

**In cooperation with the State of Michigan Department of Environmental Quality,
and the State of Michigan Department of Natural Resources**

STRMDEPL08—An Extended Version of STRMDEPL with Additional Analytical Solutions to Calculate Streamflow Depletion by Nearby Pumping Wells

Open-File Report 2008–1166

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By Howard W. Reeves

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Preface

This report describes a computer program that implements four analytical solutions to estimate streamflow depletion by a nearby pumping well. The program is named STRMDEPL08 and is an extension of the program STRMDEPL written by Barlow (2000). The major extension to STRMDEPL is the addition of the two analytical solutions presented by Hunt (1999, 2003). Technical review of this report by Paul Barlow and Randy Bayless, and editorial review and assistance by Bonnie Stich Fink and Dorothy Tepper strengthened the presentation and is gratefully acknowledged.

The program may be obtained using the Internet at http://water.usgs.gov/software/ground_water.html. The performance of the program has been tested in a variety of cases. Additional use of the program, however, may reveal errors that were not detected in the testing. Users are requested to send notification of any errors found in this report or in the computer program to:

Office of Ground Water
U.S. Geological Survey
411 National Center
Reston, VA 20192
(703) 648-5001

HWR – May 2008

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Conversion Factors and Symbols

Multiply	By	To obtain
Length		
foot (ft)	0.3048	meter (m)
Flow rate		
cubic foot per second (ft ³ /s)	0.02832	cubic meter per second (m ³ /s)
gallon per minute (gal/min)	0.000063	cubic meter per second (m ³ /s)
Hydraulic conductivity		
foot per day (ft/d)	0.3048	meter per day (m/d)
Transmissivity*		
foot squared per day (ft ² /d)	0.09290	meter squared per day (m ² /d)

*Transmissivity: The standard unit for transmissivity is cubic foot per day per square foot times foot of aquifer thickness [(ft³/d)/ft²]ft. In this report, the mathematically reduced form, foot squared per day (ft²/d), is used for convenience.

List of Symbols

Roman characters

$$a = \frac{(K''/B')t}{\sigma} (1 - \alpha^2),$$

$$b = \frac{(K''/B')t}{S} \alpha^2, \text{ in the function } G(\alpha, t) \text{ in the Hunt (2003) solution,}$$

B'	is the thickness of the aquitard (length),
B''	is the distance from the bottom of the stream to the top of the leaky aquifer (length),
b'	is the thickness of the streambed (length),
b	is the stream width (length),
d	is the distance from the well to the stream (length),
i	is the number of time intervals (dimensionless),
K	is the hydraulic conductivity of the aquifer (length per time),
K'	is the hydraulic conductivity of the streambed (length per time),
K''	is the hydraulic conductivity of the aquitard (length per time),
k	is the time interval number (dimensionless),
L	is the streambed leakance (length),
Q_s	is the rate of streamflow depletion (cubic length per time),
Q_w	is the pumping rate (cubic length per time),
$Q_s(t_i)$	is the streamflow depletion at time interval i (cubic length per time),

$Q_0(t_0)$	is the initial pumping rate of the well prior to changes in the pumping rate, which yields the depletion at time equal to t_0 (cubic length per time),
$\Delta Q_k(t_k)$	is the change in pumping rate during the interval k (cubic length per time),
$R(t_0)$	is the ratio of streamflow depletion to the initial pumping rate of the well prior to changes in the pumping rate given by either equation 1 or 2 at time equal to t_0 (dimensionless),
$R(t_i)$	is the ratio of streamflow depletion to pumping rate given by either equation 1 or 2 at time interval i (dimensionless),
S	is the storativity or specific yield of the aquifer (dimensionless),
T	is the transmissivity of the aquifer (square length per time),
t	is the time,
t_i	is the length of time from the beginning of the pumping analysis to the time of interest, and
t_0	is the length of time prior to the analysis at the initial pumping rate.

Greek characters

α	is the variable of integration (dimensionless),
λ	is the streambed conductance term (length per time), and
σ	is the specific yield of the aquitard (dimensionless).

Functions

$\binom{2n}{n}$	is the Binomial Coefficient (dimensionless),
$\operatorname{erfc}()$	is the complementary error function (dimensionless),
$\exp()$	is the exponential function (dimensionless),
$F(\alpha, t)$	is a function in the Hunt (2003) solution (dimensionless),
$G(\alpha, t)$	is a function in the Hunt (2003) solution (dimensionless),
$I_0()$	is the Modified Bessel function of zero order (dimensionless), and
$P()$	is an Incomplete Gamma function (dimensionless).

STRMDEPL08—An Extended Version of STRMDEPL with Additional Analytical Solutions to Calculate Streamflow Depletion by Nearby Pumping Wells

By Howard W. Reeves

Abstract

STRMDEPL, a one-dimensional model using two analytical solutions to calculate streamflow depletion by a nearby pumping well, was extended to account for two additional analytical solutions. The extended program is named STRMDEPL08. The original program incorporated solutions for a stream that fully penetrates the aquifer with and without streambed resistance to ground-water flow. The modified program includes solutions for a partially penetrating stream with streambed resistance and for a stream in an aquitard subjected to pumping from an underlying leaky aquifer. The code also was modified to allow the user to input pumping variations at other than 1-day intervals. The modified code is shown to correctly evaluate the analytical solutions and to provide correct results for half-day time intervals.

Introduction

This report documents modifications to the STRMDEPL (STReaMflow DEPLetion by wells) computer code (Barlow, 2000) to incorporate two additional streamflow-depletion analytical solutions. The original program evaluated the solutions for a stream that fully penetrates the aquifer with and without streambed resistance to ground-water flow. The additional solutions extend the code to simulate streamflow depletion from a partially penetrating stream with streambed resistance (Hunt, 1999) and from a stream in an aquitard subjected to pumping from an underlying leaky aquifer (Hunt, 2003). These two additional analytical solutions may be more appropriate for cases where a pumping well is potentially interacting with a small stream that does not fully penetrate the aquifer. The differences between the original solutions and the two solutions by Hunt (1999, 2003) implemented in the modified code are summarized by schematics of the conceptual models (fig. 1 and Hunt, 1999). The modified program is termed

STRMDEPL08. Test cases illustrating the performance of the modified program and documentation of the input and output files are presented in this report. The desire to modify STRMDEPL to include the Hunt (1999, 2003) solutions was identified in a cooperative research project between the U.S. Geological Survey (USGS) and the Michigan Departments of Natural Resources and Environmental Quality. The work was completed during 2007 and prepared for publication in 2008.

Analytical Solutions

The STRMDEPL computer code (Barlow, 2000) implements two analytical solutions for streamflow depletion by a well and uses superposition to allow for varying pumping rates. The first solution is for a system with a stream that fully penetrates the aquifer with no streambed resistance between the stream and the aquifer (fig. 1A) and may be expressed as (Glover and Balmer, 1954; Jenkins, 1968)

$$Q_s = Q_w \operatorname{erfc} \left(\sqrt{\frac{d^2 S}{4Tt}} \right), \quad (1)$$

where

- Q_s is the rate of streamflow depletion (cubic length per time),
- Q_w is the pumping rate (cubic length per time),
- $\operatorname{erfc}()$ is the complementary error function (dimensionless),
- d is the distance from the well to the stream (length),
- S is the storativity or specific yield of the aquifer (dimensionless),
- T is the transmissivity of the aquifer (square length per time), and
- t is the time.

