b.		RATIO	LAY	ROW	COL			
	Q1	1	1	6	4	W	Y	1
	Q2	2	0	0	0	W	Y	1
		0.5	1	6	2			
		0.5	2	6	2			
	Q3	2	0	0	0	W	Y	1
		0.75	1	4	4			
		0.25	3	4	4			
	Q4	5	0	0	0	I	Y	1
		0.20	1	1	2			
		0.20	1	1	3			
		0.20	1	1	4			
		0.20	1	2	3			
		0.20	1	2	4			

Example 2: Flow-rate decision variables for a transient model: The model grid and locations of the wells and recharge basin are the same as for example 1, but the model is now transient and consists of four stress periods that represent the four seasons of the year (Winter, Spring, Summer, and Fall). The withdrawal well at model cell (1, 6, 4) can have different pumping rates during each of the four stress periods, so four decision variables now must be used for the well site; these are specified as decision variables Q1-W, Q1-Sp, Q1-Su, and Q1-F. The withdrawal rates at the second and third well sites (decision variables Q2 and Q3) must be constant throughout the year. The recharge basin is used only during the winter and spring, and the recharge rates to the basin during the two seasons need not be the same. Two decision variables for recharge therefore are necessary for the basin, and these are specified as Q4-W and Q4-Sp.

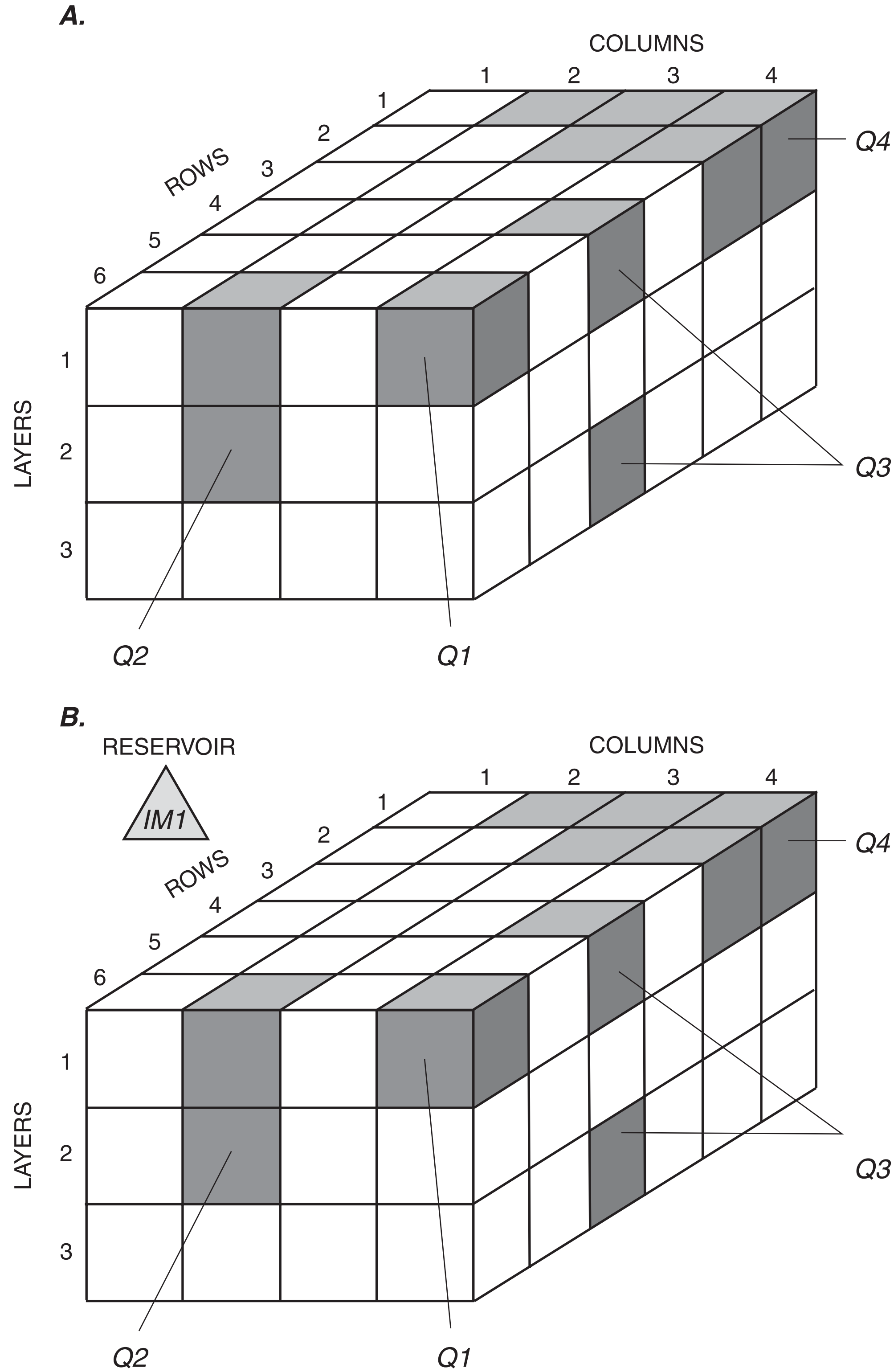


Figure 1-1. Hypothetical model grid illustrating some of the types of flow-rate and external decision variables that can be defined in GWM: *A,* four flow-rate decision variables (*Q*1, *Q*2, *Q*3, and *Q*4); and *B,* four flow-rate decision variables and an external decision variable (a surface-water reservoir, IM1) that is a source of water to the management model.