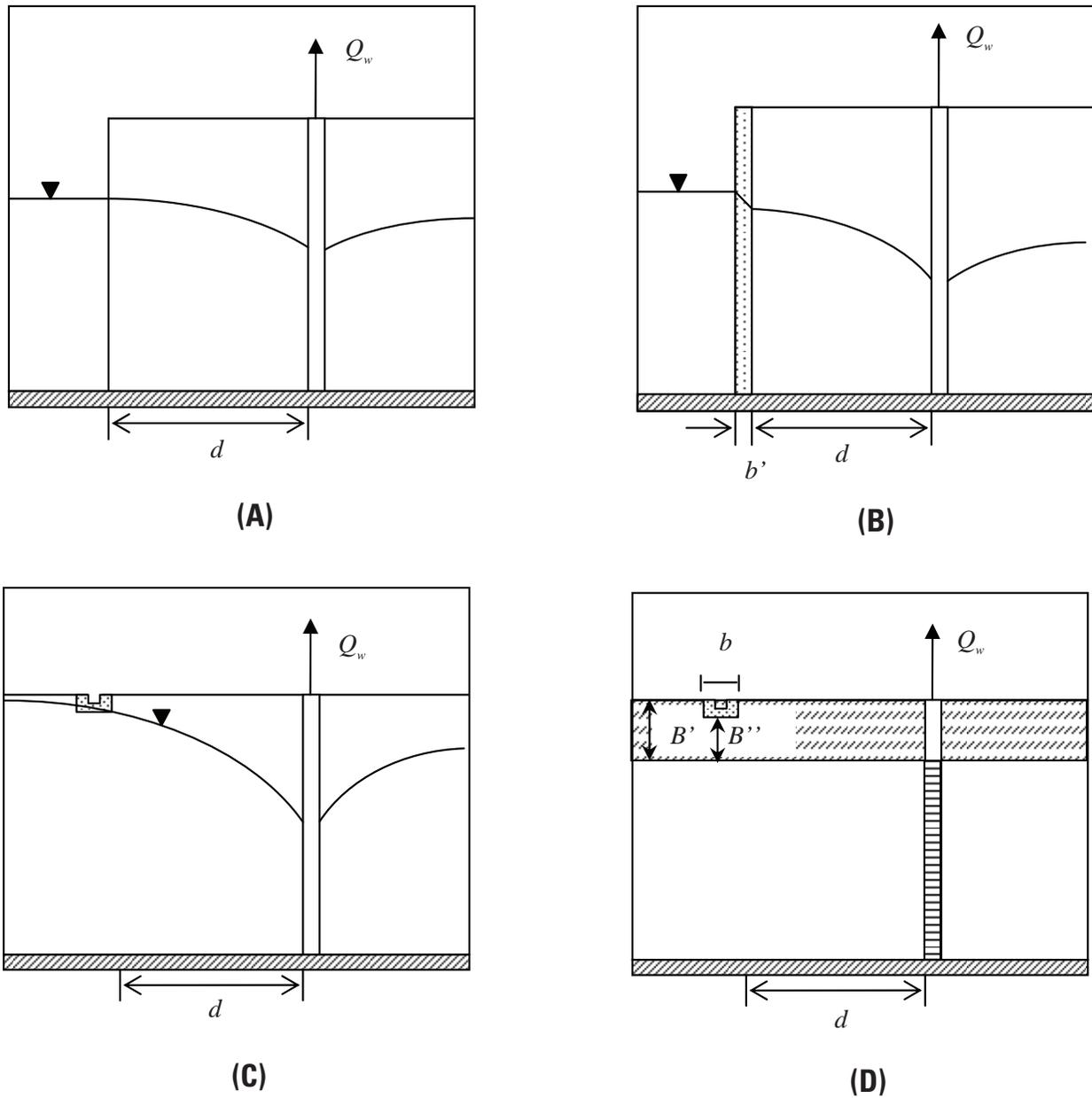


Figure 1. Alternate conceptual models for streamflow depletion by a pumping well: **(A)** fully penetrating stream with no streambed resistance, **(B)** fully penetrating stream with streambed resistance, **(C)** partially penetrating stream with streambed resistance, and **(D)** partially penetrating stream in an aquitard with pumping from underlying leaky aquifer. [d is the distance from the well to the stream, Q_w is the pumping rate from the well, b' is the thickness of the streambed, B' is the distance from the land surface to the top of the leaky aquifer, B'' is the distance from the bottom of the stream to the top of the leaky aquifer, and b is the width of the stream.]

The major assumptions used to derive equation 1 are

- horizontal flow dominates any potential vertical flow so that the Dupuit assumption is valid;
- the aquifer is homogeneous, isotropic, and has constant saturated thickness;
- the aquifer is either confined or changes in hydraulic head in the aquifer are small compared to the saturated thickness, allowing the equation describing ground-water flow to be linearized;
- the stream is straight, infinitely long, and fully penetrates the aquifer;
- the pumping does not change the stage of the stream;
- the hydraulic conductivity of the streambed is similar or greater than the aquifer and does not offer resistance to ground-water flow;
- there is no streambank storage;
- the pumping rate is constant; and
- the aquifer extends to infinity away from the stream.

Examination of equation 1 reveals that streamflow depletion depends on aquifer properties, the distance from the well to the stream, and time. Initially after pumping, streamflow depletion is small and the source of water to the well is from storage in the aquifer. At long times, determined by aquifer properties and the distance from the well to the stream, streamflow depletion approaches the pumping rate.

The second solution is for a system with a stream that fully penetrates the aquifer with streambed resistance between the stream and the aquifer (fig. 1B) and may be expressed as (Hantush, 1965)

$$Q_s = Q_w \left[\operatorname{erfc} \left(\sqrt{\frac{d^2 S}{4Tt}} \right) - \exp \left(\frac{Tt}{SL^2} + \frac{d}{L} \right) \operatorname{erfc} \left(\sqrt{\frac{Tt}{SL^2}} + \sqrt{\frac{d^2 S}{4Tt}} \right) \right], \quad (2)$$

where

L is the streambed leakance (length), and
 $\exp()$ is the exponential function (dimensionless).

The streambed leakance is defined as

$$L = \frac{Kb'}{K'}, \quad (3)$$

where

K is the hydraulic conductivity of the aquifer (length per time),

K' is the hydraulic conductivity of the streambed (length per time), and
 b' is the thickness of the streambed (length).

The assumptions used to derive equation 1 are used for this equation. In this case, the streambed also offers additional resistance to flow as described by equation 3. If streambed leakance approaches zero, then equation 2 collapses to the equation describing streamflow depletion with no streambed resistance, equation 1 (see Hantush, 1965; Hunt, 1999). The behavior with time for this equation is similar to that for equation 1. The streambed resistance slows the response of the system, but ultimately, at long times, the streamflow depletion approaches the pumping rate.

Equations 1 and 2 were programmed in the STRMDEPL computer code. The code also implemented superposition to allow for changing pumping rates, which may be expressed as (Barlow, 2000)

$$Q_s(t_i) = Q_0(t_0)R(t_0) + \sum_{k=1}^i \Delta Q_k(t_k)R(t_{i-k+1}), \quad (4)$$

where

$Q_s(t_i)$ is the streamflow depletion at time interval i (cubic length per time),
 $Q_0(t_0)$ is the initial pumping rate of the well prior to changes in the pumping rate, which yields the depletion at time equal to t_0 (cubic length per time),
 $\Delta Q_k(t_k)$ is the change in pumping rate during the interval k (cubic length per time),
 $R(t_0)$ is the ratio of streamflow depletion to the initial pumping rate of the well prior to changes in the pumping rate given by either equation 1 or 2 at time equal to t_0 (dimensionless),
 $R(t_i)$ is the ratio of streamflow depletion to pumping rate given by either equation 1 or 2 at time interval i (dimensionless),
 t_i is the length of time from the beginning of the pumping analysis to the time of interest,
 t_0 is the length of time prior to the analysis at the initial pumping rate,
 i is the number of time intervals (dimensionless), and
 k is the time interval number (dimensionless).

The use of superposition is appropriate because the underlying ground-water-flow equations used to derive the equations are linear for confined aquifers and nearly linear for unconfined aquifers meeting the assumptions required by the solutions. As discussed by Barlow (2000), the initial pumping rate prior to changes in the system is used to establish the starting conditions for the analysis. In some systems, the stream may have already been subjected to pumping and the analysis is focused on how the system changes as this initial rate is changed.

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The solution for a partially penetrating stream with streambed resistance (fig. 1C) may be written as (Hunt, 1999)

$$Q_s = Q_w \left[\operatorname{erfc} \left(\sqrt{\frac{d^2 S}{4Tt}} \right) - \exp \left(\frac{\lambda^2 t}{4ST} + \frac{\lambda d}{2T} \right) \operatorname{erfc} \left(\sqrt{\frac{\lambda^2 t}{4ST}} + \sqrt{\frac{d^2 S}{4Tt}} \right) \right], \quad (5)$$

where

λ is the streambed conductance term (length per time).

The major difference in the assumptions used to derive this equation compared to equations 1 and 2 is that the stream is assumed to be very narrow and only extend a small distance into the aquifer, allowing it to be mathematically modeled as a straight line crossing an infinite aquifer. The aquifer now is infinite in the horizontal direction, and drawdown may occur in the aquifer on the side of the stream opposite from the pumping well (Hunt, 1999). In this model, streamflow depletion is described by a simple Darcy expression describing the flux between the stream and the aquifer (Hunt, 1999). The aquifer is assumed to remain in hydraulic contact with the stream, which means that the pumping well does not cause the hydraulic head in the aquifer to be lower than the streambed (see comment by Rushton, 1999; and analysis by Peterson and Zhang (2000), Osman and Bruen (2002), and Bruen and Osman (2004) for cases where the hydraulic head in the aquifer does decrease below the streambed).

Note the similarity in form between the partially penetrating stream solution (equation 5) and the fully penetrating stream solution (equation 2). As discussed by Hunt (1999), the solutions given by equations 2 and 5 are identical if the streambed conductance is set to

$$\lambda = 2 \frac{T}{L}. \quad (6)$$

The analytical solution for a stream in an aquitard with pumping from an underlying leaky aquifer (fig. 1D) may be written as (Hunt, 2003)

$$Q_s = Q_w \left[\operatorname{erfc} \left(\sqrt{\frac{d^2 S}{4Tt}} \right) - \exp \left(\frac{\lambda^2 t}{4ST} + \frac{\lambda d}{2T} \right) \operatorname{erfc} \left(\sqrt{\frac{\lambda^2 t}{4ST}} + \sqrt{\frac{d^2 S}{4Tt}} \right) - \lambda \int_0^1 F(\alpha, t) G(\alpha, t) d\alpha \right]. \quad (7)$$

In this equation, $F(\alpha, t)$ and $G(\alpha, t)$ are functions that may be calculated as (Hunt, 2003)

$$F(\alpha, t) = \exp \left(-\frac{Sd^2}{4Tt\alpha^2} \right) \sqrt{\frac{Tt}{Sd^2\pi}} - \frac{\alpha t \lambda}{2Sd} \exp \left(\frac{\lambda d}{2T} + \frac{t\alpha^2 \lambda^2}{4ST} \right) \operatorname{erfc} \left(\frac{\alpha \lambda}{2} \sqrt{\frac{t}{ST}} + \frac{d\sqrt{S}}{2\alpha\sqrt{Tt}} \right), \quad (8)$$

$$G(\alpha, t) = \frac{1}{2} \left[1 - e^{-(a+b)} I_0(2\sqrt{ab}) + \left(\frac{b-a}{a+b} \right) \sum_{n=0}^{\infty} \binom{2n}{n} P(2n+1, a+b) \left(\frac{\sqrt{ab}}{a+b} \right)^{2n} \right], \quad (9)$$

where

$$a = \frac{(K''/B')t}{\sigma} (1 - \alpha^2), \quad (10)$$

$$b = \frac{(K''/B')t}{S} \alpha^2, \quad (11)$$

- α is the variable of integration (dimensionless),
- $I_0()$ is the Modified Bessel function of zero order (dimensionless),
- $\binom{2n}{n}$ is the Binomial Coefficient (dimensionless),
- $P()$ is an Incomplete Gamma function (dimensionless),
- K'' is the hydraulic conductivity of the aquitard (length per time),
- B' is the thickness of the aquitard (length), and
- σ is the specific yield of the aquitard (dimensionless).

In this report, the streambed conductance, λ , is given by,

$$\lambda = \frac{K''b}{B''}, \quad (12)$$

- b is the stream width (length), and
- B'' is the distance from the bottom of the stream to the top of the leaky aquifer (length).

The assumptions used to derive this solution are similar to those used for the other solutions. The drawdowns in both the aquitard and leaky aquifer are assumed to be small compared to the saturated thicknesses such that the Dupuit assumption holds for each aquifer. The aquifers are infinite, homogeneous, and have constant thickness. Flow between the aquitard and the leaky aquifer is described by Darcy's Law; pumping is constant. The stream is infinitely long, and, although stream width is used to estimate the streambed conductance (equation 12), the stream width is assumed to approach zero in the solution such that the stream may be modeled as a line that crosses the infinite aquifer. An alternate statement of this last assumption is that the well is far enough away from the stream such that the stream may be modeled as a line (Hunt, 2003).

Note that this solution is a modification of the partially penetrating stream solution (equation 5) so that modification to the code entails the evaluation of the integral containing the $F(\alpha, t)$ and $G(\alpha, t)$ functions. Methods to evaluate the functions $F(\alpha, t)$ and $G(\alpha, t)$ are discussed by Hunt (2003). Also note that the program was written to allow input of a value for streambed conductance, λ , while allowing the hydraulic conductivity of the aquitard, K'' , to be set to zero despite the definition of streambed conductance given by equation 11 (see appendix 1). The ability to set K'' to zero is helpful for testing the code because the solution collapses to the Hunt (1999) solution under that condition.

Modifications to STRMDEPL

The modifications to STRMDEPL include (1) changes to the number of input variables to accommodate the Hunt (1999, 2003) solutions, (2) the addition of a flag to indicate to the program which solution should be evaluated, (3) modifications to read in and use a constant time increment other than one day, and (4) incorporation (into the code) of the equations to evaluate the two additional solutions. Appendix 1 details the input file required to use the program. Subroutines to evaluate the Modified Bessel and Incomplete Gamma functions were based on the approximations given in Abramowitz and Stegun (1965). The integral of the functions $F(\alpha, t)$ and $G(\alpha, t)$ with respect to α was estimated using numerical integration with seven Gauss points (Gauss points and weights from Abramowitz and Stegun, 1965).

Verification of Model Results

The performance of STRMDEPL08 was tested using several test cases. The first set of test cases demonstrate performance of the code under constant pumping and illustrate the behavior of both the Hunt (1999) solution, equation 5, and the Hunt (2003) solution, equation 7. The second set of test cases shows the performance of the code under time-varying

pumping. The use of time intervals other than 1-day in input and output time series also is illustrated in the second set of tests.

Constant Pumping—First Set of Test Cases

The first set of test cases examines the performance of the model under constant pumping. The solution for the partially penetrating stream with streambed resistance (equation 5) is identical to the solution for the fully penetrating stream with streambed resistance (equation 2) if the streambed-resistance term and the streambed-leakance term are adjusted as shown in equation 6. Additionally, the solution for pumping from a leaky aquifer (equation 7) reduces to that for a partially penetrating stream with streambed resistance (equation 5) if the hydraulic conductivity of the aquitard, K'' , is set to zero. Setting the hydraulic conductivity of the aquitard to zero effectively makes the underlying leaky aquifer a strictly confined aquifer. The first test of the modified code demonstrated that the Hunt (1999) solution generated results identical to the original code using the Hantush (1965) option if λ is set to $2T/L$ (equation 6). The Hunt (2003) option was then tested by using the same suite of input data values and setting K'' to zero. For this test, $T = 1,000 \text{ ft}^2/\text{d}$, $L = 100 \text{ ft}$, $\lambda = 20 \text{ ft/d}$, $d = 500 \text{ ft}$, $S = 0.1$, and $Q_w = 0.557 \text{ ft}^3/\text{s}$ (250 gal/min). The results from the modified code for 100 days of pumping are identical for the three solution options (fig. 2). These results demonstrate that the modified code is evaluating the equations correctly.

To illustrate the behavior of the solution for varying values of the streambed-conductance value, λ , a plot similar to that presented by Hunt (1999) was generated using the modified code (fig. 3). In this figure, the ratio of streamflow depletion to pumping rate is shown plotted against a dimensionless group, $4Tt/Sd^2$, which is dimensionless time. Six curves are shown for varying values of $\lambda d/T$. Results for the no streambed-resistance (Jenkins, 1968) solution also are shown to confirm that the Hunt (1999) solution approaches the solution for no streambed resistance as λ gets large. Dimensionless groups allow this figure to be used to show the behavior of the solution for the potential range of S, T, d , and λ combinations. As shown in the figure, as the streambed conductance decreases, for a given aquifer and distance between the well and the stream, the response of the stream to the pumping is delayed. In the extreme case of streambed conductance equal to zero, the streamflow depletion becomes zero (equation 5) and the drawdown solution becomes the Theis solution for a well in an infinite confined aquifer (Hunt, 1999). Streamflow depletion increases as the streambed conductance is increased or the distance between the well and the stream is decreased. In all cases of non-zero streambed conductance, the solution will eventually reach a constant value in which the streamflow depletion is equal to the pumping rate. This steady-state condition is required because the only source of water to the well in the system other than storage in the aquifer is streamflow capture.