There are now a total of eight decision variables for the management model, and records 3a and 3b of the DECVAR file would have the following information for the 8 variables:

3a.	FVNAME	NC	LAY	ROW	COL	FTYPE	FSTAT	WSP
3b.		RATIO	LAY	ROW	COL			
	Q1-W	1	1	6	4	W	Y	1
	Q1-Sp	1	1	6	4	W	Y	2
	Q1-Su	1	1	6	4	W	Y	3
	Q1-F	1	1	6	4	W	Y	4
	Q2	2	0	0	0	W	Y	1-4
		0.5	1	6	2			
		0.5	2	6	2			
	Q3	2	0	0	0	W	Y	1-4
		0.75	1	4	4			
		0.25	3	4	4			
	Q4-W	5	0	0	0	I	Y	1
		0.20	1	1	2			
		0.20	1	1	3			
		0.20	1	1	4			
		0.20	1	2	3			
		0.20	1	2	4			
	Q4-Sp	5	0	0	0	I	Y	2
		0.20	1	1	2			
		0.20	1	1	3			
		0.20	1	1	4			
		0.20	1	2	3			
		0.20	1	2	4			

Example 3: Flow-rate and binary decision variables for a transient model: The model grid and all of the flow-rate decision variables for this example are the same as for example 2, but binary variables now are associated with the flow-rate decision variables. These binary variables are used to define the flow-rate decision variable as either active or inactive; this might be necessary, for example, when there is a minimum amount of withdrawal or injection that is required at each of the decision variables before the well or recharge basin will be constructed. The decision variables at well site (1, 6, 4) are associated with a single binary variable BV1, because if withdrawals occur at the well in at least one of the seasons, the well site must be constructed. The same situation holds for the recharge basin, so binary variable BV4 is associated with flow-rate variables Q4-W and Q4-Sp.

Records 3a and 3b of the DECVAR file would have the same information for the four flow-rate decision variables as given in example 2. Characteristics of the binary variables are specified in record 5 of the DECVAR file:

```
5.   BVNAME    NDV      BVLIST
     BV1       4        Q1-W Q1-Sp Q1-Su Q1-F
     BV2       1        Q2
     BV3       1        Q3
     BV4       2        Q4-W Q4-Sp
```

Example 4: Flow-rate and external decision variables for a transient model: The model grid and all of the flow-rate decision variables for this example are the same as for the previous example, but there is now an external source of water that is available to the management problem from a surface-water reservoir shown schematically in figure 1-1*B*. The rate at which this source of water can be imported can vary during each of the four stress periods; therefore, four external decision variables are needed to model the source: IM1-W, IM1-Sp, IM1-Su, and IM1-F. Note that these external decision variables are not associated with any of the model grid cells. Characteristics of the external decision variable are specified in record 4 of the DECVAR file:

```
4.   EVNAME    ETYPE    ESP
     IM1-W     IM       1
     IM1-Sp    IM       2
     IM1-Su    IM       3
     IM1-F     IM       4
```

APPENDIX 2: GWM Solution Algorithms

GWM includes the RMS (Response Matrix Solution) Package for solving optimization problems. This package contains three solution algorithms: linear programming, branch and bound, and sequential linear programming. Linear programming is used to solve problems with linear objective and constraint functions. When binary variables are present, the branch and bound algorithm is used. This algorithm is based on solving a series of linear programs. If the objective or constraints include nonlinear responses of system state to flow-rate variables, then the sequential linear programming (SLP) algorithm is used. SLP operates by solving a series of linear programs. Because solution of a linear program forms the core of all three solution algorithms, it is the primary focus of this section.

Linear Program Solution Algorithm

Simplex Algorithm

GWM1SIMPLEX1 is a collection of FORTRAN subroutines that solve a linear program (LP) by using a 2-phase revised simplex method (Dantzig, 1963; Gass, 1985). Sensitivity (or range) analysis is also done by GWM1SIMPLEX1 if an optimal solution to the LP exists. GWM1SIMPLEX1 assumes that the problem takes the following form:

$$\text{Minimize } Z = \boldsymbol{c}^t\boldsymbol{x} \tag{2-1}$$

$$\text{subject to } \boldsymbol{Ax} = \boldsymbol{b} \tag{2-2}$$

$$\boldsymbol{0} \le \boldsymbol{x} \le \boldsymbol{u}, \tag{2-3}$$

where $\boldsymbol{x}$ is the vector of decision variables, $\boldsymbol{c}^t$ is the transposed column vector of objective-function coefficients, $\boldsymbol{A}$ is an *m x n* matrix where *m* is the number of constraints and *n* is the number of decision variables (including slack variables), $\boldsymbol{b}$ is the total amount of resource available, and $\boldsymbol{u}$ is the upper-bound vector on the decision variables. Other LP formulations can be transformed into equations 2-1 through 2-3 through fairly simple means. For example, a maximization objective can be transformed into an equivalent minimization objective by multiplying all objective-function coefficients by (-1). The transformation from inequality constraint to equality constraint is accomplished by adding one variable to each inequality constraint. These extra variables indicate the difference between the amount of resource used by a particular solution and the amount of resource available. In that context, the variables represent the slack or surplus of a resource, and are called slack or surplus variables.

The original LP in equations 2-1 through 2-3 represents a system of *n* unknowns in *m* equations, where *n* is greater than *m*. The simplex method proceeds by fixing (*n-m*) decision variables and using the equality constraints to solve for the remaining *m* variables. At any iteration, the variables with fixed values are termed nonbasic variables, $\boldsymbol{x_N}$, while the remaining m variables are called basic variables, x_B. By using these variables and defining the matrices $\boldsymbol{B}$ and $\boldsymbol{N}$ as the columns of $\boldsymbol{A}$ corresponding to the basic and nonbasic variables, respectively, the original LP can be written as:

$$\text{Minimize } Z = \boldsymbol{c}_B^t \boldsymbol{x}_B + \boldsymbol{c}_N^t \boldsymbol{x}_N \tag{2-4}$$

$$\text{subject to } \boldsymbol{B}\boldsymbol{x}_B + \boldsymbol{N}\boldsymbol{x}_N = \boldsymbol{b} \tag{2-5}$$

$$\boldsymbol{0} \le \boldsymbol{x}_B \le \boldsymbol{u}_B \tag{2-6}$$

$$\boldsymbol{0} \le \boldsymbol{x}_N \le \boldsymbol{u}_N, \tag{2-7}$$

where $\boldsymbol{c}_B$, $\boldsymbol{u}_B$ and $\boldsymbol{c}_N$, $\boldsymbol{u}_N$ are the objective-function coefficients and upper bounds for the basic and nonbasic variables, respectively. Assuming that the list of basic variables is known and that the nonbasic variables are assigned to specific values, then the values of the basic variables are determined by rearranging equation 2-5:

$$\boldsymbol{x}_B = \boldsymbol{B}^{-1}(\boldsymbol{b} - \boldsymbol{N}\boldsymbol{x}_N)\,. \tag{2-8}$$

Furthermore, substituting equation 2-8 into 2-4, and rearranging, the objective function is calculated as

$$\text{Minimize } Z = \boldsymbol{c}_B^t \boldsymbol{B}^{-1}\boldsymbol{b} + (\boldsymbol{c}_N^t + \boldsymbol{c}_B^t \boldsymbol{B}^{-1}\boldsymbol{N})\boldsymbol{x}_N\,. \tag{2-9}$$

The rules for identifying the set of basic variables and assigning values to the nonbasics are a key component of the simplex method.