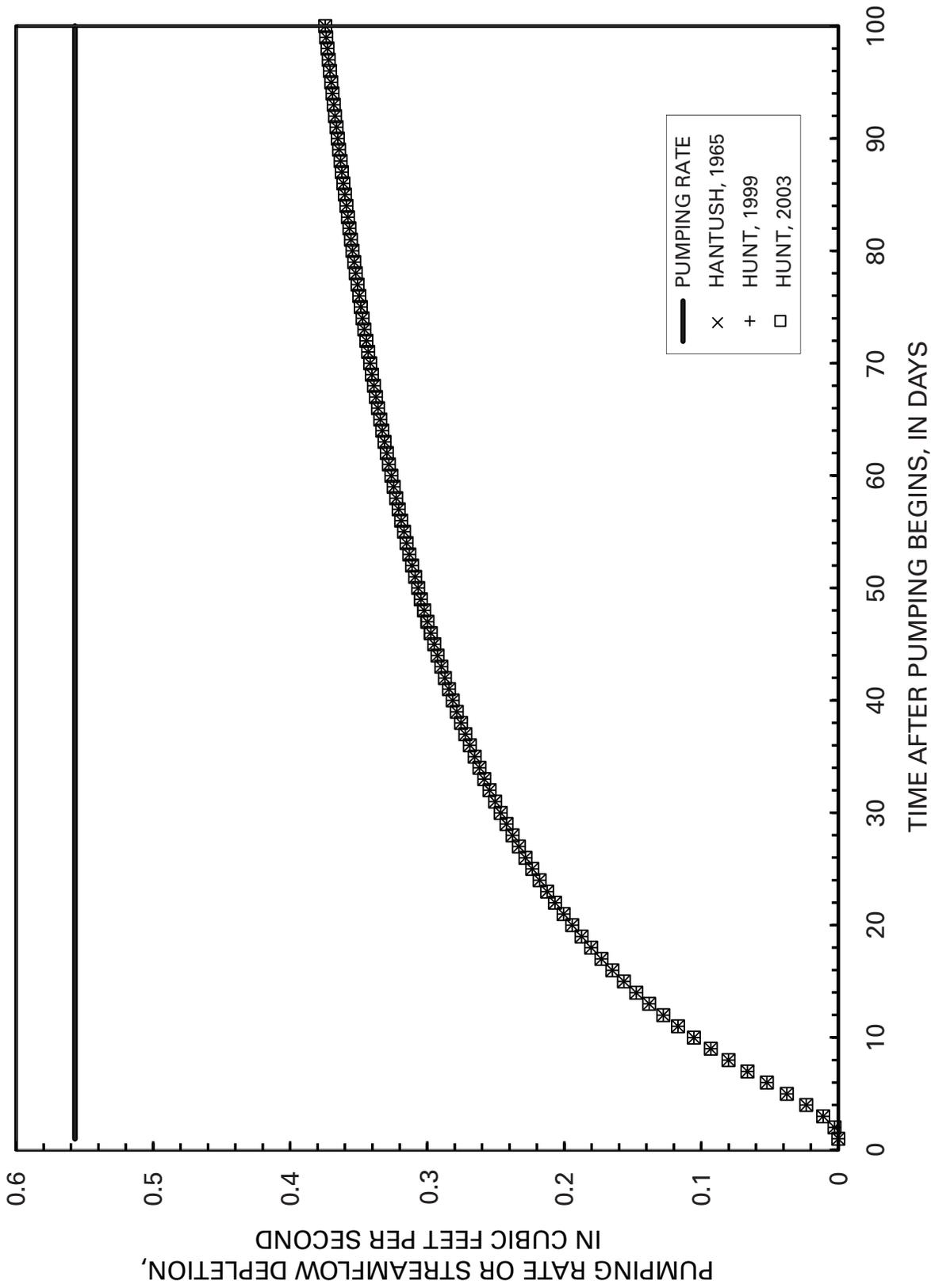


Figure 2. Results from three streamflow-depletion solutions for the special case in which the solutions should yield identical results.

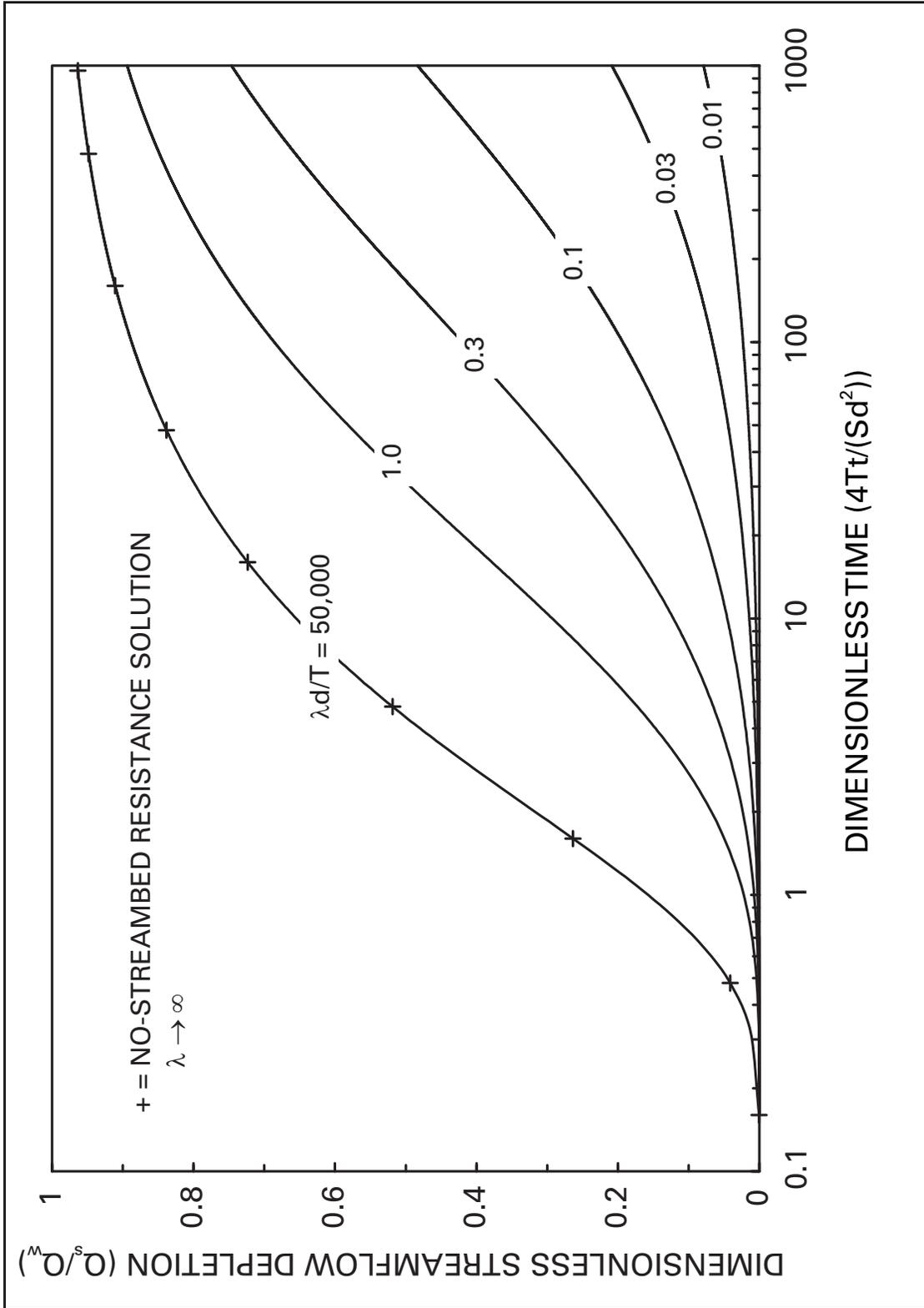


Figure 3. Hunt (1999) solution (equation 5) shown for a range of streambed-conductance values with values of no streambed resistance shown as reference. [T is the transmissivity, t is time, S is storativity or specific yield, d is the distance from the well to the stream, λ is the streambed conductance, Q_w is the pumping rate, and Q_s is the streamflow depletion.]

The nature of the solution for pumping from a leaky aquifer (equation 6) is illustrated by generating a plot similar to that presented by Hunt (2003) using the modified code (fig. 4). The correct behavior of the code is observed by (1) visual comparison to the plot from Hunt (2003) and (2) the comparison of the partially penetrating stream solution (Hunt, 1999) to the leaky-aquifer solution (Hunt, 2003) for the case where the streambed conductance, λ , is given a finite value, but the hydraulic conductivity of the aquitard, K'' , is set to zero (fig. 4). For this special case, the aquitard does not provide water to the pumping well but the stream does. Under these conditions, the system acts as a confined aquifer and the leaky-aquifer solution (Hunt, 2003) collapses to the partially penetrating stream solution (Hunt, 1999).

As the hydraulic conductivity of the aquitard is increased, it can provide water from storage to the pumped leaky aquifer, and the solution takes on a leaky-aquifer or delayed-yield behavior. The delayed-yield behavior is characterized by a double inflection of the curve and longer time required to reach specific values of streamflow depletion (fig. 4). The nature of the solution is that after pumping begins, there is some initial drawdown in the leaky aquifer and streamflow depletion begins. Depending on the aquifer and aquitard properties, however, the rate of streamflow depletion may level off because the hydraulic head in the aquitard responds to changes in head in the underlying leaky aquifer and water is released from storage in the aquitard. After some time, the head in the aquitard approaches the head in the underlying leaky aquifer and the rate of water release from the aquitard declines. As the aquitard releases less water, the rate of streamflow depletion increases until, at steady state, the streamflow depletion becomes equal to the pumping rate. Water released from storage of the aquitard only delays the attainment of steady-state streamflow depletion because, in this model, the aquitard cannot be a long-term source of water. If the hydraulic conductivity of the aquitard is great enough, the system will respond like an unconfined aquifer with the specific yield of the aquitard, and the double-inflection behavior noted at intermediate values of aquitard hydraulic conductivity will be difficult to observe. This behavior is illustrated in the series of curves shown in figure 4; as the $(K''/B')d^2/T$ ratio increases, the aquitard supplies more water and delays streamflow depletion.

Time-Varying Pumping Rates—Second Set of Test Cases

The second set of test cases demonstrates the performance of the model under time-varying pumping rates. The time-varying pumping-rate test was expected to perform with-

out any problems because the constant pumping-rate solutions for the modified code matched the results from the original code for the test cases and visually matched published results from Hunt (1999) and Hunt (2003). The only modification to the superposition algorithm programmed in STRMDEPL was that the time increment was changed from a fixed value of 1 day to an optional variable input by the user. The program still requires an input time series of pumping rates at a fixed increment, but the increment can be different than 1 day. The length of the increment in days also is read by the program. Note that because this is an analytical solution, the time increment used is only to allow variation in the pumping rate; the solution is accurate for any time interval evaluated. The output series produced by the code is identical to the input series.