The simplex method proceeds by finding a sequence of feasible solutions to the system of equations in equations 2-1 through 2-3 until an optimal solution is found. At each iteration, the nonbasic variables are considered fixed at either their upper or lower bound. The basic variables are then computed from equation 2-8 to ensure feasibility. Information from equation 2-9 is used to determine if a given solution is optimal. If the current solution is not optimal, then a new simplex iteration begins and one basic variable becomes nonbasic while one nonbasic variable becomes basic.

The nonbasic variable that becomes basic (that is, enters the basis) is chosen on the basis of its impact on the objective function. The objective function, as written in equation 2-9, includes a term that only depends on $\boldsymbol{x}_N$ and a constant term. The coefficient on $\boldsymbol{x}_N$ in equation 2-9 is called the reduced cost. The nonbasic variable that will have the most advantageous impact on the objective function, that is, the best reduced cost, is chosen as the entering nonbasic variable. The basic variable that first goes to either its upper or lower bound as the nonbasic variable is changed, leaves the basis, and becomes nonbasic.

The simplex method finds a sequence of feasible solutions by altering the set of basic variables one variable at a time until the reduced costs indicate that no further improvement is possible. At this point, the optimal solution has been found. In many LPs, it is a nontrivial problem to find an initial feasible solution from which to begin the search for an optimal solution. This problem is addressed in GWM1SIMPLEX1 by use of the 2-phase revised simplex method, which proceeds in Phase I by altering the original problem (eqs. 2-1 through 2-3) in such a way that an initial feasible solution to the altered problem is trivial and the optimal solution of the Phase I problem is a feasible solution to the original problem.

Algorithm Specifics

Input to GWM1SIMPLEX1: GWM1SIMPLEX1 is a self-contained implementation of the 2-phase revised simplex method. Augmentation of the original LP with slack variables so that the problem takes the form in equations 2-1 through 2-3 is required prior to calling GWM1SIMPLEX1. Variables related to solving the LP are defined in the module GWM1RMS1, whereas variables used exclusively by GWM1SIMPLEX1 and its subroutines are defined in module GWM1RMS1LP. Calls to GWM1SIMPLEX1 should be of the form

CALL GWM1SIMPLEX1(M, NV, NDV, AMAT, COST, BNDS, RHS, OBJ, IFLG,
LPITMAX),

where

M	is	the number of constraints;
NV	is	the number of variables (decision variables + slacks);
NDV	is	the number of decision variables + 1;
AMAT(M,NDV)	is	the original $\boldsymbol{A}$ matrix augmented with slack-variable coefficients in the last column;
COST(NV)	is	the vector of objective-function coefficients for each variable, including slacks;
BNDS(NV)	is	the array of upper bounds on each variable, including slacks;
RHS(M)	is	the original right-hand side values of the constraints; and
LPITMAX	is	the maximum number of iterations allowed.

If an optimal solution to the original LP is found, GWM1SIMPLEX1 stores the optimal solution, shadow prices, reduced costs, and range-analysis information in space that contained the original LP information, so that output from GWM1SIMPLEX1 includes

IFLG	the output-status flag indicating optimal solution found (0), problem is infeasible (1), or problem is unbounded (2);
COST(NV)	the values of the decision variables at the optimal solution;
BNDS(NV)	the reduced costs of the decision variables;
RHS(M)	the shadow prices of the constraints (that is, the dual variables); and
OBJ	the value of the objective function at the optimal solution.

If specified by the user, range analysis is done on the objective-function coefficients ($\boldsymbol{c}$) and the right-hand-side values (RHS, $\boldsymbol{b}$). During range analysis, one parameter at a time is perturbed about its original value until the optimal basis changes. When this occurs, the variables that would enter and leave the basis are determined by means of a ratio test. For the objective-function coefficient ranges, the optimal values of the decision variables will not change provided the coefficients remain within the stated ranges. Conversely, when an RHS value is modified, the value of the decision variables will change (see eq. 2-8), but the optimal basis and shadow prices will not be affected. Upon completion of range analysis, program control is then passed back to the calling program.

Phase I and II: The Phase I procedure is needed to obtain an initial feasible solution to the LP. Phase I begins by augmenting the original problem (eqs. 2-1 through 2-3) with artificial variables, $\boldsymbol{x}'$, such that the artificial LP becomes

$$\text{Minimize } Z = \mathbf{1}^t\boldsymbol{x}' + \mathbf{0}^t\boldsymbol{x} \tag{2-10}$$

$$\text{subject to } \boldsymbol{Ax} + \boldsymbol{Ix}' = \boldsymbol{b} \tag{2-11}$$

$$\mathbf{0} \le \boldsymbol{x} \le \boldsymbol{u} \tag{2-12}$$

$$\mathbf{0} \le \boldsymbol{x}', \tag{2-13}$$

where $\mathbf{1}$ is an $(m \text{ x } 1)$ vector of ones and $\boldsymbol{I}$ is an $(m\ x\ m)$ identity matrix. The Phase I problem has the form of a linear program. An initial feasible solution to this artificial problem is obtained by setting all original variables $\boldsymbol{x} = \mathbf{0}$ and using the system of equality equations in the constraints to solve for the values of $\boldsymbol{x}'$ (see Bradley and others, 1977, for more detail about the 2-phase simplex method). Because the objective-function coefficients on the original variables have been set to zero in the artificial problem, application of the simplex algorithm to equations 2-10 through 2-13 will tend to drive the artificial variables from the problem.

If a feasible solution to the original problem exists, then all artificial variables will be driven to zero and the optimal solution in Phase I represents a feasible solution to the original problem. The original LP is then solved during Phase II from the initial feasible solution obtained from the artificial LP. If the solution to equations 2-10 through 2-13 retains any artificial variables, the original LP must be infeasible and the algorithm terminates.