The first test uses the same parameters as the first constant-rate pumping test: $T = 1,000$ ft²/d, $L = 100$ ft, $\lambda = 20$ ft/d, $d = 500$ ft, $S = 0.1$, and $Q_w = 0.557$ ft³/s (250 gal/min). The pumping begins after 31 days and continues for 28 days. The fully penetrating stream solution (Hantush, 1965) and the partially penetrating stream solution (Hunt, 1999) were evaluated using the modified code. Two input time intervals were tested: a half-day interval and a 1-day interval. The parameters were set so that the fully penetrating solution and the partially penetrating solution should yield identical results. The results demonstrate that the modified code evaluates the Hunt (1999) solution correctly, and that the modifications to allow different time intervals in the solution were implemented correctly (fig. 5).

The behavior of time-varying pumping on streamflow depletion is shown in the second test using the modified code and the partially penetrating stream solution (equation 5). For this case, $T = 2,000$ ft²/d, $\lambda = 10$ ft/d, $d = 250$ ft, and $S = 0.05$. The pumping begins after 244 days and continues for 91 days and then is followed by another 91-day pumping cycle that begins after 274 days. This time series was used to represent summer irrigation pumping and a time series that begins on October 1 of a calendar year. The aquifer conditions and distance to the stream were selected to be representative of conditions encountered in the field and to yield different streamflow-depletion estimates depending on the pumping scenario used. Three pumping scenarios were tested. In the first scenario, the well is pumped at 1 ft³/s for 12 hours followed by 12 hours with no pumping for the 91-day pumping cycles (half-day pumping in fig. 6). In the second scenario, the pumping rate is set to 1 ft³/s for 3-1/2 days followed by no pumping for 3-1/2 days for the 91-day pumping cycles (half-week pumping in fig. 6). In the third scenario, the pumping rate is equal to 0.5 ft³/s for 24 hours of continuous pumping for the 91-day pumping cycles (half-rate pumping in fig. 6).

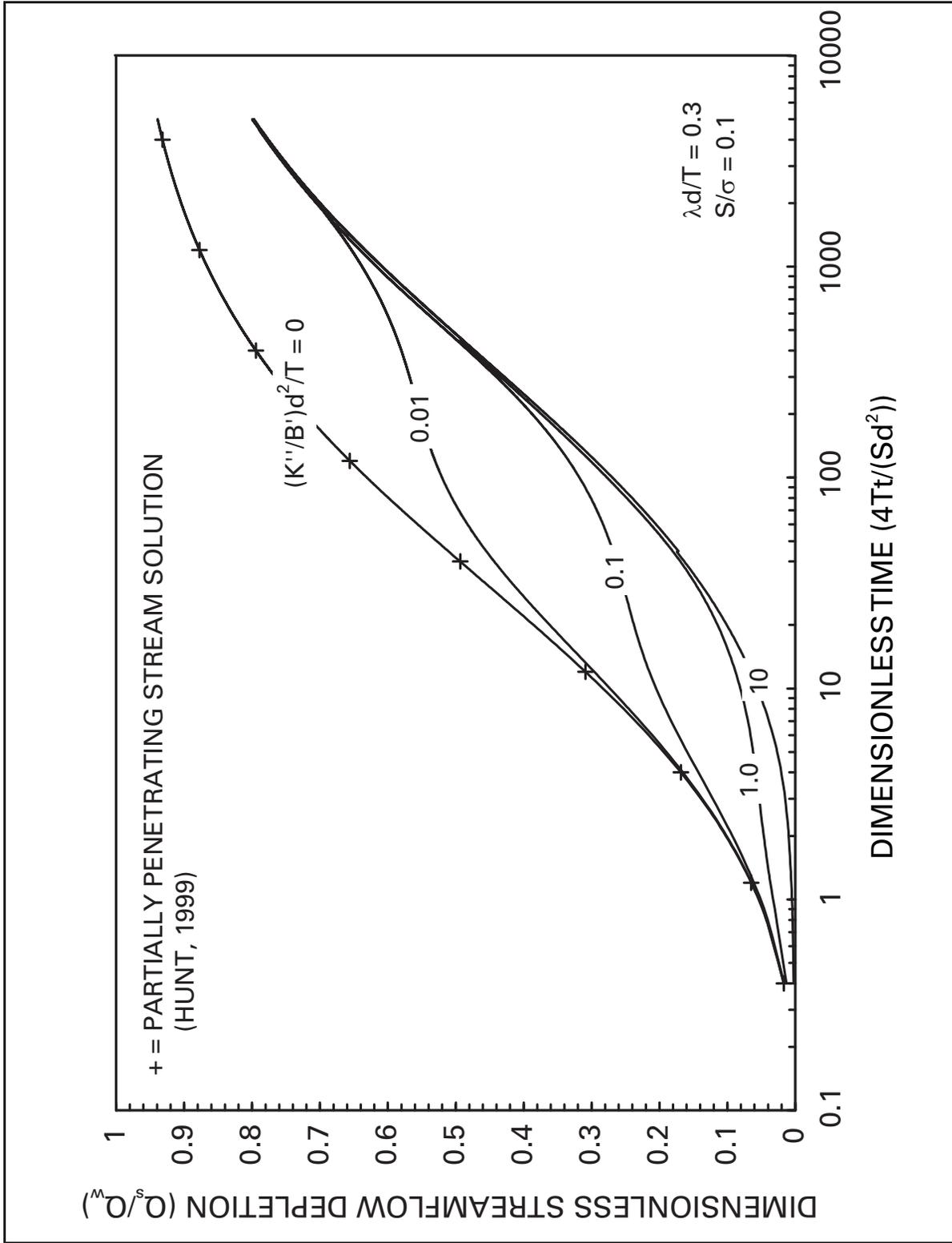


Figure 4. Hunt (2003) solution for pumping from a leaky aquifer underlying a stream in an aquitard with varying hydraulic properties and Hunt (1999) solution showed as reference. [T is the transmissivity, t is time, S is storativity of the leaky aquifer, σ is the specific yield of the aquitard, K'' is the hydraulic conductivity of the aquitard, B' is the thickness of the aquitard, d is the distance from the well to the stream, λ is the streambed conductance, Q_w is the pumping rate, and Q_s is the streamflow depletion.]

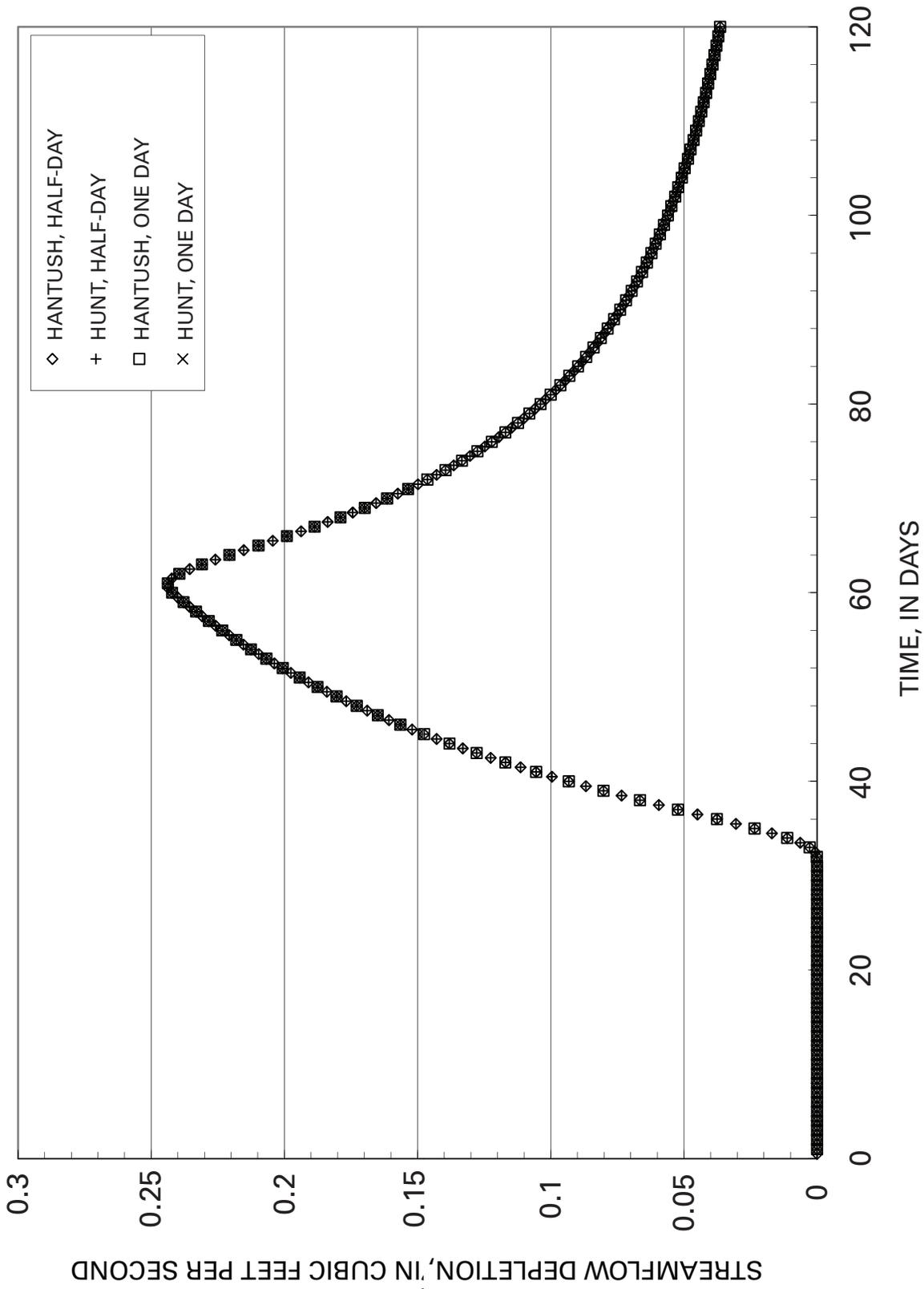


Figure 5. Comparison of fully penetrating stream (Hantush, 1965) and partially penetrating stream (Hunt, 1999) solutions for case where solutions should yield identical results. [Pumping begins after 31 days and lasts for 28 days. The results shown are for two time intervals: half-day and 1 day.]

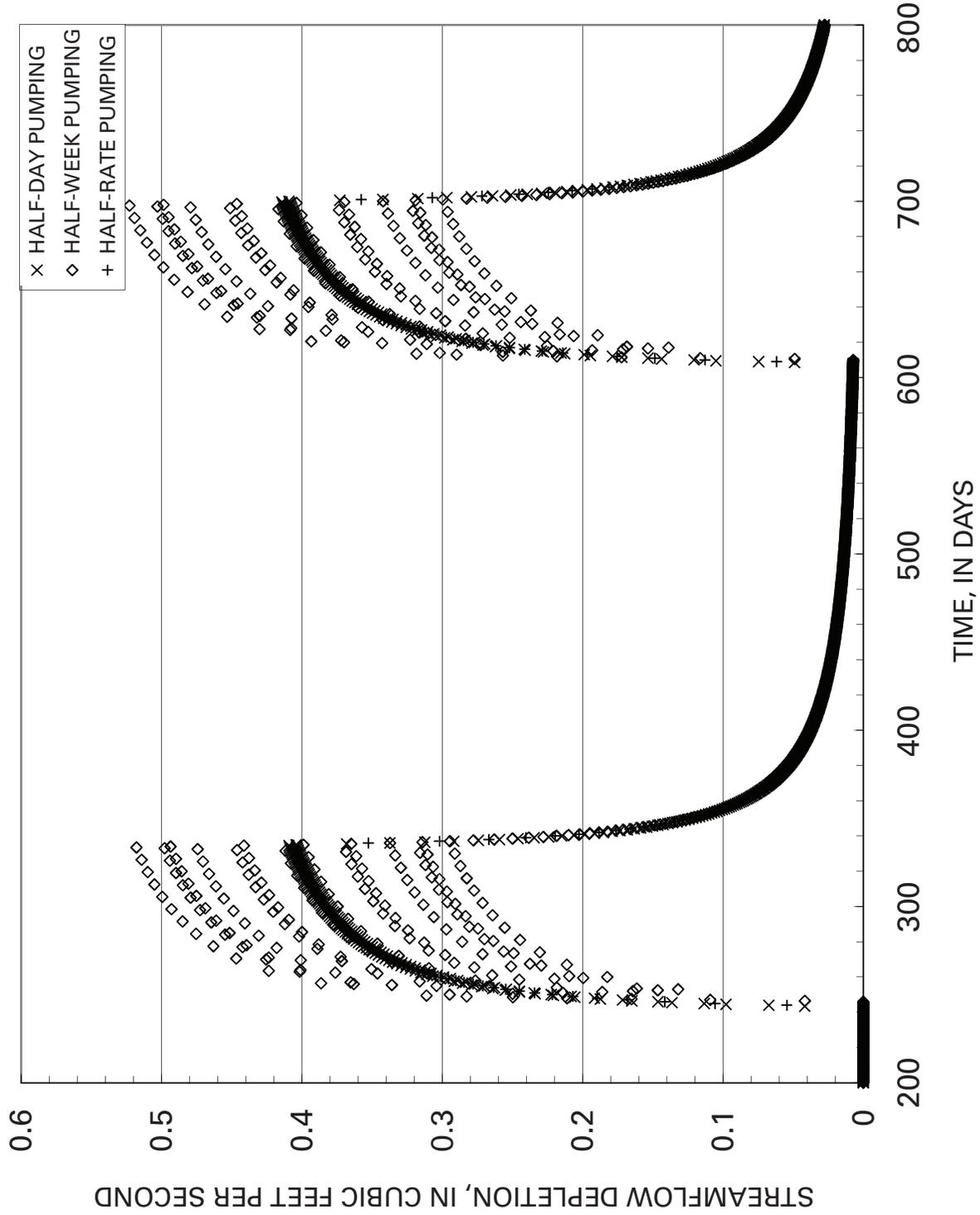


Figure 6. Response to time-varying pumping comparing three pumping scenarios: 3 months of pumping at 1 cubic foot per second with 12 hours of pumping followed by 12 hours of no pumping (half-day pumping); 3 months of pumping at 1 cubic foot per second with 3 ½ days of pumping followed by 3 ½ days of no pumping (half-week pumping); and 3 months of continuous pumping at 0.5 cubic feet per second (half-rate pumping).

The total volume of water pumped for the three scenarios is the same. The important feature of these solutions is that although the total volume of pumping is the same for the three scenarios, the maximum streamflow depletion differs between the scenarios. In this case, the 3-½ day pumping at 1 ft³/s followed by 3-½ days of no pumping impacts the stream more in terms of the calculated maximum streamflow depletion compared to either 12 hours of pumping followed by 12 hours of no pumping or continuous pumping at 0.5 ft³/s. In this case, the 12-hour cycle at 1 ft³/s is nearly identical to continuous pumping at 0.5 ft³/s. The behavior results from the close proximity of the well to the stream, the high aquifer dispersivity because of the high value for transmissivity, and the low value for storativity. This test illustrates the findings by Wallace and others (1990) that if maximum streamflow depletion in response to cyclic pumping is of interest, then, in some cases, actual pumping schedules must be used because continuous pumping at the average value for the cyclic schedule may under-estimate the streamflow depletion.

Summary of Modified Code

The computer program evaluating the analytical solutions for streamflow depletion by a pumping well by Barlow (2000) was modified to include two additional analytical solutions as part of a cooperative research project with the Michigan Departments of Natural Resources and Environmental Quality. The original program, STRMDEPL, evaluates the solution for streamflow depletion for a fully penetrating stream with no streambed resistance (Glover and Balmer, 1954; Jenkins, 1968) and the solution for streamflow depletion for a fully penetrating stream with streambed resistance (Hantush, 1965). The modified code, STRMDEPL08, also evaluates solutions for streamflow depletion from a partially penetrating stream with streambed resistance (Hunt, 1999) and from a stream in an aquitard subjected to pumping from an underlying leaky aquifer (Hunt, 2003). These two additional analytical solutions may be more appropriate for cases where a pumping well is potentially interacting with a small stream that does not fully penetrate the aquifer. The required input file for the modified code and example output files produced by the code are shown in appendix 1. The code itself is provided on the website listed in the Preface of this report. This code was tested and satisfactorily evaluated four analytical solutions for streamflow depletion by a pumping well. The nature of the solutions for varying parameter values was discussed.

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Appendix

Appendix 1. Input Instructions and Example Files

One input file is required to run STRMDEPL08. The name of the file is prompted from the code. It also may be supplied at the command line. The program may be run from the command line:

> **STRMDEPL08.exe**

Enter name of file containing input data:

example.dat

Enter name of file for program results:

example.out

Enter name of plot file (return for no plot file):

example.plt

File names have been read.

>

Alternatively,

> **STRMDEPL08.exe example.dat example.out**

example.plt

>

Bold font indicates user input. The format for the input file was taken from the original STRMDEPL program (Barlow, 2000), and existing data files should be able to be used with only minor changes. The input is read in free format. Each input line may be up to 140 characters in length and the input variables may be separated by blanks or commas.

Line 1: Title

Line 2: Well identifier

Line 3: XWELL, DIFFUS, ISOLN, SLEAK, STOR, SIGMA, BPP, BP, RKPRIME, STRWTH, DELT

Line 4: INTIME, QWINIT

Line 5: NPD

Lines 6 – NPD: CDATE(I), QWELL(I)

The inputs are:

Title: Title of the simulation that may be up to 100 characters in length. The title is printed on the output file. Leave blank if no title is desired.