Pricing: As discussed above, each iteration of the simplex method proceeds by substituting one basic variable with one nonbasic variable. The decision process in this step requires first determining which nonbasic variable will enter the basis. This decision is made during the pricing step. The reduced cost of all nonbasic variables is determined and the entering variable is chosen as the one that results in the largest marginal improvement in the objective function:

$$Min\begin{cases} (\boldsymbol{c_N} - \boldsymbol{c_B^t B^{-1} N}) & \forall \boldsymbol{x_N} = \mathbf{0} \\ -(\boldsymbol{c_N} - \boldsymbol{c_B^t B^{-1} N}) & \forall \boldsymbol{x_N} = \boldsymbol{u} \end{cases} . \tag{2-14}$$

When the nonbasic variable is at its lower bound, the most negative reduced cost is advantageous. Conversely, when the nonbasic variable is at its upper bound, the most positive reduced cost is advantageous.

The Ratio Test: Once the entering variable is identified in the pricing step, the leaving variable must be determined. At each iteration, the set of equality constraints in equations 2-1 through 2-3 must be satisfied to maintain feasibility. Thus, the constraints are used to determine which variable will leave the basis. When the entering variable x_j increases from its lower bound (that is, it is currently equal to zero and will increase in value upon entering the basis), the ratio test

$$Min\begin{cases} \dfrac{[\boldsymbol{B}^{-1}(\boldsymbol{b} - \boldsymbol{N}\boldsymbol{x}_N)]_i}{(\boldsymbol{B}^{-1}\boldsymbol{N})_{ij}} \Bigg| (\boldsymbol{B}^{-1}\boldsymbol{N})_{ij} > 0 \\ \dfrac{u_{\boldsymbol{B},i} - [\boldsymbol{B}^{-1}(\boldsymbol{b} - \boldsymbol{N}\boldsymbol{x}_N)]_i}{-(\boldsymbol{B}^{-1}\boldsymbol{N})_{ij}} \Bigg| (\boldsymbol{B}^{-1}\boldsymbol{N})_{ij} < 0 \\ u_j \end{cases} \tag{2-15}$$

is done, where i and j refer to elements in the associated vector or matrix. If one of the first two ratios provides the limiting value, then the variable x_i leaves the basis. If the third argument, u_j, is the minimum, however, then the entering variable hits its upper bound prior to a basic variable hitting one of its bounds. Consequently, the variable remains nonbasic but its value is set to its upper bound.

Similarly, when the entering variable enters from its upper bound and decreases in value upon entering the basis, the next iteration is determined by

$$Min\begin{cases} \dfrac{[\boldsymbol{B}^{-1}(\boldsymbol{b} - \boldsymbol{N}\boldsymbol{x}_N)]_i}{-(\boldsymbol{B}^{-1}\boldsymbol{N})_{ij}} \Bigg| (\boldsymbol{B}^{-1}\boldsymbol{N})_{ij} < 0 \\ \dfrac{u_{\boldsymbol{B},i} - [\boldsymbol{B}^{-1}(\boldsymbol{b} - \boldsymbol{N}\boldsymbol{x}_N)]_i}{(\boldsymbol{B}^{-1}\boldsymbol{N})_{ij}} \Bigg| (\boldsymbol{B}^{-1}\boldsymbol{N})_{ij} > 0 \\ u_j \end{cases} . \tag{2-16}$$

Once the entering and leaving variables are determined, a pivot operation is done in which the basis pointer array is updated and the basis matrix is updated and factorized. The process of substituting the appropriate columns of $\boldsymbol{A}$ into the matrices $\boldsymbol{B}$ and $\boldsymbol{N}$ is termed pivoting. After each pivot operation, the basis matrix is factorized by the method of LU decomposition, by use of routines from LAPACK (Anderson and others, 1999).

The steps for determining the entering and leaving variables followed by pivoting and factorization are done iteratively until one of three stopping criteria is met:

1. An optimal solution is found. This is determined during the pricing operation: if all prices are greater than or equal to zero for nonbasics at their lower bounds or less than or equal to zero for nonbasics at their upper bounds, then the solution cannot be improved and the optimal solution has been found.
2. The iteration limit, LPITMAX, is exceeded. When this occurs, an error message is printed and GWM is terminated.
3. The problem is determined to be unbounded. If a leaving variable cannot be found in the ratio test, then the problem is unbounded.

Range analysis: When an optimal solution is found, range analysis can be performed. The goal during range analysis is to determine the range over which a specified parameter can vary without changing the optimal basis. At the limit of these ranges, the new optimal basis is also determined. Range analysis is applicable for strictly linear programs and should be used with caution if the problem is a mixed-binary linear program or is nonlinear but being solved by using sequential linear programming.

Objective-function coefficient ranges are determined by considering the value of the reduced cost for each variable that would cause the variable to enter the basis. For nonbasic variables, objective-function coefficients can vary in the range

$$\begin{cases} -[c_j - (\boldsymbol{c}_{\boldsymbol{B}}^t \boldsymbol{B}^{-1}\boldsymbol{N})_j] \le \Delta c_j \le \infty & \forall \boldsymbol{x}_{\boldsymbol{N}} = \boldsymbol{0} \\ -\infty \le \Delta c_j \le -[c_j - (\boldsymbol{c}_{\boldsymbol{B}}^t \boldsymbol{B}^{-1}\boldsymbol{N})_j] & \forall \boldsymbol{x}_{\boldsymbol{N}} = \boldsymbol{u} \end{cases} . \tag{2-17}$$

When the range limit is exceeded, x_j becomes a candidate to enter the basis.

For basic variables, the objective-function coefficient values that would cause the variable to leave the basis are determined by imposing the optimality condition on all reduced costs. Letting the nonbasic variable be variable j, the objective-function coefficient range for the basic variable i is determined by

$$\underset{j}{Max}\left[\frac{(c_j - (\boldsymbol{c}_{\boldsymbol{B}}\boldsymbol{B}^{-1}\boldsymbol{A})_j)}{(\boldsymbol{B}^{-1}\boldsymbol{A})_{ij}} \; for \; \begin{matrix} (\boldsymbol{B}^{-1}\boldsymbol{A})_{ij} < 0, x_j = 0 \\ (\boldsymbol{B}^{-1}\boldsymbol{A})_{ij} > 0, x_j = u \end{matrix}\right] \le \Delta c_i$$

$$\le \underset{j}{Min}\left[\frac{(c_j - (\boldsymbol{c}_{\boldsymbol{B}}\boldsymbol{B}^{-1}\boldsymbol{A})_j)}{(\boldsymbol{B}^{-1}\boldsymbol{A})_{ij}} \; for \; \begin{matrix} (\boldsymbol{B}^{-1}\boldsymbol{A})_{ij} > 0, x_j = 0 \\ (\boldsymbol{B}^{-1}\boldsymbol{A})_{ij} < 0, x_j = u \end{matrix}\right] . \tag{2-18}$$

The nonbasic variable producing the limiting ratio in each case enters the basis. For all objective-function coefficient ranges, the leaving variables are determined by the standard ratio test, as described above. Because the optimal solution is calculated as $\boldsymbol{x}_B = \boldsymbol{B}^{-1}(\boldsymbol{b} - \boldsymbol{N}\boldsymbol{x}_N)$, the optimal solution is unaffected by objective-function coefficients that vary within the ranges determined above.