Well identifier: Identifying string for the pumping well that may be up to 20 characters in length. The identifying string is printed on the output file. Leave blank if no well identifier is desired.

XWELL: Distance from well to stream, feet.

DIFFUS: If ISOLN (see next input variable) ≤ 1 : diffusivity of aquifer, square feet per second, and if ISOLN > 1 , transmissivity of aquifer, square feet per second.

ISOLN: Flag to specify choice of analytical solution used:

0 = fully penetrating with no streambed resistance, equation 1 (Jenkins, 1968),

1 = fully penetrating stream with streambed resistance, equation 2 (Hantush, 1965),

2 = partially penetrating stream with streambed resistance, equation 5 (Hunt, 1999), and

3 = stream in an aquitard with pumping from an underlying leaky aquifer, equation 7 (Hunt, 2003).

SLEAK: If ISOLN = 1, streambed leakage, feet, if ISOLN = 2, streambed conductance, feet per second, if ISOLN = 3, default streambed conductance if RKPRIME=0, feet per second. This value is not used if ISOLN=0 and should be entered as 0.

STOR: If ISOLN > 1 , storativity or specific yield, dimensionless. This value is not used if ISOLN ≤ 1 and should be entered as 0.

SIGMA: If ISOLN = 3, specific yield of aquitard containing stream, dimensionless. This value is not used if ISOLN ≤ 2 and should be entered as 0.

BPP: If ISOLN = 3, distance from streambed to top of the leaky aquifer, feet. This value is not used if ISOLN ≤ 2 and should be entered as 0.

BP: If ISOLN = 3, thickness of aquitard containing stream, feet. This value is not used if ISOLN ≤ 2 and should be entered as 0.

RKPRIME: If ISOLN = 3, hydraulic conductivity of aquitard, feet per second. This value is not used if ISOLN ≤ 2 and should be entered as 0. If RKPRIME is set to 0.0 and ISOLN = 3, then the value of SLEAK is used for the streambed conductance in the solution. If RKPRIME is not zero, then equation (12) is used to calculate streambed conductance. $\lambda = (\text{RKPRIME})(\text{STRWTH})/(\text{BPP})$.

- STRWTH:** If ISOLN = 3, stream width, feet. This value is not used if ISOLN ≤ 2 and should be entered as 0.
- DELT:** Time interval size, days. If 0.0 is read, the time interval is set to 1 day. Because this is an analytical solution, the time interval should match the interval of pumping rate changes. Solution accuracy does not depend on the interval used. The time interval must be uniform for the input pumping record. The maximum time interval is 28 days. The date string would have to be converted to Julian days for longer time intervals.
- INTIME:** Number of pumping days prior to start of the analysis.
- QWINIT:** Pumping rate prior to start of analysis in cubic feet per second. As discussed by Barlow (2000), the variables INTIME and QWINIT allow the user to establish an initial streamflow depletion before the analysis period. Streamflow depletion calculated for the time intervals and pumping rates specified below are added to the initial streamflow depletion. The values for INTIME and QWINIT should be adjusted to reflect the conditions of the system before the analysis period.
- NPD:** Number of time intervals for the analysis. This may be set to 1 for a single evaluation of the solution using the value entered for DELT as the time for the evaluation.
- CDATE(I):** Date for time interval (I) in YYYYMMDDHH format (10 character string).
- QWELL(I):** Pumping rate for time interval (I), cubic feet per second.
- Note that the solution is evaluated for each input CDATE(I), QWELL(I) pair. Output is produced for each of these evaluations. The time interval, DELT, is used in the computations and must match the interval implied by the CDATE entries.

Example Input File

Example Intermittent Pumping Problem with one-day time interval

WELL EXAMPLE 1

500, 0.0115740740740741, 2, 0.000231481481481481, 0.1, 0, 0, 0, 0, 0, 1

3650, 0.0

120

2001010100	0.0000	2001020700	0.5570
2001010200	0.0000	2001020800	0.5570
2001010300	0.0000	2001020900	0.5570
2001010400	0.0000	2001021000	0.5570
2001010500	0.0000	2001021100	0.5570
2001010600	0.0000	2001021200	0.5570
2001010700	0.0000	2001021300	0.5570
2001010800	0.0000	2001021400	0.5570
2001010900	0.0000	2001021500	0.5570
2001011000	0.0000	2001021600	0.5570
2001011100	0.0000	2001021700	0.5570
2001011200	0.0000	2001021800	0.5570
2001011300	0.0000	2001021900	0.5570
2001011400	0.0000	2001022000	0.5570
2001011500	0.0000	2001022100	0.5570
2001011600	0.0000	2001022200	0.5570
2001011700	0.0000	2001022300	0.5570
2001011800	0.0000	2001022400	0.5570
2001011900	0.0000	2001022500	0.5570
2001012000	0.0000	2001022600	0.5570
2001012100	0.0000	2001022700	0.5570
2001012200	0.0000	2001022800	0.5570
2001012300	0.0000	2001030100	0.0000
2001012400	0.0000	2001030200	0.0000
2001012500	0.0000	2001030300	0.0000
2001012600	0.0000	2001030400	0.0000
2001012700	0.0000	2001030500	0.0000
2001012800	0.0000	2001030600	0.0000
2001012900	0.0000	2001030700	0.0000
2001013000	0.0000	2001030800	0.0000
2001013100	0.0000	2001030900	0.0000
2001020100	0.5570	2001031000	0.0000
2001020200	0.5570	2001031100	0.0000
2001020300	0.5570	2001031200	0.0000
2001020400	0.5570	2001031300	0.0000
2001020500	0.5570	2001031400	0.0000
2001020600	0.5570	2001031500	0.0000

2001031600	0.0000	2001040800	0.0000
2001031700	0.0000	2001040900	0.0000
2001031800	0.0000	2001041000	0.0000
2001031900	0.0000	2001041100	0.0000
2001032000	0.0000	2001041200	0.0000
2001032100	0.0000	2001041300	0.0000
2001032200	0.0000	2001041400	0.0000
2001032300	0.0000	2001041500	0.0000
2001032400	0.0000	2001041600	0.0000
2001032500	0.0000	2001041700	0.0000
2001032600	0.0000	2001041800	0.0000
2001032700	0.0000	2001041900	0.0000
2001032800	0.0000	2001042000	0.0000
2001032900	0.0000	2001042100	0.0000
2001033000	0.0000	2001042200	0.0000
2001033100	0.0000	2001042300	0.0000
2001040100	0.0000	2001042400	0.0000
2001040200	0.0000	2001042500	0.0000
2001040300	0.0000	2001042600	0.0000
2001040400	0.0000	2001042700	0.0000
2001040500	0.0000	2001042800	0.0000
2001040600	0.0000	2001042900	0.0000
2001040700	0.0000	2001043000	0.0000

Example Output File

```

*****
*
*          ****  U.S. GEOLOGICAL SURVEY  ****          *
*
*          ***  STRMDEPL08: PROGRAM OUTPUT  ***          *
*
*          ONE-DIMENSIONAL MODEL OF STREAMFLOW DEPLETION  *
*
*          BY WELLS, BASED ON ANALYTICAL SOLUTIONS          *
*
*          DEVELOPED BY JENKINS (1968) AND HANTUSH (1965)  *
*
*          MODIFIED TO INCLUDE HUNT (1999, 2003) SOLUTIONS *
*
*          VERSION 1.0, FEBRUARY, 2008                      *
*
*****

```

Example Intermittent Pumping Problem with one-day time interval

SUMMARY OF INPUT DATA

WELL IDENTIFIER:	WELL EXAMPLE 1
WELL DISTANCE TO STREAM (XWELL):	0.500D+03 feet
TRANSMISSIVITY:	0.116D-01 square feet per second
STORATIVITY:	0.100D+00
STREAMBANK CODE (ISOLN):	2 (partially penetrating stream with resistance, Hunt 1999)
STREAMBED CONDUCTANCE:	0.231D-03 feet per second
INITIAL TIME (INTIME):	3650 days
INITIAL PUMPING RATE (QWINIT):	0.000D+00 cubic feet per second
NUMBER OF PUMPING STEPS (NPD):	120
TIME STEP FOR PUMPING (DELT):	0.100D+01 days

RESULTS

STREAMFLOW DEPLETION AT BEGINNING OF ANALYSIS:

0.0000 cubic feet per second

DAY	PUMPING RATE (cubic feet per second)	STREAMFLOW DEPLETION (cubic feet per second)	DAY	PUMPING RATE (cubic feet per second)	STREAMFLOW DEPLETION (cubic feet per second)
2001010100	0.0000	0.0000	2001020600	0.5570	0.0522
2001010200	0.0000	0.0000	2001020700	0.5570	0.0665
2001010300	0.0000	0.0000	2001020800	0.5570	0.0802
2001010400	0.0000	0.0000	2001020900	0.5570	0.0932
2001010500	0.0000	0.0000	2001021000	0.5570	0.1055
2001010600	0.0000	0.0000	2001021100	0.5570	0.1170
2001010700	0.0000	0.0000	2001021200	0.5570	0.1278
2001010800	0.0000	0.0000	2001021300	0.5570	0.1380
2001010900	0.0000	0.0000	2001021400	0.5570	0.1475
2001011000	0.0000	0.0000	2001021500	0.5570	0.1564
2001011100	0.0000	0.0000	2001021600	0.5570	0.1649
2001011200	0.0000	0.0000	2001021700	0.5570	0.1728
2001011300	0.0000	0.0000	2001021800	0.5570	0.1804
2001011400	0.0000	0.0000	2001021900	0.5570	0.1875
2001011500	0.0000	0.0000	2001022000	0.5570	0.1942
2001011600	0.0000	0.0000	2001022100	0.5570	0.2006
2001011700	0.0000	0.0000	2001022200	0.5570	0.2067
2001011800	0.0000	0.0000	2001022300	0.5570	0.2125
2001011900	0.0000	0.0000	2001022400	0.5570	0.2180
2001012000	0.0000	0.0000	2001022500	0.5570	0.2233
2001012100	0.0000	0.0000	2001022600	0.5570	0.2283
2001012200	0.0000	0.0000	2001022700	0.5570	0.2331
2001012300	0.0000	0.0000	2001022800	0.5570	0.2378
2001012400	0.0000	0.0000	2001030100	0.0000	0.2421
2001012500	0.0000	0.0000	2001030200	0.0000	0.2437
2001012600	0.0000	0.0000	2001030300	0.0000	0.2394
2001012700	0.0000	0.0000	2001030400	0.0000	0.2309
2001012800	0.0000	0.0000	2001030500	0.0000	0.2206
2001012900	0.0000	0.0000	2001030600	0.0000	0.2097
2001013000	0.0000	0.0000	2001030700	0.0000	0.1990
2001013100	0.0000	0.0000	2001030800	0.0000	0.1886
2001020100	0.5570	0.0001	2001030900	0.0000	0.1789
2001020200	0.5570	0.0028	2001031000	0.0000	0.1698
2001020300	0.5570	0.0112	2001031100	0.0000	0.1614
2001020400	0.5570	0.0235	2001031200	0.0000	0.1535
2001020500	0.5570	0.0376	2001031300	0.0000	0.1463

DAY	PUMPING RATE (cubic feet per second)	STREAMFLOW DEPLETION (cubic feet per second)	DAY	PUMPING RATE (cubic feet per second)	STREAMFLOW DEPLETION (cubic feet per second)
2001031400	0.0000	0.1395	2001040700	0.0000	0.0606
2001031500	0.0000	0.1333	2001040800	0.0000	0.0590
2001031600	0.0000	0.1275	2001040900	0.0000	0.0575
2001031700	0.0000	0.1221	2001041000	0.0000	0.0560
2001031800	0.0000	0.1170	2001041100	0.0000	0.0547
2001031900	0.0000	0.1123	2001041200	0.0000	0.0533
2001032000	0.0000	0.1080	2001041300	0.0000	0.0521
2001032100	0.0000	0.1038	2001041400	0.0000	0.0508
2001032200	0.0000	0.1000	2001041500	0.0000	0.0497
2001032300	0.0000	0.0964	2001041600	0.0000	0.0485
2001032400	0.0000	0.0930	2001041700	0.0000	0.0475
2001032500	0.0000	0.0897	2001041800	0.0000	0.0464
2001032600	0.0000	0.0867	2001041900	0.0000	0.0454
2001032700	0.0000	0.0839	2001042000	0.0000	0.0444
2001032800	0.0000	0.0812	2001042100	0.0000	0.0435
2001032900	0.0000	0.0786	2001042200	0.0000	0.0426
2001033000	0.0000	0.0762	2001042300	0.0000	0.0417
2001033100	0.0000	0.0739	2001042400	0.0000	0.0409
2001040100	0.0000	0.0717	2001042500	0.0000	0.0401
2001040200	0.0000	0.0696	2001042600	0.0000	0.0393
2001040300	0.0000	0.0676	2001042700	0.0000	0.0385
2001040400	0.0000	0.0657	2001042800	0.0000	0.0378
2001040500	0.0000	0.0639	2001042900	0.0000	0.0371
2001040600	0.0000	0.0622	2001043000	0.0000	0.0364

Example Plot File

DATE	QWELL	QS	DATE	QWELL	QS
2001010100	0.0000	0.0000	2001020800	0.5570	0.0802
2001010200	0.0000	0.0000	2001020900	0.5570	0.0932
2001010300	0.0000	0.0000	2001021000	0.5570	0.1055
2001010400	0.0000	0.0000	2001021100	0.5570	0.1170
2001010500	0.0000	0.0000	2001021200	0.5570	0.1278
2001010600	0.0000	0.0000	2001021300	0.5570	0.1380
2001010700	0.0000	0.0000	2001021400	0.5570	0.1475
2001010800	0.0000	0.0000	2001021500	0.5570	0.1564
2001010900	0.0000	0.0000	2001021600	0.5570	0.1649
2001011000	0.0000	0.0000	2001021700	0.5570	0.1728
2001011100	0.0000	0.0000	2001021800	0.5570	0.1804
2001011200	0.0000	0.0000	2001021900	0.5570	0.1875
2001011300	0.0000	0.0000	2001022000	0.5570	0.1942
2001011400	0.0000	0.0000	2001022100	0.5570	0.2006
2001011500	0.0000	0.0000	2001022200	0.5570	0.2067
2001011600	0.0000	0.0000	2001022300	0.5570	0.2125
2001011700	0.0000	0.0000	2001022400	0.5570	0.2180
2001011800	0.0000	0.0000	2001022500	0.5570	0.2233
2001011900	0.0000	0.0000	2001022600	0.5570	0.2283
2001012000	0.0000	0.0000	2001022700	0.5570	0.2331
2001012100	0.0000	0.0000	2001022800	0.5570	0.2378
2001012200	0.0000	0.0000	2001030100	0.0000	0.2421
2001012300	0.0000	0.0000	2001030200	0.0000	0.2437
2001012400	0.0000	0.0000	2001030300	0.0000	0.2394
2001012500	0.0000	0.0000	2001030400	0.0000	0.2309
2001012600	0.0000	0.0000	2001030500	0.0000	0.2206
2001012700	0.0000	0.0000	2001030600	0.0000	0.2097
2001012800	0.0000	0.0000	2001030700	0.0000	0.1990
2001012900	0.0000	0.0000	2001030800	0.0000	0.1886
2001013000	0.0000	0.0000	2001030900	0.0000	0.1789
2001013100	0.0000	0.0000	2001031000	0.0000	0.1698
2001020100	0.5570	0.0001	2001031100	0.0000	0.1614
2001020200	0.5570	0.0028	2001031200	0.0000	0.1535
2001020300	0.5570	0.0112	2001031300	0.0000	0.1463
2001020400	0.5570	0.0235	2001031400	0.0000	0.1395
2001020500	0.5570	0.0376	2001031500	0.0000	0.1333
2001020600	0.5570	0.0522	2001031600	0.0000	0.1275
2001020700	0.5570	0.0665	2001031700	0.0000	0.1221

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DATE	QWELL	QS	DATE	QWELL	QS
2001031800	0.0000	0.1170	2001040900	0.0000	0.0575
2001031900	0.0000	0.1123	2001041000	0.0000	0.0560
2001032000	0.0000	0.1080	2001041100	0.0000	0.0547
2001032100	0.0000	0.1038	2001041200	0.0000	0.0533
2001032200	0.0000	0.1000	2001041300	0.0000	0.0521
2001032300	0.0000	0.0964	2001041400	0.0000	0.0508
2001032400	0.0000	0.0930	2001041500	0.0000	0.0497
2001032500	0.0000	0.0897	2001041600	0.0000	0.0485
2001032600	0.0000	0.0867	2001041700	0.0000	0.0475
2001032700	0.0000	0.0839	2001041800	0.0000	0.0464
2001032800	0.0000	0.0812	2001041900	0.0000	0.0454
2001032900	0.0000	0.0786	2001042000	0.0000	0.0444
2001033000	0.0000	0.0762	2001042100	0.0000	0.0435
2001033100	0.0000	0.0739	2001042200	0.0000	0.0426
2001040100	0.0000	0.0717	2001042300	0.0000	0.0417
2001040200	0.0000	0.0696	2001042400	0.0000	0.0409
2001040300	0.0000	0.0676	2001042500	0.0000	0.0401
2001040400	0.0000	0.0657	2001042600	0.0000	0.0393
2001040500	0.0000	0.0639	2001042700	0.0000	0.0385
2001040600	0.0000	0.0622	2001042800	0.0000	0.0378
2001040700	0.0000	0.0606	2001042900	0.0000	0.0371
2001040800	0.0000	0.0590	2001043000	0.0000	0.0364