Right-hand-side range analysis is done by determining the range of RHS values for which the optimal basis remains unchanged. Note, however, that the optimal values of the basic variables (given by eq. 2-8) will change with a change in the RHS. RHS range analysis proceeds by first determining the leaving variables as those that provide the limiting ratios

$$\underset{i}{Max}\left[\begin{matrix} -\dfrac{(x_{\boldsymbol{B}})_i}{\beta_{ik}}\Bigg| \beta_{ik} > 0 \\ \dfrac{(u_{\mathbf{B}})_i - (x_{\boldsymbol{B}})_i}{\beta_{ik}}\Bigg| \beta_{ik} < 0 \end{matrix}\right] \le \Delta b_k \le \underset{i}{Min}\left[\begin{matrix} -\dfrac{(x_{\boldsymbol{B}})_i}{\beta_{ik}}\Bigg| \beta_{ik} < 0 \\ \dfrac{(u_{\mathbf{B}})_i - (x_{\boldsymbol{B}})_i}{\beta_{ik}}\Bigg| \beta_{ik} > 0 \end{matrix}\right], \tag{2-19}$$

where

Δb_k	is	the change in the kth RHS (k = 1, ..., m);
$(x_{\mathbf{B}})_i = (\boldsymbol{B}^{-1}(\boldsymbol{b} - \boldsymbol{N}\boldsymbol{x}_N))_i$	is	the optimal value of the ith basic variable;
β_{ik}	is	the ith row, kth column element of $\boldsymbol{B}^{-1}$; and
$(u_{\mathbf{B}})_i$	is	the upper bound of the ith basic variable.

The entering variable is determined by the optimality condition, which requires that the reduced costs of nonbasic variables at their lower bounds remain non-negative while the reduced costs of nonbasics at their upper bounds stay non-positive. The nonbasic variable that first violates this condition becomes the candidate to enter the basis. Let the leaving variable identified above be basic in row *i*. If the leaving variable leaves at its lower bound, then the entering variable provides the limiting value in the following test:

$$\underset{\boldsymbol{x_N}}{Min}\left\{-\frac{[c_j - (\boldsymbol{c_B^t B^{-1} N})_j]}{(\boldsymbol{B^{-1}N})_{ij}}\middle|(\boldsymbol{B^{-1}N})_{ij} < 0, \boldsymbol{x_{N,j}} = \boldsymbol{0};\ (\boldsymbol{B^{-1}N})_{ij} > 0, \boldsymbol{x_{N,j}} = u_j.\right. \tag{2-20}$$

Likewise, if the leaving variable leaves at its upper bound, the entering variable is determined by

$$\underset{\boldsymbol{x_N}}{Min}\left\{\frac{[c_j - (\boldsymbol{c_B^t B^{-1} N})_j]}{(\boldsymbol{B^{-1}N})_{ij}}\middle|(\boldsymbol{B^{-1}N})_{ij} > 0, \boldsymbol{x_{N,j}} = \boldsymbol{0};\ (\boldsymbol{B^{-1}N})_{ij} < 0, \boldsymbol{x_{N,j}} = u_j\ .\right. \tag{2-21}$$

Output from GWM1SIMPLEX1: Output from GWM1SIMPLEX1 consists of the optimal solution, shadow prices, reduced costs, and range-analysis information. Shadow prices represent the sensitivity of the objective-function value to changes in the right-hand-side value of a particular constraint [that is, $(\partial Z)/(\partial b)$] (Bradley and others, 1977; Ahlfeld and Mulligan, 2000; Hillier and Lieberman, 2001). Shadow prices also represent the optimal value of the dual variables. The slack, or the difference between the right-hand side (b) and the constraint value at the optimum, indicates the amount of a resource not utilized at the optimal solution.

Branch and Bound Solution Algorithm

The branch and bound algorithm is used by GWM to solve problems that include binary variables. The algorithm consists of solving a series of linear programs. Within each linear program, the binary variables are either assigned, a priori, a binary value (0 or 1) or allowed to take a noninteger value between 0 and 1. A binary variable that is not required to take a binary value is referred to as relaxed. By assigning values to some binary variables and relaxing others, a problem is created that no longer has decision variables that are required to take binary values. Such a problem is referred to as a subproblem and can be solved by using the simplex algorithm for linear programs. By solving a series of subproblems, the branch and bound algorithm can identify the optimal solution to the original mixed-binary problem.

The branch and bound algorithm begins by relaxing all binary variables and solving the resulting linear program. If the fully relaxed problem has no feasible solution, then the full binary problem will also be infeasible. The solution to the fully relaxed problem provides a starting point for the creation of additional subproblems by branching.

The algorithm proceeds by selecting one of the relaxed binary variables from the initial subproblem and branching on it to create two new subproblems. In the first new subproblem, the selected variable is forced to take a value of 0 while the remaining binary variables are still relaxed. In the second new subproblem, the selected variable is forced to 1. Each of these subproblems is solved. Additional subproblems are created by branching on other relaxed binary variables. The implementation of the branch and bound algorithm in GWM uses a modified depth-first search with backtracking (Nemhauser and Wolsey, 1988).

For each parent subproblem, two children are created, branching from the first available nonspecified variable. Branching proceeds down the tree along the path with binary variables set to one. A notable step in the algorithm occurs when a feasible linear program solution is found in which all binary variables have binary values. This is referred to as a binary solution and is a feasible, although not necessarily optimal, solution to the original mixed-binary problem. Once a binary solution is determined, additional branching is accomplished by backtracking, with the most recently generated subproblem branched from first.

Branching on each relaxed binary variable in each generated subproblem would eventually lead to enumeration of all possible combinations of 0/1 values for each binary variable. Even for moderately sized problems, the number of linear programs that would be solved can be substantial. The branch and bound algorithm avoids enumeration of all possible combinations of binary values by eliminating subproblems and all their successive branching possibilities by using a series of fathoming tests. If a subproblem is fathomed, then additional branching from that subproblem will not yield the optimal solution and the subproblem and all it subsidiary subproblems can be dropped from further consideration.

The fathoming test has three parts:

1. A binary solution is found and thus the subproblem is fathomed, because no additional subproblems are possible. The objective function value for this subproblem is compared with any other subproblem that has a fully binary solution to identify the current best binary solution. The objective value for the current best solution provides an upper bound on the objective function for the original problem. That is, the optimal solution to the original problem will have an objective function value that is less than or equal to the one found from the subproblem with the best binary solution.
2. The subproblem is fathomed if it is infeasible. If a subproblem, with some binary variables relaxed, is infeasible, then imposing additional restrictions on binary variables will not produce a feasible solution. Hence, no successive subproblems can be feasible, and the subproblem is not worth further branching.
3. The subproblem is fathomed if the objective-function value for the optimal solution of the subproblem is larger than the upper bound from the current best binary solution. This applies even if the subproblem has some variables that are relaxed. It follows that imposition of additional restrictions on binary variables will not improve the objective function, so that all successive subproblems will have objective values that are inferior to the current best binary solution.

The optimal solution to the problem is found when all subproblems are fathomed. The subproblem with the best binary solution at that point is the solution to the original problem.

One extension to the branch and bound algorithm is used by GWM for the case in which some binary variables are specified with values of 1 and some relaxed binary variables are determined by the linear program to have 0 optimal values.

Experience with ground-water flow management problems has shown that it is common for candidate-well pumping rates to take 0 values at the optimal solution even when binary variables are not used. This characteristic results from the geometry of the ground-water simulation problem and the proximity of the flow-rate variables to constraints.

At the completion of the solution of a subproblem, GWM checks the value of the relaxed binary variables and their associated non-binary variables. If the values of all associated non-binary variables are 0, then the relaxed binary variable can be treated as having taken a 0 value. That is, the same solution to the subproblem would be obtained if that binary variable were forced to take a 0 value. If this condition is true for all relaxed variables in the subproblem, then all relaxed variables are reset to redefine the subproblem as if the relaxed variables had been specified. Implementation of this rule will reduce the number of subproblems that require solution. In effect, redefinition of the subproblem jumps ahead of the branching process to a subproblem that would have been identified after many additional iterations.

Two GWM input parameters control the branch and bound algorithm. BBITMAX specifies the maximum number of subproblems that will be considered by the algorithm. The RMS implementation of the branch and bound algorithm stores certain information about each subproblem. BBITMAX serves to define the dimension of the arrays that are used to store this information. BBITPRT controls the output directed to the global output file. Full output consists of details of the progress of the algorithm at each subproblem.

APPENDIX 3: Programmers' Guide to GWM

The Ground-Water Management (GWM) Process solves an optimization problem to minimize or maximize a specified objective function subject to specified constraints. The current implementation of GWM uses a numerical method for solution that is based on construction of a response matrix and its use in a linear program. GWM does not interact with any process, other than the Ground-Water Flow (GWF) Process, in MODFLOW. GWM is organized into a number of packages that define the objective function, various constraint types, and the solution algorithm. A number of new procedures were defined for MODFLOW that are required for completion of the GWM Process.

Procedures Used by GWM

Five new procedures have been added to MODFLOW for use by GWM. Figure 3-1 depicts the placement of these procedures in the overall structure of MODFLOW-2000. The core computations of the GWF Process are carried out in the Stress, Time Step, and Iteration Loops. The GWM Process requires repeated runs of the GWF Process; these runs are controlled with two levels of loops that surround the GWF Process—the GWM Iteration Loop and the Flow-Process Loop. For the Response Matrix Solution (RMS) Package, the Flow-Process Loop performs the perturbations required to construct a response matrix, whereas the GWM Iteration Loop performs iterations required by the sequential linear programming algorithm.

AR Procedure—Allocate and Read: This procedure combines the functions of the AL (Allocate) and RP (Read and Prepare) Procedures found elsewhere in MODFLOW. Because GWM packages use FORTRAN allocatable arrays, it is not necessary to have a separate allocation procedure. The GWM AR Procedure is located after GWF AL and RP Procedures in figure 3-1.

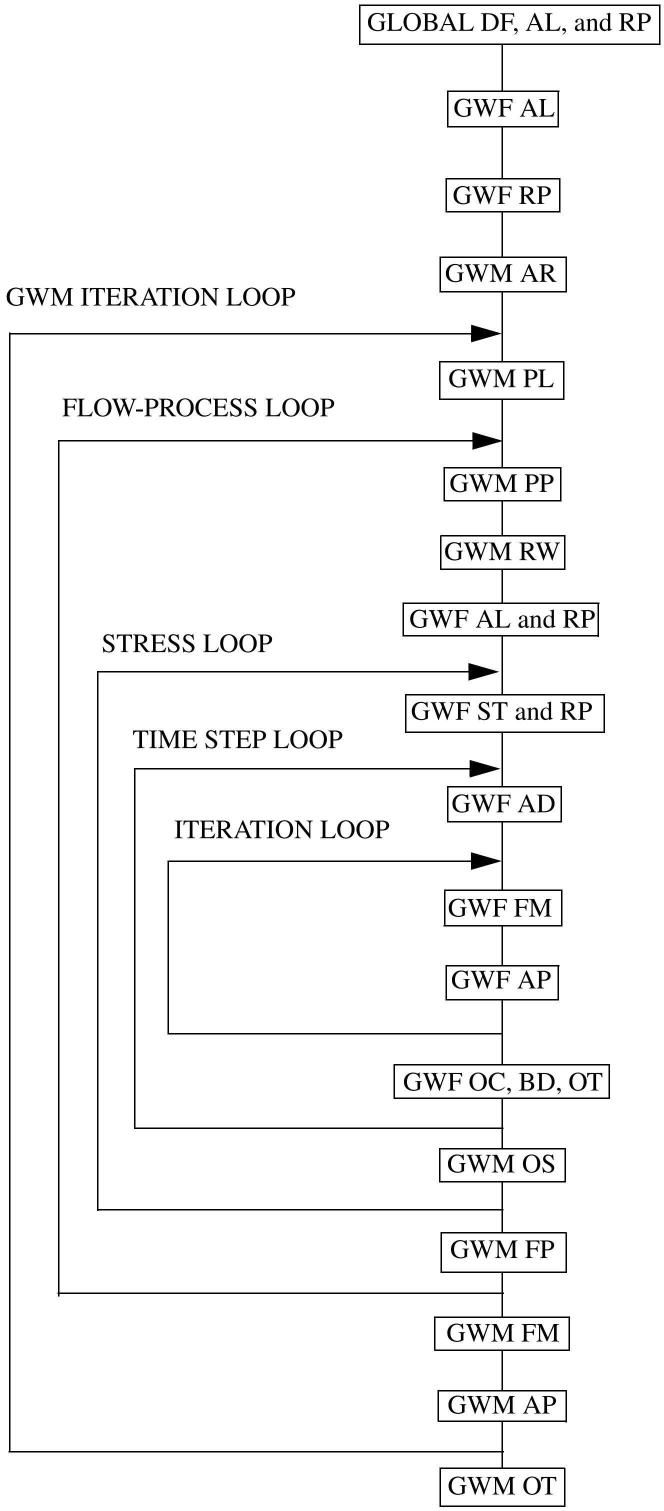
PL Procedure—Prepare Loop: This procedure processes information from the GWM Iteration Loop and prepares information for the Flow-Process Loop. For the RMS Package, the PL Procedure sets indices and counters, sets perturbation size, and sets base flow-rate variable values.

PP Procedure—Prepare Process: This procedure prepares information for a single run of the flow process. For the RMS Package, the PP Procedure determines if the flow-process run is for a base run or a perturbation run and sets the flow-rate variables accordingly. The GWM PP Procedure is followed by a GWM RW Procedure (fig. 3-1). The RW Procedure rewinds appropriate GWF Process files that will be reread during the Flow-Process Loop.

OS Procedure—Observe State: This procedure obtains the state of the system, as simulated by the GWF Process, at selected locations at the completion of a stress period. The specific locations at which the state is observed are determined by the needs of the objective and constraint packages.

FP Procedure—Formulate Process: This procedure processes the results of a completed flow-process run. It can be used to formulate the portions of the optimization problem that depend upon the flow process. For the RMS Procedure, the response coefficients are computed as part of the FP Procedure.

Figure 3-1 concludes with three additional GWM Procedures. The FM procedure formulates the equations that will be solved. For the RMS Package, the FM procedure assembles all the elements of the linear program. The AP Procedure solves the optimization problem at an iteration. Finally, the OT Procedure writes relevant output at the solution.



GWF and GWM Procedures:

DF — Define
AL — Allocate
RP — Read and Prepare
RW — Rewind
ST — Stress
AD — Advance a time step
FM — Formulate equations
AP — Solve equations
OC — Output Control
BD — Calculate volumetric budget
OT — Write output

GWM-Specific Procedures:

AR — Allocate and read
PL — Prepare Loop
PP — Prepare Process
OS — Observe State
FP — Formulate Process

Figure 3-1. Simplified flow chart showing Ground-Water Flow (GWF) and Ground-Water Management (GWM) Processes.

GWM Packages

The Ground-Water Management (GWM) Process is organized into a series of packages. Table 3-1 summarizes the packages used by GWM, the procedures used by each package, and primary module names assigned to each package/procedure combination.

DCV Package—Decision Variables: This package processes information regarding the flow-rate, external, and binary decision variables of the optimization problem. Because the flow-rate decision variables affect the flow process, two new GWF modules are needed; the GWF1DCV1FM and GWF1DCV1BD modules add managed flows to the GWF formulation and budget calculations, respectively.

OBJ Package—Objective Function: This package processes information describing the objective function for the optimization problem.

HDC (Head Constraints), STC (Streamflow Constraints), SMC (Summation Constraints), DCC (Decision-Variable Constraints) Packages—These packages all have the same structure. Information about the constraints is read and processed in the AR procedure. For the head and streamflow constraints, the state of the flow-process simulated system is recorded in the OS procedures. All constraints are placed in the GWM equations in FM procedures and constraint status is described in the OT procedures.

RMS Package—Response Matrix Solution: This package uses the response-matrix method to solve the optimization problem. RMS incorporates the simplex algorithm, the branch and bound algorithm, and the sequential linear programming algorithm as the numerical solver for the response-matrix method. The response matrix consists of the coefficients of the responses of head and streamflow at the constraint locations to changes in the flow-rate decision variables. RMS controls creation of the response matrix for these constraints through the PL, PP, and FP procedures. The full linear program coefficient matrix includes the response matrix. Additional columns are added for the external and binary decision variables. Additional constraint rows are added for summation constraints and constraints controlling binary variables. This assembly of the linear program is accomplished in procedure FM. The problem is solved, assuming linearity, in the AP procedure. If the problem is nonlinear, then it is solved iteratively.

BAS Package—Basic Package for GWM: This package contains the AR procedure for reading the GWM names file and several utility subroutines for file handling and input/output.

Table 3-1. Outline of packages, procedures, and modules for GWM.

Package	Procedures	Module names
DCV—Decision Variables	AR—read, allocate, and echo	GWM1DCV1AR
	FM—place flow rates in GWF equations	GWF1DCV1FM
	BD—add flow rates to GWF budget calculations	GWF1DCV1BD
	OT—write optimal solution of decision variables	GWM1DCV1OT
OBJ—Objective Function	AR—read, allocate, and echo	GWM1OBJ1AR
	FM—place into GWM equations	GWM1OBJ1FM
	OT—write final status of objective	GWM1OBJ1OT
HDC, STC, SMC, DCC—Constraints	AR—read, allocate, and echo	GWM1__1AR[1]
	OS—observe state	GWM1__1OS[1]
	FM—place into GWM equations	GWM1__1FM[1]
	OT—write final status of constraints	GWM1__1OT[1]
RMS—Response Matrix Solution Package	AR—read, allocate, and echo	GWM1RMS1AR
	PL—prepare perturbation loop	GWM1RMS1PL
	PP—prepare flow-process run by setting perturbation of flow rate	GWM1RMS1PP
	FP—formulate flow process by calculating response-matrix entries	GWM1RMS1FP
	FM—formulate objective and all constraints through GWM1___1FM[1] modules	GWM1RMS1FM
	AP—solve the optimization problem at an iteration and test convergence	GWM1RMS1AP
	OT—write optimization output	GWM1RMS1OT
BAS—GWM Basic	AR—read GWM name file	GWM1BAS1AR
	Various input/output utility routines	

[1]Underscore indicates three-character designation of constraint type (HDC, STC, SMC, or DCC).

Adding New GWM Packages

GWM is organized to make the addition of new objective or constraint packages or solver packages relatively easy. Adding new decision-variable types, however, is difficult.

The response-matrix method and the associated use of a linear-program formulation for the optimization problem inherently leads to mingling of decision variables and constraints within the solver. GWM is organized so that each constraint package operates independently of other constraint packages. This independence means that new constraint packages can be added without making any modifications to other constraint packages; however, all constraint packages are restricted to using the existing set of decision-variable types—the flow-rate, external, and binary decision variables. Adding new decision-variable types would require modification of every constraint package.

The same reasoning applies to the addition of new objective-function packages; namely, the objective-function package operates independently of the constraints but is closely tied to the three types of decision variables available in GWM.

Adding new solver packages is also possible. Presumably, any solver package will require repeated runs of the flow process followed by an assessment of results of the run, rules for the subsequent run, and assembly of information for ultimate solution of the optimization problem. By using the two loops depicted in figure 3-1 and appropriate content of the PL, PP, FP, FM, and AP procedures, many different optimization solver methods can be accomodated in the GWM structure.

Some Programming Details

Dimensioning of GWM Arrays: GWM uses FORTRAN-90 allocatable arrays for all internal storage. Arrays are defined in FORTRAN Modules at the beginning of each package file. Arrays and variables are accessed within each subroutine with the USE command. During execution, the size required for the arrays is determined from the relevant input files, and the arrays are dimensioned with the ALLOCATE command. In some cases, memory is allocated for arrays within a package and then later deallocated when the information is no longer needed. GWM reports to the output file the number of kilobytes of memory allocated for each of the packages.

Integration of GWM into MODFLOW MAIN program: GWM controls the repeated runs of the GWF Process through two loops depicted in figure 3-1—the flow-process loop and the GWM iteration loop. Both loops use a FORTRAN DO-WHILE structure. In both cases, the number of executions of the loop that will be required cannot be determined a priori. For example, when the flow-process loop is used to calculate perturbations, the number of successfully completed flow-process runs is known, but if any run should fail (perhaps because of a lack of flow-process convergence), then the loop will have to be executed an additional time.

GWM makes use of three logical variables in the MODFLOW MAIN program to track the current flow-process run. These are FIRSTSIM, LASTSIM, and FINISH. FIRSTSIM is set to indicate that the flow process is being run for the first time, whereas LASTSIM indicates that the flow process is being run for the final time. If GWM is not activated, then both FIRSTSIM and LASTSIM are set to TRUE so that only one flow-process run is executed. If GWM is active, FIRSTSIM is used to initialize variables and perform other GWM setup functions, whereas LASTSIM is set once a solution to the optimization problem has been found. FINISH is set at the completion of the last flow-process loop to terminate the GWM iteration loop.

Repeated Execution of the GWF Process: The GWM Process requires repeated execution of the GWF Process. At each execution of the GWF Process, GWM performs a rewind of all GWF files by using module GWM1BAS1RW. For many situations, each subsequent run will use a set of managed flow rates that are very close to those used in a prior run. For steady-state problems, this means that the solution to the prior run is an excellent first estimate of the solution to the subsequent problem. To facilitate this, a new module, GWM1BAS1RPP, is called if GWM is active and it is not the first run. If the problem is not steady-state, the head information is read from the original files; the IBOUND array is read and the initial heads are read into array STRT and assigned to HNEW. If the problem is steady-state, however, neither STRT nor IBOUND are read. HNEW retains its value from the prior flow-process run and automatically becomes the initial head for the next run.

GWM dependence on MODFLOW variables: GWM uses several MODFLOW variables to control progress of the solution algorithm. When modifying MODFLOW, consideration should be given to the definition of these variables. ICNVG is used to determine if the flow process has converged for a given flow-process run. If it has not converged, then GWM takes steps to perform the run again. HCLOSE is used as a measure of the precision of computed head values. It is assumed that HCLOSE has a value that reflects the head convergence criterion, regardless of the flow-process solver used.

Sign conventions on flow variables: The GWF Process treats withdrawals of water as negative values and injections of water as positive values. The GWM linear-program solver requires that all decision variables be lower bounded by zero, so that variables that represent withdrawals must be redefined as positive-value variables.

The following conventions are used to keep track of sign conventions. From the input perspective, all variables are treated as positive. This means that the bounds on withdrawals and injections are treated as positive values, and the variables are assumed to take nonnegative values when they appear in the objective function or summation constraints. Within the code, the flow-rate decision variables are stored by using the flow-process convention; for example, the array FVBASE stores the current base flow rates. The values in this array will be nonpositive for withdrawal variables and nonnegative for injection variables. Calculation of the response coefficients proceeds by adding to the FVBASE value a number stored in DELINC. This perturbation value may either be positive or negative. For the withdrawal variable, a negative value of DELINC implies a forward difference and a positive value implies a backward difference. The opposite is true for injection variables.

The difference in sign conventions is reconciled in subroutine SGWM1RMS1AP, where the sign on the response-matrix coefficients for withdrawal decision variables is switched immediately before the call to the linear-program solver. Because all other coefficients and bounds on the variables already have the positive sign convention, the problem is ready for solution. After return from the optimization solver, the signs of withdrawal decision variables are switched back to be consistent with the flow-process sign convention.

Cover: Hypothetical model grid illustrating some of the types of flow-rate and external decision variables that can be defined in the Ground-Water Management Process (GWM) for MODFLOW-2000. Flow-rate decision variables *Q*1, *Q*2, and *Q*3 represent ground-water withdrawal wells; flow-rate decision variable *Q*4 represents a recharge basin; and external decision variable IM1 represents a surface-water reservoir that is a source of water to the management model. (See figure 1-1 of this report and related discussions for more details.)

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